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COMPLEAT SYSTEM

OF

OPTICKS

In Four BOOKS, viz.

A Popular, a Mathematical, a Mechanical, and a Philoſophical Treatiſe.

To which are added,

REMARKS upon the Whole.

BY

ROBERT SMITH LL. D.

Profeſſor of Aſtronomy and Experimental Philoſophy at CAMBRIDGE, and Maſter of Mechanicks to his MAJESTY.

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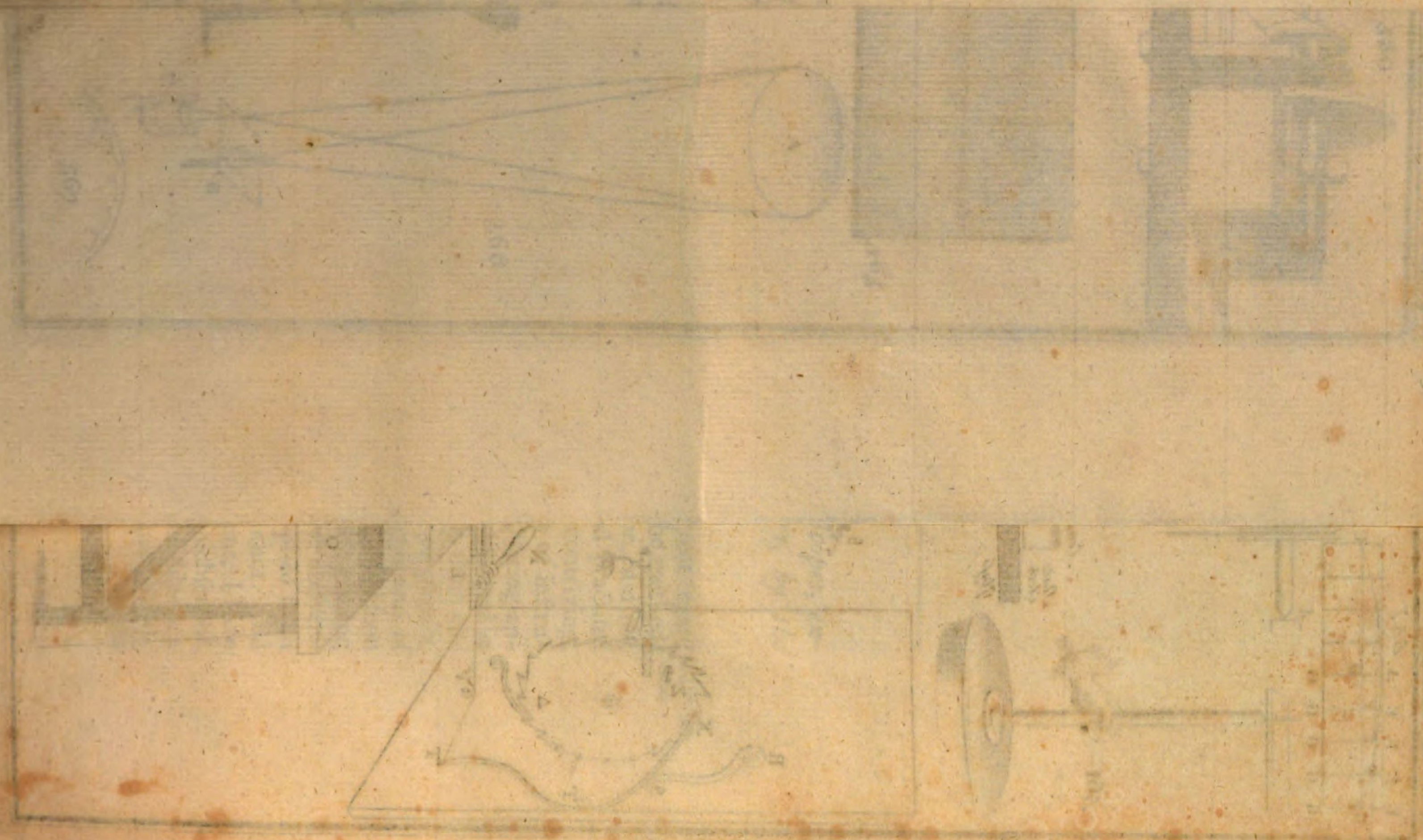
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A COMPLEAT SYSTEM OF OPTICKS.

BOOK III.

A

Mechanical Treatiſe.

CHAPTER I.

The method of grinding and poliſhing glaſſes for Teleſcopes, extracted from Mr. Huygens *and other Authors, by the Honourable* Samuel Molyneux *Eſquire.*

Advertiſement.

THIS ingenious and honourable Author, out of his great regard for the improvement of Aſtronomy, by perfecting the methods of making teleſcopes both by refraction and reflection, did not only collect and conſider what had been written and practiſed by others, but alſo made ſeveral new experiments of his own contriving, after he had procured a moſt compleat Apparatus of all ſorts of inſtruments for that purpoſe. But in the midſt of theſe thoughts, being appointed a Lord Commiſſioner of the Admiralty, he became ſo engaged in publick affairs, that he told me he had not leiſure, to purſue theſe enquiries any farther. Then it was that he gave me the following papers, and was pleaſed to invite me to make uſe of his houſe and apparatus of inſtruments, in order to finiſh what he had left imperfect. But he dyed ſoon after, and ſo I loſt that opportunity, and a moſt worthy friend. Having therefore ſeen nothing of the practice of grinding glaſſes, I durſt not venture to add any thing of my own relating to it; but have ſupplied from Mr. *Huygens* what was left unfiniſhed by our honourable Author. And as Mr. *Huygens*'s treatiſe is eſteemed the beſt of any yet extant, I have taken care that nothing material therein contained ſhould paſs unobſerved. If I have miſtaken his meaning any where, it is the more excuſable conſidering the difficulty of the ſubject, and that it was a poſthumous trea-

tiſe written originally in Dutch and tranſlated into Latine by another hand. To diſtinguiſh Mr *Molyneux*'s papers from what I have tranſlated from Mr. *Huygens*, I have put their names in the margine at the beginning of every piece.

§. 1. *Of making and poliſhing the Tools.*

Proportions in the object-glaſs.

742. It is eaſier to make an object-glaſs of equal convexities than of any other figure; becauſe the ſame tools will ſerve for forming both its ſurfaces. And a glaſs of this figure will make as perfect an image in its focus as any other, becauſe the aberrations of the rays cauſed by the ſphericalneſs of the figures of the ſurfaces, whatever be the proportion of their ſemidiameters, are inconſiderable in long teleſcopes, in compariſon to the aberrations cauſed by the different refrangibility of the rays[a]; and theſe latter aberrations are the ſame whether the ſemidiameters of the ſurfaces be equal or unequal, ſuppoſing the aperture and focal diſtance to be the ſame[b]. If it be propoſed then to make a glaſs of equal convexities, that ſhall have a given focal diſtance, the radius of the ſpherical ſurface will be found by taking it in proportion to the given focal diſtance as 12 to 11, as will appear by art. 232, putting the ſine of incidence to the ſine of refraction out of air into glaſs as 17 to 11, as Sir *Iſaac Newton* has accurately determined it[c]. The focal diſtance of the glaſs being given, its aperture is alſo given by the Table in art. 364; and becauſe its figure cannot be formed exactly true to the very edges, the breadth of the glaſs may be taken about half an inch more than the diameter of its aperture, or even 3 quarters or a whole inch more, if its focal diſtance be between 50 and 200 feet.

a Art. 340.

b Art. 678.

c Newt. Opt. p. 86.

Proportions in the tools.

743. Mr. *Huygens* directs in general to make the breadth of the concave tool, or plate or diſh or form, in which an object-glaſs muſt be ground, almoſt three times the breadth of the glaſs. Though in another place, he ſpeaks of grinding a glaſs whoſe focal diſtance was 200 feet and breadth $8\frac{1}{4}$ inches in a plate only 15 inches broad[d]. But for eye-glaſſes and others of leſſer ſpheres, the tool muſt be broader in proportion to the breadth of theſe glaſſes, to afford room enough for the motion of the hand in poliſhing. Mr. *Huygens* made his tools of copper or caſt braſs; which for fear they ſhould change their figures by bending, can hardly be caſt too thick; nevertheleſs he found by experience that a tool 14 inches broad and half an inch thick was ſtrong enough for forming glaſſes to a ſphere of 36 feet diameter; when the tool was ſtrongly cemented upon a cylindrical ſtone an inch thick, with hard cement made of pitch and aſhes.

d Dioptr. p. 280.

Patterns for the tools.

744. In order to make moulds for caſting ſuch tools as are pretty much concave, he directs that wooden patterns ſhould be turned in a lathe, a little thicker and broader than the tools themſelves. But for tools that be-

long

long to ſpheres above twenty or thirty feet diameter, he ſays it is ſufficient to make uſe of flat boards turned circular to the breadth and thickneſs required. When the plates are caſt, they muſt be turned in a lathe, exactly to the concavity required. And for this purpoſe it is requiſite to make a couple of braſs gages in the manner following.

Gages for the tools. Mr. *Molyneux*

745. Take a wooden pole a little longer than the radius of the ſpherical ſurface of the glaſs to be formed, and through the ends of it ſtrike two ſmall ſteel points, at a diſtance from each other equal to the radius of the ſphere intended; and by one of the points hang up the pole againſt a wall, ſo that this upper point may have a circular motion in a hole or ſocket made of braſs or iron fixt firmly to the wall. Then take two equal plates of braſs or copper, well hammered and ſmoothed, whoſe length is ſomewhat more than the breadth of the tool of caſt braſs; and whoſe thickneſs may be about a tenth or a twelfth of an inch, and whoſe breadth may be two or three inches. Then having faſtened theſe plates flat againſt the wall in an horizontal poſition, with the moveable point in the pole ſtrike a true arch upon each of them. Then file away the braſs on one ſide exactly to the arch ſtruck, ſo as to make one of the braſs edges convex and the other concave; and to make the arches correſpond more exactly, fix one of the plates flat upon a table and grind the other againſt it with emery. Theſe are the gages to be made uſe of in turning the braſs tools exactly to the ſphere required.

Mr. *Huygens*. Fig. 557.

But if the radius of the ſphere be very great the gages muſt be made in this manner. Imagine the line *AE* drawn upon the braſs plate to be the tangent of the required arch *AFB*, whoſe radius for example is 36 feet and diameter 72. From *A* ſet off the parts *AE*, *EE*, &c. ſeverally equal to an inch, and let them be continued a little beyond half the breadth of the tool required. Then as 72 feet or 864 inches is to 1 inch, ſo let 1 inch be to a fourth number; this will be the number of decimal parts of an inch in the firſt line *EF* reckoning from *A*. Multiply this fourth number ſucceſſively by the numbers 4, 9, 16, 25, &c. the ſquares of 2, 3, 4, 5, &c; and the ſeveral products will be the number of parts contained in the 2d, 3d, 4th, 5th *EF* reſpectively. But becauſe theſe numbers of parts are too ſmall to be taken from a ſcale by a pair of compaſſes, ſubſtract them ſeverally from one inch, repreſented by the lines *EG*, and the remainders being taken from a ſcale of an inch divided into decimal parts and transferred by the compaſſes from *G* to *F*, will determine the points *F*, *F*, &c. of the arch required. And the ſame being done on the other ſide of the line *AD*, the braſs plate muſt be filed away exactly to the points of this arch, and poliſhed as before.

Of turning the braſs tools in a lathe. Mr *Huygens*.

746. To apply the braſs tool to a turning lathe in order to turn the concave ſurface of it exactly ſpherical, let the 558th figure repreſent a view of ſome part of the lathe, taken from a point directly over it; and here

Fig. 558 to 560.

here let *ab* repreſent a ſtrong flat diſk of braſs, half an inch thick at leaſt, having a ſtrong iron ſkrew-pin fixt firmly in the center of it and ſtanding out exactly perpendicular to one ſide of it; by which it may be ſkrewed into the end *c* of the mandrel or axis of the lathe, repreſented by *c d*. This diſk is repreſented ſeparately in fig. 559. and muſt be well ſodered to the back ſide of the tool *ef*, which therefore in the middle of it muſt be made plane, and exactly parallel to the circumference of its oppoſite ſurface; to the intent that the circumference may be carried round the axis of the lathe in a plane perpendicular to it. The mandrel or axis *c d* turns upon a point *d* in the puppet head of the lathe, and in an iron collar repreſented by *s t*, as uſual.

Let *g h i k* repreſent a board nailed faſt upon the other puppet head; and let the concave gage *g h* be laid upon this board, with its concave arch parallel to the concavity of the tool *ef*, and be ſkrew'd down to the board with flat-headed ſkrews ſunk into the braſs. Let *l m n o* repreſent ſuch another board lying upon the former, with the convex gage *l m* ſkrewed to the under ſide of it, ſo that by moving this upper board, the arch of the convex gage may be brought to touch the concave one and ſlide againſt it. The turning tool *p q* is laid upon this moveable board, and is held faſt to it by a broad headed ſkrew at *r*, to be turned or unturned by the hand upon occaſion. To know whether the concave gage be exactly parallel to the concavity of the tool *ef* ſkrewed faſt to the mandrel, direct the point *p* of the turning tool *p q* to touch any point of the tool *ef* near its circumference; then having fixt the turning tool *p q* by the ſkrew *r*, turn the braſs tool *ef* half round, and move the upper board till the point *p* of the turning tool be brought over againſt the ſame mark upon the tool *ef*; and if it juſt touches it as before when the gages coincide, all is right. If not, the poſition of the head of the lathe may be altered a little by ſtriking it with a mallet. But the beſt way is to make this examination of the ſituation of the concave gage, when only one end of it is fixt to the head of the lathe by a ſingle tack or ſkrew, about which it may eaſily be moved into its true poſition. And while the tool or plate *ef* is turning, the ſame examination of its paralleliſm to the gage muſt be frequently repeated; otherwiſe its ſurface will take a falſe figure. It is convenient that the upper board *l m n o* ſhould project over both the gages; and to keep its ſurfaces parallel to that of the under board, two round headed nails, or a plate of braſs, as thick as the gages, muſt be fixt to its under ſurface, towards the oppoſite ſide *n o*. Care muſt be taken to drill the holes in the gages, through which they are ſkrewed to the boards, not too near the poliſhed arches, for fear of altering their figure, by the yielding of the braſs. The tool and all the parts of the lathe muſt be fixt very firm; becauſe any trembling motion will cauſe the graving tool *p q* to indent

the

the braſs. After the tool is well turned it muſt be ſeparated from the braſs back *a b* by melting the ſoder with live coals laid upon it.

It is eaſy to underſtand, that by tranſpoſing the gages, a convex tool may be turned in the ſame manner.

747. Mr. *Huygens* would have his plates or tools firſt formed in a turning lathe, and then ground together with emery; that is to ſay the concave and convex tool of the ſame ſphere together. But the tools of very large ſpheres, he would have ground at firſt quite plane by a ſtone-cutter; and then ground hollow with a round flat ſtone and emery to the deſired gage. And he preſcribes to uſe for this grinding firſt a ſtone half as broad as the tool, and after that another ſtone very nearly as broad as the tool; and in this way of forming the tools it is convenient to tye a little frame of thick paper, or rather of thin paſtboard about an inch high round about the tool to keep in the emery; and in grinding, the whole muſt be made extreamly firm and ſtable. And when the tool is to be poliſhed, it muſt ſtill remain upon the ſtone pedeſtal; otherwiſe it will be in danger of bending a little in the operation.

Another way of forming large tools. Mr. *Molyneux*.

748. For poliſhing theſe tools thus ground, Mr. *Huygens* takes ſome ſoap and daubs the concave tool therewith, then he takes the laſt mentioned round ſtone, ſomewhat leſs than the tool, (or the convex tool it ſelf) and heats it, and then he pours upon it ſome hot melted cement (made of pitch and fine powdered and ſifted aſhes as much as he can mix with it) and then he turns over the ſtone and cement upon the concave tool, into which alſo he had poured a good quantity of the ſame cement, having firſt laid three little pieces of braſs of equal thickneſſes on the circumference thereof in order to preſs and keep this cruſt of cement of an exact equal thickneſs in all its parts; and thus he lets them cool together. Then taking the ſtone from the tool and turning it up, he ſifts upon the cement that ſticks to it, a cruſt of very fine emery, and with a flat iron ſpatula about one third of an inch thick, gently warmed, he preſſes lightly the emery, to ſtick to and incruſtate upon the cement; then he again gently warms the whole ſtone (or convex tool) cement and emery, and he again replaces it upon the concave tool and leaves it again to cool; ſo that he has by this means a cruſt of emery exactly of the figure of his tool; with this he poliſhes the tool dry, without the addition of any wet, preſſing it down hard on the ſurface of the tool. And to preſs it the harder he places upon it a long pole, a little bent to make it ſpring, whoſe upper end is fixt to the cieling of the room, or elſe is preſſed downwards by a ſtrong iron ſpring; and he ſays it will be neceſſary to have two perſons to rub the ſtone upon the tool. And here it is to be noted that great care muſt be taken in this and all caſes where this way of grinding by a pole is made uſe of, to fix the point of preſſure exactly and truly in the middle,

To poliſh the tool by an incruſtation of pitch and emery.

as

as ſhall be more particularly explained hereafter when the glaſs comes to be ground in this tool.

Several ſorts of cement.

749. It is to be noted that this cement, and indeed all cements for ſticking glaſſes, is made in different manners by different perſons. *Pere Cherubin* [a] ſays it is uſually made of common black pitch and fine ſifted vine aſhes, but he himſelf made it of roſin and ocre, or of roſin and ſpaniſh white, pounding the roſin firſt and mixing it with a due quantity of the powder, and then ſifting the mixture upon hot melted pitch, and while hot, well mixing and incorporating the whole. The cement that Mr. *Scarlet* uſes is made of common pitch, and common coal-aſhes ſifted fine. In all caſes it is harder or ſofter as more or leſs of the aſhes or other powder is put into it; and in this preſent caſe for poliſhing theſe tools it muſt be made as hard as poſſible, by putting in as great a quantity of the aſhes as is poſſible: for otherwiſe if the cement be not hard enough, the particles of the emery will be looſened by the heat in grinding, and then will only run round upon the tool, and will not work out the little inequalities and roughneſſes thereof. If the emery ſhould be found to grow blunt, a very little more of it may be duſted dry upon the tool, by which its ſharpneſs and cutting quality will be a little recovered; but if the cement be ſufficiently hard at firſt, as is preſcribed, the emery will always remain ſharp enough.

[a] La Dioptrique oculaire P. 352.

To perfect the poliſh of the tool with blue hones.

750. To bring the concave tool ſtill to greater perfection, and ſtill to preſerve its true form, take equal pieces, about an inch ſquare, of blue hone, ſuch as engravers poliſh their copper withal, and place as many of them, laying the grain ſome one way and ſome another, as you can, upon the ſurface of the tool to be poliſhed; ſticking and ſteadying the ſame, as cloſe as you can to one another, with ſoap or common white ſtarch; then fill up all the interſtices between the ſaid pieces of hone with clean dry ſand, to about two thirds of the thickneſs of the ſaid hones; then having a border of paper or paſtboard put round the tool as before, ſhake the tool gently that the ſand may every where equally ſubſide, and blow it to an equal depth with a pair of bellows. Then take ſome hard cement extreamly hot and pour it all over the hones, and having cleaned the ſtone (or the convex tool) which before was incruſtated with pitch and emery, place this ſtone (or convex tool) warmed on the top of this cement and let all cool together. Then rubbing the tool with this poliſher made with hones, by applying your pole to the top of the ſtone as before, you ſhall know when the tool is brought to perfection, if wiping off the dirt and filth you find, by looking obliquely upon it againſt the light, that it ſhines equally bright and ſtrong in all parts. If you would uſe this poliſher again, it muſt be kept in a cool cellar, leaving the hones uppermoſt; otherwiſe in warm weather they will change their ſituation in the cement, even with their own weight.

751.

751. And here it is to be obſerved that this method of grinding theſe tools by an incruſtation of emery on hard cement, formed to a convexity by the very tool it ſelf, and alſo the method of poliſhing the ſame by the blue hones fixt in hard cement, may probably as well be uſed and applyed to the grinding and poliſhing of a reflecting metal for a teleſcope. But in that caſe I believe it is beſt not to uſe a ſpringing pole, but one that ſhall turn upon a point above, being nearly equal in length to the radius of the ſphere intended.

Applyed to the grinding and poliſhing an object-metal for a teleſcope.

§. 2. *Of chooſing glaſs.*

752. The beſt ſort of glaſs has generally a yellowiſh, reddiſh, or a greeniſh colour, when we look through it againſt the ſky-light, or lay it upon a ſheet of white paper. That which is perfectly white, though it tranſmits the moſt light, is generally fuller of veins, and is often ſubject to grow moiſt in the air, which in time deſtroys its poliſh. In theſe parts there is no better glaſs to be had, than pieces of broken looking-glaſſes. But after ſome time I found pretty good glaſs at a glaſs-houſe; and made uſe of the ſame matter with which they make drinking-glaſſes, which I found was always the beſt after the matter had reſted for two or three days, when the workmen kept holydays. I ordered pieces of glaſs to be made for my own uſe in the ſame manner as they make looking-glaſſes, that is by cutting off the top and bottom of the round globe they blow up, and by ſlitting the ſide of it, and then by flatting it upon the hearth of the furnace. Theſe pieces, which were about half an inch or three quarters thick, I ordered to be ground to an equal thickneſs by a ſtone-cutter, in the machine they poliſh marble with.

Several ſorts of glaſs. Mr. *Huygens.*

753. To diſcover the veins in glaſs one ſhould look very obliquely againſt a ſmall light in a room otherwiſe dark. Thus one may examine the pieces of a poliſhed looking-glaſs. But becauſe they are ſeldom thick enough for object-glaſſes, it is neceſſary to take ſome pieces of the ſame ſort of glaſs before it is poliſhed, and to get it ground to an equal thickneſs and poliſhed a little by the common glaſs-grinders, in order to judge what pieces are fit for uſe. Sometimes little veins will appear like fine hairs or threads, which ſcarce do any harm. I have ſeveral very good glaſſes of this ſort. Sometimes their imperfections cannot be diſcovered by the former way of trial, and yet after the glaſs is well formed and poliſhed they will appear by reflection in this manner. In a dark room place the glaſs upright upon a table, turning that ſurface from you which is ſuſpected to be faulty, then holding a lighted candle in your hand, ſo that the middle of the broad light reflected from the firſt ſurface may fall upon your eye, recede from the glaſs till the rays reflected from the back-ſurface ſhall juſt begin to invert the candle; then the whole glaſs will appear

Veins in glaſs how diſcovered.

appear all over bright, and then you will diſcover its defects and the imperfections of the poliſh. When the glaſs is a portion of a large ſphere, we uſe a ſmall perſpective 3 or 4 inches long to magnify the defects.

§. 3. *Of preparing the glaſſes before they can be ground and poliſhed.*

To round the glaſs. Mr. *Huygens.*

754. The pieces of glaſs above mentioned, which we directed to be planed to an equal thickneſs and poliſhed a little by a glaſs-grinder, ſhould be much broader than the intended object-glaſs, that there may be room enough for chooſing the beſt part of them. For planing and ſmoothing thoſe large pieces of glaſs, I ordered the workmen to make uſe of plates of caſt iron, ſuch as are ſold at the ironmongers ſhops, after they had been ground and planed in a ſtone-cutters engine. Upon the plate of glaſs, with a diamond-pointed compaſs, ſtrike a circle repreſenting the circumference of the object-glaſs; and alſo another concentrick circle with a radius about a tenth or twelfth part of an inch bigger. And alſo two other ſuch circles, on the other ſide of the glaſs, directly oppoſite to the former: which may be done by means of a Circular glaſs to be deſcribed by and by. The larger parts of the glaſs may be ſeparated from the outward circle by a red hot iron, or by a ſtrong broad vice, opened exactly to the thickneſs of the glaſs. The remaining inequalities may be taken off by a grind-ſtone, beginning with the largeſt firſt, and taking care they do not ſplinter. Then having warmed the glaſs, cement a wooden handle to it, and in a common deep tool for eye-glaſſes, making uſe of white clear ſand and water, grind the circumference of the glaſs exactly true to the innermoſt circle on each ſide of it.

To grind it to an equal thickneſs.

755. Then having made a great many ſmall cavities with a punch upon one ſide of a round copper plate, and having fixt the other ſide of it upon the middle of the round glaſs, by cement made with two parts of roſin or hard pitch and one part of wax, place the ſteel point of the ſpringing pole above deſcribed, being 14 or 15 feet long, into that cavity of the copper plate, which lyes neareſt to the thickeſt part of the glaſs. Then work the glaſs by the pole with ſand and water upon a flat plate of caſt iron, of a round figure, the plate having been planed with ſand and water by a ſtone-cutter. Then having examined the thickneſs of the glaſs in ſeveral places by a hand vice, which is better than a pair of callippers, by repeating the ſame operation it will ſoon be reduced to an equal thickneſs in all its parts.

Towards the end of this operation it is convenient to make uſe of ſifted emery, becauſe the ſand will ſcratch too deep; and then it will alſo be neceſſary to place the ſteel point of the pole exactly over the center of the under ſurface of the glaſs; otherwiſe that ſurface will take a cylin-

cylindrical or ſort of a convex figure inſtead of a plane one, even though it was exactly plane before you began to grind it. The reaſon of which is well worth obſerving. And when convex glaſſes are to be poliſhed, it is alſo abſolutely neceſſary to place the point of preſſure exactly over the center of the under ſurface of the glaſs.

The circular glaſs deſcribed.

756. Therefore to bring one of the little cavities in the copper plate exactly over that center, we make uſe of a circular glaſs formed from a broken looking-glaſs, the quickſilver being rubbed off; having deſcribed upon it, with a diamond-pointed compaſs, 8 or 10 concentrick circles at the diſtance of about a quarter of an inch from each other, ſo that the larger circles may be ſomewhat bigger than the circumference of the glaſs to be poliſhed. Lay this circular glaſs upon the ſurface of the glaſs to be poliſhed, and move it to and fro till you perceive the circumference of the glaſs to be poliſhed is exactly parallel to the neareſt circle upon the circular glaſs; then having inverted both the glaſſes, lay the circular glaſs upon a table, and having laid a ſmall live coal upon the copper plate, to make it moveable upon the cement, place one point of a pair of compaſſes in one of the little cavities, and move the copper till a circumference deſcribed with the other point coincides exactly with any one circle upon the circular glaſs, and the buſineſs is done. With ſtarch it is convenient to paſte three ſlender ſhreds of fine linen, directed towards the center of the circular glaſs, that the other glaſs may not ſlide too eaſily upon it, and that they may not ſcratch one another.

The object-glaſs muſt be exactly circular.

757. The cavities punched in the copper plate, and alſo the point of the pole, ſhould be triangular, to hinder the rotation of the glaſs; which is ſtill more neceſſary in perfecting the poliſh of the glaſs. And here it muſt be obſerved again whether the circumference of the glaſs remains exactly circular on both ſides of it, which muſt be tryed with a pair of compaſſes; and if it be not, it muſt be corrected again by grinding it exactly circular in a common tool for making eye-glaſſes; which will contribute very much to its taking an exact ſpherical ſurface when it comes to be ground in its proper tool. For if any part of the circumference be protuberant; it will hinder the adjoining parts of the ſurface from wearing ſo much as they ſhould do; and by conſequence will ſpoil the ſpherical figure of the ſurface.

§. 4. *Of grinding the glaſſes.*

Mr. *Molyneux.*

758. The glaſs being planed and rounded as above, take away the plate with ſeveral cavities, and with ſome of the ſame cement fix on a ſmaller round piece of braſs or rather ſteel, truly flat and turned about the bigneſs of a farthing but thicker, having firſt made in the center thereof, with a triangular ſteel punch, a hole about the bigneſs of a

gooſe quill, and about the depth of $\frac{1}{12}$ of an inch; and at the very bottom of this triangular hole, a little ſmall round hole muſt be punched ſomewhat deeper with a very fine ſmall ſteel punch. A ſmall ſteel point of about an inch long muſt be truly ſhaped and fitted to this triangular hole, and at the very apex to the ſmall round deeper impreſſion. Nevertheleſs it muſt not be fitted ſo exactly to the ſame but that it may have ſome liberty to move a little to and fro; the apex always continuing to touch and preſs upon the ſurface of the round hole below. This ſteel triangular point muſt be fixed to the end of a pole, to the other end of which another round iron point muſt be fixed, of about five or ſix inches long, to play freely up and down in a round hole in a piece of braſs let into a board fixed againſt the ceiling for that purpoſe, perpendicularly over the bench, and over the center of the tool, which muſt be ſtrongly and truly fixed horizontally thereon.

Fig. 561.

759. Now here it is to be noted that Mr. *Huygens* preſcribes to fix his braſs plate to the glaſs by the means of cement, and takes no notice of any other method whatever; though a very ſmall experience in theſe affairs will convince any body that it is hardly poſſible, in this or any other caſe, to bring the cement to a fluidity ſufficient to fix two plane ſurfaces exactly parallel one to the other, without heating the glaſs and the braſs alſo to a great degree, and ſo as to endanger the figure of the glaſs conſiderably. To avoid this in fixing glaſſes to braſs or wood or the like, ſome have done it with plaiſter of Paris: Mr. *Scarlet* does it by cementing another intermediate glaſs to the braſs (or wood) and then fixing the glaſs, to be ground, to the outward ſurface of the cemented glaſs with common glew. Without all this trouble I have done it only with common Icthyocolla or fiſh glew, which will run very fluid, and will fix the glaſs and the braſs it ſelf ſtrongly together; and round the edges of the braſs I ſtick on ſome common ſoft red wax, ſuch as is uſed for the privy ſeal, to keep the wet from getting to the glew.

760. For grinding glaſſes truly plane, upon a plane tool, by this method, Mr. *Huygens* preſcribes this pole to be about fifteen feet long; but in grinding upon a concave plate, the pole had beſt be made equal to the radius of the ſphere of the tool; though I believe it would not be material if made conſiderably ſhorter, according as the height of the room will allow.

It is neceſſary to have lying by one an ordinary piece of courſe glaſs, ground in the ſame tool, called a bruiſer; whereby when any new emery is neceſſary to be laid on the tool in grinding your glaſs, the ſaid emery is to be conſtantly firſt run over and ſmoothed, for fear any little courſe grains ſhould remain and ſcratch the glaſs to be ground.

761. Having theſe things prepared together with ſome pots of emery of various fineneſſes, take of your rougheſt ſort a ſmall half pugil, wetting the

the ſame and daubing it pretty equably on the tool; then lay on your glaſs and fix up your pole and continue to grind for a quarter of an hour, not preſſing upon the pole, but barely carrying the glaſs round thereby; then take the like quantity of ſome finer emery and work another quarter of an hour therewith; then take the like quantity of emery ſtill finer and work for the ſame time; laſt of all take a leſs quantity of ſome of the very fineſt you have, which will be ſufficient for a glaſs of five inches diameter, and work therewith for an hour and half, taking away by little and little ſome of the emery with a wet ſponge. Do not keep it too wet nor too dry, but about the conſiſtence of pap; for much depends on this. If it is too dry, your emery will clog and ſtick and incorporate, ſo as for the moſt part to cut little or not at all, unleſs here and there where its body chances to be broke, and there it will ſcratch and cut your glaſs irregularly; and if it is too wet, and too much diluted, it will from the irregular ſeparation of its parts cut in ſome places more than others juſt as in the other caſe. Fig. 561.

762. But Mr. *Huygens* tells us this method of uſing various ſorts of freſh emery is not good, finding by experience that the ſurfaces of large glaſſes are often ſcratched. And therefore he ſays it is beſt to take a large quantity of the firſt or ſecond ſort of emery, and ſo work with the ſame from the firſt to the laſt; taking away by little and little every half hour, or quarter of an hour, more and more of the emery with a wet ſponge; by which means he could bring the glaſs extreamly ſmooth and fine, ſo as to ſee pretty diſtinctly a candle or the ſaſh windows well defined through it; which is a mark when it is ground enough to be ready to receive a poliſh. But if the glaſs has not acquired this degree of tranſparency, it is certain, ſays Mr. *Huygens*, that too much emery remains; and therefore it muſt ſtill be diminiſhed and the operation continued. He found it beſt to make uſe of common well water in this grinding; and he took care to move the glaſs in circles, taking in an inch beyond the center of the tool and ſomewhat beyond the outſide of the tool; and he found in a glaſs of two hundred feet whoſe diameter was 8 inches $\frac{3}{4}$, which he ground in a tool of 15 inches diameter, that the figure of the tool in grinding would alter conſiderably, unleſs he carried the glaſs round an inch beyond the center of the tool one way, and 3 inches $\frac{1}{4}$ beyond the ſkirts of it another way; but if he carried it no more than a ſtraws breadth beyond the ſkirts of the tool, and accordingly farther beyond the center, the glaſs would always grind falſly, too much being taken off on the outſides ſo that he could never after bring the outſides of the glaſs to a true and fine poliſh.

763. When you firſt begin to grind and the emery begins to be ſmooth, the glaſs will ſtick a little to the tool and run ſtiff; then freſh emery is to be added. When it afterwards comes to be poliſhed it will, if

large, require a conſiderable ſtrength to move it, but this inconvenience will happen leſs in grinding by the pole than in grinding by hand. For the warmth of the hand makes the ſubſtance of the glaſs ſwell, and not only increaſes the ſticking of the glaſs, but in ſome meaſure may alſo ſpoil the figure of it and alſo of the tool. When it is ground with the pole it never ſticks very ſtrongly, unleſs when you take the glaſs off from the tool and keep it from it for ſome time, and then apply it to the tool again; and this in large glaſſes; for by this means, ſays Mr. *Huygens*, the glaſs gets from the air a greater warmth than it had on the tool; and being again applyed to the tool, its lower ſurface is ſuddenly contracted by the coldneſs of the tool and ſo ſticks to it. Wherefore ſaith he you muſt, in that caſe, wait till the glaſs and the tool come to be of one temper. The like effect is obſerveable in grinding large glaſſes when there is a fire in the room. Perhaps the cauſe of theſe effects may be more truly deduced from the attractive qualities of warm glaſs. But whatever is the cauſe, we may from hence perceive the great nicety of grinding large glaſſes, and the neceſſity there is of grinding them ſlowly, and with the greateſt caution in the moſt minute circumſtances.

764. The method hitherto deſcribed of grinding with emery, is what is recommended by Mr. *Huygens*. Le Pere *Cherubin* preſcribes another material, and it is the grit of a hard grind-ſtone well beaten into a fine powder and ſifted pretty fine. And here in England the ſame thing was uſed to be performed by Mr. *Cox* with common clean fine white ſand, taking away by little and little the ſaid grit and ſand as it ground finer and finer. Nay Mr. *Cox* was uſed to continue his grinding till the matter of the ſand came to be ſo fine, and ſo little of it to remain in the tool, that he could, and frequently did uſe to poliſh off his glaſſes therein, without the uſe of any other material whatſoever; and I my ſelf have been preſent, while Mr. *Scarlet* ground and poliſhed, or dryed off, a glaſs of 16 feet in this manner. They call this way drying off on ſand, becauſe as the matter grows finer and finer, they wet it leſs and leſs, till for the laſt quarter of an hour (the whole work laſting near two hours) they only wet it by breathing upon it, and at the very laſt not at all.

765. It ſeems this method is now quite diſuſed: perhaps the violent labour requiſite at the laſt may be a reaſon of it. A better reaſon may be the great improbability of grinding or poliſhing true by this method, by the uncertain and unequal force of the hand, and if this be the true reaſon of its diſuſe, I cannot well ſee but that this method of grinding and poliſhing out and out in the ſame tool, and with the ſame material, *viz.* white ſand, might perhaps be again reſtored, and greatly improved by adding to the old way Mr. *Huygens*'s method of grinding and poliſhing with a pole and ſpring to preſs down the pole; or ſome analogous contrivance. And in relation to grinding by all or any of the methods above deſcribed,

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this one general remark muſt be made, that the artiſt muſt allow time and patience to bring his glaſs by grinding to the ſmootheſt and fineſt ſurface, that he poſſibly can, before he attempts to poliſh. For this and this only makes his glaſs poliſh truly ſmooth, well and eaſily: and the ſmoother you bring it in grinding, the leſs labour you will have in poliſhing; in which conſiſts not only the greateſt difficulty but the greateſt danger too of ſpoiling all you have done before.

§. 5. *Of giving glaſſes the laſt and fineſt poliſh.*

766. Having removed the little braſs plate from the glaſs, take ſays Mr. *Huygens* a very thick ſlate, or rather a block of blue or grey ſtone; let it be half an inch thick, and let it be ground true and round at the ſtone-cutters; its diameter being ſomewhat ſmaller than the diameter of your glaſs, leaving a hole, quite through, in the center, of about an inch diameter. Then make ſome cement two parts roſin or hard pitch and one part wax; and taking a piece of thick kerſey cloth, truly and equally wrought, cut this cloth round, and leave a like hole, one inch diameter, in the middle. Then warming the ſtone and alſo warming the glaſs, and ſpreading thinly and equably upon them ſome of this cement, lay on the cloth and thereupon lay on alſo the glaſs, having left in the middle a ſpace the breadth of a ſhilling uncemented and blackt with a candle. Then provide an hollow conical plate of iron or ſteel (ſhaped like an high crowned hat) having the baſis of the cone 1 inch diameter, and having round the baſis a flat border about $2\frac{1}{2}$ inches diameter, and having the depth or altitude of the cone exactly of the thickneſs of the ſlate, cloth and cement, to which the glaſs is fixt. The vertex of this cone muſt go down through the ſlate and cloth, ſo that being cemented on the ſlate, the ſaid vertex may approach to the glaſs within a hair's breadth, and lye perpendicularly over the center of the lower ſurface of the glaſs: and this muſt be adjuſted by the circular glaſs deſcribed above. Within the vertex of this hollow cone, the lower point of the pole is to be applyed in poliſhing; but it may be firſt proper to be obſerved, that perhaps fiſh glew and a braſs plate, in lieu, and of the dimenſions, of the aboveſaid ſlate, may perhaps be better. Mr. *Huygens* obſerves alſo that the angle of the cone ſhould be 80 or 90 degrees, and that the hollow vertex of it ſhould be ſolid enough to receive a ſmall impreſſion from a round ſteel punch, to put the point of the pole into, which might otherwiſe have too much liberty and ſlip from the vertex. The deſign of the black ſpot in the middle of the glaſs, is to diſcover by the light of a candle obliquely reflected from your glaſs, after it has been poliſhed ſome time, whether it be perfectly clear, and free from the appearance of any bluiſh colour like that of aſhes.

Fig. 562. Mr. *Molyneux.*

767.

767. Before the work of poliſhing is begun, it is proper to ſtretch an even well wrought piece of linen over the tool, duſting thereupon ſome very fine tripoly. Then taking the glaſs in your hand, run it round 40 or 50 times thereupon; and this will chiefly take off the roughneſs of the glaſs about the border of it, which otherwiſe might too much wear away the lower parts of the tool, in which the glaſs is chiefly to obtain its laſt poliſh. If I underſtand Mr. *Huygens* right, this cloth is then to be removed, and the glaſs to be begun to be poliſhed upon the very naked tool it ſelf. But firſt there is to be prepared ſome very fine tripoly, and alſo ſome blue vitriol, otherwiſe called cyprion, engliſh and hungarian vitriol finely powdered: mix four parts of tripoly with one of vitriol: 6 or 8 grains of this mixture (which is about the quantity of two large peas) is ſufficient for a glaſs 5 inches broad. This compound powder muſt be wetted with about 8 or 10 drops of clear vinegar in the middle of the tool; and it muſt be mixed and ſoftened throughly with a very fine ſmall mullet. Then with a courſe painting bruſh take great care to ſpread it thinly and equably upon the tool, or at leaſt upon a much larger ſpace in the middle of it, than the glaſs ſhall run over in the poliſhing. This coat muſt be laid on very thin (but not too thin neither) otherwiſe it will waſte away too much in the poliſhing, and the tool will be apt to be furrowed thereby, and to have its figure impaired; inſomuch that ſometimes a new daubing thereof muſt be laid on, which it is not eaſy to do ſo equably as at firſt. This daubing muſt be perfectly dryed by holding over it a hot clean frying pan, or a thin pan of iron, with lighted charcoal therein for that purpoſe; then leave all till the tool is perfectly cold. Then having ſome other very fine tripoly very well waſhed and ground with a mullet, and afterwards dryed and finely powdered, take ſome of the ſame and ſtrow it thinly and equably on the tool ſo prepared; then take your courſe glaſs which lay by you, and ſmooth all the ſaid tripoly, very equably and finely: then take your glaſs to be poliſhed and wipe it thoroughly clean from all cement, greaſe or other filth which may ſtick to it, with a clean cloth dipped in water, a little tinged with tripoly and vitriol; then taking your glaſs in your hand apply it on the tool, and move it gently twice or thrice, in a ſtraight line backwards and forwards; then take it off and obſerve whether the marks of the tripoly, ſticking to the glaſs, ſeem to be equably ſpread over the whole ſurface thereof; if not, it is a ſign that either the tool or the glaſs is too warm; then you muſt wait a little and try again till you find the glaſs takes the tripoly every where alike. Then you may begin boldly to poliſh, and there will be no great danger of ſpoiling the figure of the glaſs; which in the other caſe would infallibly happen. If the tool be warmer than the glaſs, it will touch the glaſs harder in the middle than towards its circumference; becauſe the upper ſurface of the tool being ſwelled by heat will become

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too flat. On the contrary if the glaſs be warmer than the tool, it will bear harder towards its circumference than at the center; becauſe the inferior ſurface of the glaſs is contracted by the coldneſs of the plate, more than the ſuperior.

768. Mr. *Huygens* ſays that if the work of poliſhing were to be performed by ſtrength of hand only, it would be a work of very great labour, and even could not be performed in glaſſes of 5 or 6 feet focal diſtance: and he ſeems to think it abſolutely neceſſary that an extraordinary great force or preſſure ſhould be applyed upon the glaſs. For this purpoſe he has therefore contrived and deſcribed two methods for ſufficiently increaſing the preſſure; for the explanation of which, recourſe muſt be had to the book it ſelf and his figures [a]; it may ſuffice here to ſay that they chiefly conſiſt in applying the force of a ſtrong ſpring to preſs down the center of the glaſs upon the poliſher.

[a] See Art. 774.

769. This operation of poliſhing, as it is one of the moſt difficult and nice points of the whole, hath been very variouſly attempted and deſcribed by various Authors. Sir *Iſaac Newton*, Pere *Cherubin*, Mr. *Huygens* and the common glaſs grinders, have taken different methods in this matter. Sir *Iſaac* is the only perſon who ſeems not to inſiſt on the neceſſity of a very violent and ſtrong preſſure. In the engliſh 8° edition of his Opticks page 95. he hath theſe words "An object-glaſs of a fourteen foot teleſcope, made by an artificer at *London*, I once mended conſiderably, by grinding it on pitch with putty, and leaning very eaſily on it in the grinding, leſt the putty ſhould ſcratch it. Whether this may not do well enough for poliſhing theſe reflecting glaſſes, I have not yet tried. But he that ſhall try either this or any other way of poliſhing which he may think better, may do well to make his glaſſes ready for poliſhing by grinding them without that violence, wherewith our *London* workmen preſs their glaſſes in grinding. For by ſuch violent preſſure, glaſſes are apt to bend a little in the grinding, and ſuch bending will certainly ſpoil their figure.

770. As to his own method of poliſhing glaſs, I do not know that he any where expreſly deſcribes it, but his method of poliſhing reflecting metals he doth, and it was thus in his own words p. 92. "The poliſh I uſed was in this manner. I had two round copper plates each ſix inches in diameter, the one convex the other concave, ground very true to one another. On the convex I ground the object-metal or concave which was to be poliſhed, till it had taken the figure of the convex and was ready for a poliſh. Then I pitched over the convex very thinly, by dropping melted pitch upon it, and warming it to keep the pitch ſoft, whilſt I ground it with the concave copper wetted to make it ſpread evenly all over the convex. Thus by working it well, I made it as thin as a groat, and after the convex was cold I ground it again, to give it as true a figure as

as I could. Then I took putty which I had made very fine by washing it from all its grosser particles, and laying a little of this upon the pitch, I ground it upon the pitch with the concave copper till it had done making a noise; and then upon the pitch I ground the object-metal with a brisk motion for about two or three minutes of time, leaning hard upon it. Then I put fresh putty upon the pitch and ground it again till it had done making a noise, and afterwards ground the object-metal upon it as before. And this work I repeated till the metal was polished, grinding it the last time with all my strength for a good while together, and frequently breathing upon the pitch to keep it moist, without laying on any more fresh putty. The object-metal was two inches broad and about one third part of an inch thick, to keep it from bending. I had two of these metals, and when I had polished them both I tried which was best, and ground the other again to see if I could make it better than that which I kept. And thus by many trials I learned the way of polishing, till I made those two reflecting perspectives I spake of above. For this art of polishing will be better learned by repeated practice than by my description. Before I ground the object-metal on the pitch, I always ground the putty on it with the concave copper till it had done making a noise, because if the particles of the putty were not by this means made to stick fast in the pitch, they would by rolling up and down grate and fret the object-metal and fill it full of little holes. It seems not improbable but that glass may also be polished with proper care by the same method.

771. The method of polishing described by Pere *Cherubin* seems to be chiefly thus: he polishes with tripoly or putty or first with tripoly and afterwards with putty. But what he seems most to approve of is putty alone. He polishes in the same tool he grinds in, and he very verbosely describes various ways of doing it. He prescribes to stretch very tight a very fine thin leather or fine english fustian, or fine holland, or any fine linen or fine silk taffety or satin, all of an equable thickness as near as may be, upon the tool; then he daubs thinly on this surface thus stretched a streak of putty, sufficiently wetted to the consistence of thick syrrup, about as broad as the glass or a little more, passing through the center of the tool directly from him; then smoothing the putty by running his bruiser and pressing it backwards and forwards to him and from him, he at length lays on the glass cemented to its handle; and giving it always the same motion, strongly pressing to him and from him along the streak of putty, and by such pressure forcing the surface of the silk, already somewhat stretched, close to the surface of the tool, to which the figure of the glass was exactly adapted, he says that he could by that means obtain an excellent fine polish on any of the abovementioned substances. Before every stroke he turned the glass a little on its axis, and its handle was on this occasion considerably heavier than usual in grinding, which he commends

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as very uſeful in this buſineſs; and if new putty was wanting he made no difficulty of laying it on, as often as neceſſary, always carefully ſmoothing it thereon with the bruiſer before the glaſs was applyed.

772. This method I am of opinion might be improved by moving the glaſs not by hand but by the pole and ſpring, ſomewhat after the method of Mr. *Huygens*, eſpecially if the pole were contrived not to move looſe in a round braſs hole above, but on a ſtrong point preſſed down by ſome ſpring; the length of the pole being equal to the radius of the tool, and the point where the ſpring preſſes the upper end of the pole, being truly perpendicular over the center of the tool, and exactly in the center of its ſphere.

773. Another method which the ſame author preſcribes for poliſhing in the tool is thus; he takes a ſheet of very fine paper and examining it very carefully by looking upon it and through it, he takes off with a fine penknife all the little lumps, hard parts, roughneſſes and inequalities that he can find; then he ſoaks it in clean water, then he takes it and dries it between two clean linen cloths, though not ſo much as to make it quite dry, but to leave it dampiſh; then with ſome very thin ſtarch or paſte he daubs equably all over the ſurface of his tool as thin as poſſible, but ſome every where; then he lays on his paper very gently and ſlowly, letting it touch and ſtick firſt at one ſide, and by degrees more and more towards the middle, and by degrees to cover the whole; and he does this ſlowly to let all the air get out; then with the palm of his hand he preſſes the center and every where round about it towards the circumference to make the paper ſtick every where; and this he does 3 or 4 times while it is drying to get out all the air; he lets it dry of its ſelf, then he reviſes it again with his knife as before; then he hath a very coarſe bruiſer of glaſs whoſe circumference is ſharply ground round and at right angles to its ſurface, which he had coarſely ground before in the ſame tool; with this and a very heavy handle he ſmooths and poliſhes and rubs off all the remaining inequalities of the paper, and when this is done he lays on a ſtreak of tripoly and poliſhes as in his other method.

Mr. *Huygens*'s deſcription of his machines for poliſhing. Fig. 563.

774. At *CC* is repreſented a ſquare beam of wood a little longer than the diameter of the tool, and about 1 $\frac{1}{2}$ inch thick; the two extremities of it at *C* and *C* are bent downwards, and then are again directed parallel to the whole length, and ſerve for handles for the workman to lay hold upon. In the middle of this beam there is fixt an iron ſpike, ſo long that when the lower ſurfaces of the handles *C*, *C* are placed upon a plane, the point of the ſpike ſhall juſt touch the plane. This point preſſes upon the apex of the hollow cone, which deſcends through the hole in the ſlate, which by the interpoſition of a cloth was cemented to the glaſs *B* lying upon the tool *A*. To increaſe this preſſure a ſort of a bow *DED* is ſhaped out of a deal board, half an inch thick and 5 feet long, being 7 inches broad

broad in the middle, and tapered narrower towards its extremities, ſo as to end almoſt in a ſharp point. The middle of the bow is fixt to the floor by an iron ſtaple at *E* driven croſs it; and is bent into an arch by a rope *FIIF*; to which two other ropes are tied at *I* and *I*; the interval *II* being equal to the length of the beam *CC*. One of theſe ropes *ICCG* goes over the back of the beam *CC*, paſſing through a hole in each handle at *C* and *C*, and then is lapped round a cylindrical pegg at *G*, that paſſes through two wooden chaps, to the bottom of which the other rope is tied that comes from the other *I*. So that by turning the pegg *G*, to lap the rope about it, the bow *D D* may be bent as much as you pleaſe. The tool *A* is placed upon a ſtrong ſquare board fixt to the table *O* on one ſide, and ſupported on the other ſide by the poſt *P*. Then the workman ſits down, and taking hold of the handles *CC*, he draws the glaſs to him and from him over the tool *A*, with a moderate motion. And after every 20 or 24 ſtrokes he turns the glaſs a little about its axis. This way of poliſhing took up two or three hours, and was very laborious as well as tedious, becauſe the glaſs being ſo much preſſed downwards was moved very ſlowly.

Fig. 564. 775. Inſtead of the bow *D D*, afterwards I invented another ſpring, by ſloping the flat ends of a couple of deal boards $\alpha\beta$, $\alpha\gamma$ and by nailing the flat ſlopes together very firmly that the boards might make an acute angle $\beta\alpha\gamma$. One of theſe boards ſo joined was laid upon the floor under the poliſhing table, the ends $\beta\gamma$ being under the middle of the tool *A*. So that they lay quite out of the way of the workman, who before was a little incommoded by the ends of the bow *D D*. The boards at the end α were 8 or 10 inches broad, and from thence went tapering almoſt to a point at β and γ. The board $\alpha\gamma$ lying upon the floor, the end β, of the upper board, was pulled downwards by a rope $\beta\varepsilon\zeta$ that paſſed under a pully ε, fixt to the floor, and then was lapped round a ſtrong pegg ζ that turned ſtiff in a hole in the floor. Under the end γ the middle of a ſtrong ſtick $\delta\gamma\delta$ was fixt at right angles to the board $\alpha\gamma$, and cords were tied to each end of this ſtick at δ, δ, which went over the poliſhing beam *C*, *C* as in the former machine. This ſtick was lifted up but very little from the floor at the time of poliſhing; and by conſequence the ropes δC, δC were long enough to give liberty of motion to the poliſhing beam *CC*. Two iron pins θ, θ paſſing through the ends of the boards at α, were ſkrewed into the floor, but the heads of the pins ſtood up above the boards, to give them liberty to riſe up when the rope $\beta\varepsilon\zeta$ was ſtretched.

Fig. 563. 776. To fecilitate the labour of moving the glaſs backwards and forwards in the tool, I made this addition to the former machine. At *M* is repreſented a ſtrong hand made of wood or iron, having a ſquare cavity cut through the bottom of it, for the poliſhing beam *CC* to paſs through, not tight but with ſome liberty. To one ſide of this hand *M* a long board *L L* is annexed, ſome way or other, by means of an iron bolt. The breadth

breadth of the lower ſurface of this board LL is equal to the breadth of the hand M, being 2 inches and a half, its thickneſs is half an inch, and its length is equal to three ſemidiameters of the tool. The board LL muſt be drawn length ways backward and forward over a block H fixt firmly to a table O; the thickneſs of the block being ſuch that the board LL may lye an inch higher than the ſurface of the tool A. The wooden hooks at Π and the pins at Σ keep the motion of the board in the ſame direction by hindering it from ſlipping either ſideways or upwards. Over this board, at right angles to it, and over the middle of the block H there lyes a wooden roller, having a ſtrong iron axis which turns in the holes of two iron plates fixt to the ends of the block. The thickneſs of the roller is about an inch and a half. Through two holes bored through this roller and made wider at one end of them, two ſtrong cords are made to paſs with knots at one end of them, to be drawn into the wider parts of the holes, that they may neither ſlip through nor ſtand out from the roller. Then each cord is lapped round the cylinder ſeveral times; and one end of each is pegged firmly into the board LL at the end towards M, and the other ends of them are lapped round a pegg at N; which being turned round will ſtretch the cords as much as you pleaſe. At one end of the axis of this roller there is a handle $\mathcal{Q}$ which being turned round, backwards and forwards alternately, the board LL with the glaſs annexed to it is moved to and fro, ſo far, that about a third part of its diameter ſhoots both ways over the margin of the tool. The ſpike in the middle of the beam CC preſſes the glaſs a little obliquely, becauſe the hand M holds the beam CC not tight but ſomewhat looſely, to the end that the glaſs may paſs over the tool without trembling. Nevertheleſs this inclination of the ſpike muſt be but very ſmall; and may eaſily be increaſed or diminiſhed ſeveral ways. Two pins or ſtops muſt be fixt to the under ſurface of the board LL, to determine the length of the ſtroke. The tool A, or rather the ſtone to which it is cemented, is ſqueezed faſt between the block H and a ſtrong ſtop on the oppoſite ſide of the ſtone, by the interpoſition of a wedge. The workman ſits upon a round ſtool, and when one hand is tired with turning the roller, he applies the other; and therefore is not tired ſo ſoon as with the other machine, which required both hands, and alſo a reciprocating motion of his whole body. Sometime after, I cauſed a longer handle $\mathcal{Q}\,2$ to be made that turned at both ends, for the convenience of uſing both hands at once.

777. After every 20 or 24 ſtrokes it is neceſſary to give the glaſs a ſmall turn about its axis; which is eaſily done by laying hold of the ſlate, fixt to it, with one hand, while the other hand goes on with the poliſhing motion. The tool muſt alſo be moved a little after every 25 or 50 ſtrokes, by drawing it half a ſtraws breadth towards that part of it which the glaſs has left; and by drawing it back again after as many more ſtrokes. At the

the beginning of the work the tripoly will be gathered into little lumps in ſome places of the tool, but will be diſperſed again in a little time; and then the area of the tool will become perfectly ſmooth.

778. If the tripoly does not appear to ſtick equally to the glaſs in all parts, and to be diffuſed over it in ſlender ſtraight ſtreaks, the frying-pan, with coals in it, muſt be held over the tool again; till you perceive the the area or coat of tripoly is not quite ſo cold as the other parts of the tool. Then let tripoly be rubbed upon the tool again, and let the glaſs be preſſed over it with your hand, to try whether it ſticks equally to the glaſs in every place. When it does you may proceed in the work of poliſhing. But after I began to make uſe of vitriol inſtead of verdigreaſe, all that I have ſaid about warming the tool may be omitted. Becauſe theſe coats always touch the glaſs as they ſhould do, and ſtick better than before. Alſo let the tool be rubbed with tripoly over the coat without warming the tool, that the coat may be preſerved more entire and that the glaſs may touch it better; which muſt always be repeated after 200 or 400 ſtrokes in poliſhing. The glaſs ſhould alſo be taken from the tool after 200 ſtrokes, by withdrawing the bolt *L* which connects the hand *M* to the board *LL*; and by removing the beam *CC*. Then rub your finger upon the glaſs, or a clean rag or a bit of leather, to examine how much it is poliſhed.

779. To ſave the trouble of counting the ſtrokes, there is a wooden wheel Δ*X*, 7 or 8 inches broad, placed flat againſt a board, fixt to the ſide of a wall. It turns eaſily about an axis, and has 24 teeth, like thoſe of a ſaw, which are puſhed round by a bended wire *TYX*, in this manner. The wire turns about a center *Y*, and while one end of it is pulled by the ſtring *TV*, tyed to the end of the board *LL*, the oppoſite end *YX* puſhes back a long ſpring *RS* fixt to the board at *R*; which by preſſing upon the wire at *S* cauſes the part *YX* to bend a little, and ſo the point *X*, in returning to the wheel (the ſtring being relaxed) falls a little lower, into the next tooth, and puſhes it forward into the poſition repreſented in the figure. There is a ſpringing catch at Δ which ſtays the wheel, after every ſtroke at *X*. Laſtly there is a pin fixt in the circumference of the wheel at *Z*, which by preſſing the tail of a hammer and letting it go again, cauſes a bell to ſound after every revolution of the wheel, and gives notice that the glaſs muſt be turned a little about its center. It is eaſy to underſtand that another piece of wheel work, having three or four indexes, whoſe revolutions are in decimal progreſſion, may be fixt to the block *H* and impelled by the ſtrokes of the board *LL*; by which means, without any trouble of counting, one may be informed how many ſtrokes go to poliſh a glaſs.

780. A glaſs 5 or 6 inches broad requires about 3000 ſtrokes upon each ſurface, to bring it to perfection. And you muſt carefully examine, the middle

middle of the glaſs oppoſite to the blacking, whether any place appears darkiſh or of an aſh-colour; or whether any ſmall ſpots appear by an oblique reflection of the light of a candle or of a ſmall beam of light let into a dark room. For the other parts of the glaſs will appear perfectly fine much ſooner than the middle.

781. After the glaſs has been ſufficiently poliſhed, let the ſtone, the cloth and the cement be warmed over a pan of charcoal, till the cement grows ſo ſoft that the glaſs may be ſeparated from it by a ſide motion. Then whatever cement remains upon the glaſs muſt be wiped off with a hot cloth dipt in oil or tallow, and laſt of all with cleaner cloths. Then if it does not appear perfectly poliſhed (for we are often deceived in this point) the work muſt be repeated again, by glewing the glaſs to the ſlate as before; then it muſt be wiped very clean and be made a little rough as we ſaid before. We may alſo lay a new fund or coat upon the tool if the old one be ſpoiled; provided no other glaſs has been poliſhed in the tool in the mean time. The old fund may be waſhed off from the tool with a little vinegar. Laſtly take care always to chooſe the thickeſt and the cleareſt pieces of glaſs, to avoid a great many difficulties that ariſe from the unequal preſſure in poliſhing.

CHAPTER II.

The method of caſting, grinding and poliſhing metals for reflecting teleſcopes, begun by the Honourable Samuel Molyneux *Eſquire, and continued by* John Hadley *Eſquire, Vice-Preſident of the* Royal Society.

Hiſtorical preface.

IT is chiefly to the invention of teleſcopes that we owe all the late diſcoveries and moſt of the preſent accuracy in Aſtronomy. An invention which certainly, in its firſt original, was put in practice by an Engliſhman *Frier Bacon*; although its firſt application to aſtronomical purpoſes may be juſtly attributed to *Galileo*. The greateſt improvement, which this invention has ever received, is indiſputably and ſingly owing to Sir *Iſaac Newton*: to whoſe extraordinary ſagacity, and very judicious experiments, the world firſt owes the diſcovery of the different refrangibility of the rays of light, and the inſuperable difficulties ariſing from thence in perfecting any refracting teleſcope. This led him to the practice of making teleſcopes by reflection; which having attempted with his own hands, he perfected ſome of ſix inches length about the year 1670. Whoſoever therefore would throughly underſtand the method of making theſe teleſcopes, will find it very proper in the firſt place, to peruſe and throughly conſider the account which the celebrated author of this invention hath himſelf given of it in his writings; which are to be met with in the Philoſophical

loſophical Tranſactions, N°. 80, 81, 82, 83, 88, 96, 97; and in his Treatiſe of Light and Colours, beginning at the 89th page of the engliſh edition in 8°. It will be neceſſary here to note an error of the printer in this 89th page; where it is ſaid " that the apertures of the object-glaſſes, and the charges or magnifying powers, ought to be as the cubes of the *ſquare* roots of the lengths of the teleſcopes; the word *ſquare* is falſe printed; it ſhould be as the cubes of the ſquare-ſquare roots of the lengths.

782. As great an improvement as this was to teleſcopes, I do not find that it was ever effectually proſecuted from that time till about the year 1719 or 20; when a very ingenious gentleman Mr. *Hadley* of *Eſſex*, attempted it, and ſucceeded very well in making two inſtruments of this kind, of about 5 foot 3 inches long; one of which he has been pleaſed to give to the Royal Society. A very particular and curious deſcription of this inſtrument, and of the apparatus for managing thereof, hath been given by Mr. *Hadley*, with a figure of it alſo, in the Philoſophical Tranſactions for the months of March and April 1723 N°. 376. It will be neceſſary for any perſon that would attempt to make this inſtrument, to conſider that account given by Mr. *Hadley*; for although he gives therein no account of the manner of caſting, grinding and poliſhing the ſpecula, yet as to the proportion and compoſition of the different parts of the inſtrument, and of the apparatus for moving it, the reader will there find ſeveral uſeful inſtructions. What is contained in the following papers is alſo chiefly owing to the communicative genius of that gentleman: and had he ever given himſelf the trouble to reduce to writing what he knows and hath practiſed in the abovementioned particulars, as to the conſtruction of this inſtrument, the following account had been altogether unneceſſary. Upon his encouragement and inſtructions, the Reverend Mr. *James Bradley* Profeſſor of Aſtronomy in *Oxford*, attempted the ſame about three years ago; and having ſucceeded pretty well, would probably have perfected one of them, had he not been obliged ſuddenly to remove from the place where he then dwelt, and been ſince diverted from it by other avocations. Soon after this Mr. *Bradley* and I began our endeavours at *Kew* to perform the ſame, and our firſt attempt was to make them about 26 inches long. Notwithſtanding Mr. *Bradley*'s former tryals and Mr. *Hadley*'s frequent inſtructions, we were a long while before we could tolerably ſucceed. The firſt good one that we finiſhed was in May 1724, of the aforeſaid length of 26 inches. I have ſince made a pretty good one of about 7 inches, and we are now about one of 8 foot.

Deſign. 783. The main drift of all our tryals hath been if poſſible to reduce the method of making theſe inſtruments to ſome degree of certainty and eaſe; to the intent that the difficulty in making them, and the danger in miſcarrying, might no longer diſcourage any workman from attempting the ſame for publick ſale; which no body but Mr. *Haukſbee* in *Crane Court*

Court hath ever ventured upon. He has made a good one of about 3 ½ foot, and is now about one of 6 foot and another of 12 foot, and deſerves very well to be encouraged, being the firſt perſon who hath attempted it without the aſſiſtance of a fortune, which could well bear the diſappointment. About the beginning of the laſt winter being pretty well ſatisfied as to moſt of the circumſtances in this performance, and being deſirous that theſe inſtruments might become cheap and of publick ſale, we acquainted Mr. *Scarlet* near St. *Anne*'s Church, and Mr. *Hearne* a Mathematical Inſtrument-maker in *Dogwel Court*, *White Friers*, with the whole proceſs of the operation as we had practiſed the ſame; and they have ſince ſucceeded in making theſe inſtruments. However as they are not yet become ſo common, ſo cheap and ſo univerſally made and uſed, as one would wiſh an inſtrument of this nature to be, we have been encouraged to give this following account, for the general information of all perſons who would make the ſame for their own uſe or for ſale.

To make the gages.

784. Having in the firſt place conſidered of what length one would propoſe the inſtrument to be, and conſequently what diameter it will be neceſſary to give to the large ſpeculum, for which there are ample inſtructions by Sir *Iſaac Newton*'s table in the Philoſophical Tranſactions aforeſaid*, allowing about an inch more than the aperture in the table for the falſe figure of the edges, which very often happens; I ſay having determined theſe things, take a long pole of fir deal or any wood, of a little more than double the length of the inſtrument intended, and ſtrike through each end of it two ſmall ſteel points, and by one of them hang up the ſame againſt a wall perpendicularly; then take two pieces of thin plate braſs well hammered, a little thicker than a ſix-pence; theſe may be about an inch and a half broad, and let their length be in reſpect of the diameter of the ſpeculum as 3 to 2; *viz.* if the ſpeculum be 8 inches diameter, theſe may be about 12. Fix each of theſe ſtrongly with rivets between two thin bits of wainſcot, ſo that a little more than a quarter of an inch in the breadth, may ſtand out from between the boards. Then fix up theſe pieces horizontally againſt the wall under your pole, and therewith, as with a beam compaſs, ſtrike an arch upon each of them; then file each of them with a ſmooth file to the arch ſtruck, ſo as one may be a convex and the other a concave arch of the ſame circle. Theſe braſſes are the gages to keep the ſpeculum, and the tools on which it is ground, always to the ſame ſphere. And that they may be therefore perfectly true to each other, it is neceſſary to grind them with fine emery one againſt the other, laying them on a flat table for that purpoſe, and fixing one of them to the table.

Patterns for the ſpecula.

785. When the gages are perfectly true, let a piece of wood be turned about 2 tenths of an inch broader than the intended ſpeculum, and ſome-

* See another table Art. 364. and alſo Art. 362.

what

what thicker, which it is beſt to caſt in no caſe leſs than 2 tenths of an inch thick, and for ſpecula of 6, 8, or 10 inches broad, this ſhould be at leaſt three or 4 tenths thick when finiſhed[a]. This board being turned, take ſome common pewter and mix with it about $\frac{1}{10}$ of regulus of antimony, and with that wooden pattern caſt one of this pewter, which will be conſiderably harder than common pewter. Let this pewter pattern be truly turned in a lathe, and examined by means of the gages aforeſaid, as a pattern for caſting the ſpecula themſelves; and take care when it is turned that it be at leaſt $\frac{1}{20}$ of an inch thicker, and about $\frac{1}{10}$ of an inch broader than the ſpeculum intended to be caſt therefrom.

To make the moulds for caſting the patterns and the ſpecula.

786. The manner of making the moulds for caſting is now to be explained, and will ſerve for a direction as well for caſting this pewter pattern, as afterwards for caſting thereby the ſpeculum it ſelf. The flaſks had beſt be of iron and muſt be at leaſt 2 inches wider every way than the ſpeculum intended. In each flaſk there ſhould be the thickneſs at leaſt, of one inch of ſand. The caſting ſand which the common founders uſe from *Highgate*, will do as well as any; and any ſand will do which is naturally mixt with a ſmall proportion of clay, to make it ſtick. The ſand ſhould be as little wet as may be, and well beaten but not too hard. The ingates ſhould be cut ſo as to let the metal flow in, in four or five ſtreams over the whole upper part of the mould; otherwiſe whatever pores happen in the metal, will not be ſo equally diſperſed as they ſhould be over the whole face of the metal; theſe pores generally falling near the ingate ſtreams. Let the flaſks dry in the ſun for ſome hours or near a very gentle fire, otherwiſe they will warp and give the ſpeculum, when caſt, a wrong figure. For beſides ſaving the trouble in grinding, it is beſt on many accounts to have the ſpeculum caſt of a true figure; and it is for this reaſon, that it is beſt to caſt it from a hard pewter pattern, and not from a wooden one as founders uſually caſt.

The compoſition of the metal and manner of caſting it.

787. The next point that is to be conſidered is the compoſition or ingredients of the metal it ſelf for the ſpecula. As to this it may be ſaid in general, that any hard white metal that will take a poliſh will do more or leſs well. We have made trials of above 150 different mixtures and found none of them entirely free from all faults. Three parts of copper and one part and a quarter of tin will make a very hard white metal, but it is very liable to be more porous than it ſhould be, eſpecially if the metal be too much heated in melting. Six parts of good ſhruff braſs[b] and one part of tin, will make a whiter and harder metal; but the fume of

a The metal of my reflecting teleſcope, made by that excellent workman Mr. *Hearne* in *Dogwel-Court*, *White Friers* by *Fleet-Street*, is 6 inches broad and between 6 and 7 tenths thick; beſides a braſs plate ſoldered to its back an inch and half broad, with 4 holes in it to fix a ſhort handle to it mentioned in art. 790; having alſo a ſocket in the middle to ſkrew in another handle when finiſhed. Its focal diſtance is five foot.

b Plate-braſs, caſt and milled; the beſt comes from *Hamburgh*.

the

the calamy ſtone in braſs, leaves very often ſtreaks of ſcabrous parts in the ſurface of the metal; which if many utterly ſpoil it. Take two parts of the former mixt metal of copper and tin, and one part of the latter braſs and tin, this alſo will make a good metal; let the copper and the braſs be firſt melted together, and keep them in fuſion for half an hour or more; then clear the pot and put in the proper quantity of tin for both; which will inſtantly melt; ſtir it well about and pour it off immediately. This mixture may be melted over and over again in caſe of neceſſity, provided always care be taken that the fire is not ſuffered to be too violent. A common bellows furnace has been found moſt convenient for governing the fire, and ſome metals have ſucceeded which have been caſt by a common braſs-founder in their ordinary way of caſting; the compoſition having been firſt made and melted together for the ſpeculum and delivered to them only for caſting. There hath been tried alſo another mixture and another manner of caſting, which ſucceeded better than any of the abovementioned: it was copper, ſilver, regulus of antimony, tin and arſenick; and the metal was caſt in very hot moulds of braſs; but as this method is very expenſive and will never become common, it need be no further inſiſted upon in this place.

Of rough grinding the metal.

788. The metal being duly caſt, grind the ſurface of it quite bright upon a common grindſtone; keeping it by means of your convex gage as near to your intended figure as may be. When all the outward ſurface and all the ſand holes and falſe parts and inequalities are ground off, then provide a good thick ſtone; a common ſmall grinding ſtone will do very well. Let its diameter be to the diameter of the ſpeculum as 6 to 5; with another courſe ſtone and ſharp ſand or courſe emery, rub this ſtone till it fits the concave gage; and then with water and courſe emery at firſt and afterwards with finer rub your ſpeculum upon this ſtone until it forms it ſelf into a true portion of a ſphere fitting your convex gage. A different manner of moving the metal upon the ſtone, will incline it to form it ſelf ſomewhat of a ſmaller or larger ſphere. If it be ſtruck round and round, after the manner of glaſs-grinders, the ſtone will wear off at the outſides, and the metal will form it ſelf into a portion of a leſs ſphere. If it be ſtruck croſs and croſs the middle, it will flat the ſtone and become ſomewhat of a larger ſphere. There ſhould be uſed but very little emery at a time, and often changed; otherwiſe the metal, will always be of a ſmaller ſphere than the ſtone and will hardly take a true figure, eſpecially at the outſide. For the better grinding the metal it is neceſſary that this ſtone ſhould be placed firm on a ſtrong round board fixed firmly on a poſt to the floor, as is uſual with glaſs-grinders, and the ſame table or pillar will ſerve for the further grinding and poliſhing the ſpeculum.

Mr. *Hadley*'s continuation of this chapter.

789 When the metal is caſt and rough figured (which ſhould be done with taking off as little of the ſurface of the metal as is poſſible, becauſe

that cruſt ſeems to be generally harder and more ſolid than the inner parts,) the ſides and back of it ſhould be ſmoothed and finiſhed; leaſt the doing that afterwards ſhould make the metal caſt and ſpoil the figure of the foreſide.

Provide a concave braſs tool.

790. A round braſs plate muſt be caſt of ſufficient breadth and thickneſs; (for a ſpeculum 6 inches diameter I uſed a plate 8 or 9 inches broad, and $\frac{1}{2}$ an inch thick.) Let one ſide be turned to the concavity you deſign your ſpeculum ſhould have; on the other ſide let it have ſuch a handle faſtened as may make it eaſily manageable. This handle ſhould be as ſhort as conveniently it can, and fixt to the plate's back rather by ſome other manner than either by ſkrewing it into a hole in the metal, or by a broad ſhoulder ſkrewed againſt the back of it, for fear of bending the plate.

And a convex marble tool covered with whetſtones.

791. Have ready a round marble about $\frac{1}{8}$ or $\frac{1}{10}$ part broader than the braſs plate, and an inch or inch and $\frac{1}{4}$ thick: let this be cut by a ſtone-cutter to the ſame convexity on one ſide as the concavity of the plate; and then grind it with the plate and emery till all the marks of the chiſſel are out. This marble is to be covered with pieces of the fineſt blue hone or whetſtone, chooſing thoſe that are neareſt of a breadth and thickneſs, but chiefly thoſe that when wetted appear moſt even and uniform in their colour and grain. They are to be cut into ſquare bits, and theſe, having each one ſide ground concave on the convex marble with emery or fine ſand, are to be fixt cloſe down on it with ſome tough and ſtrong cement in the manner of a pavement, leaving a ſpace of a ſmall ſtraws breadth between each; their grain being likewiſe placed in an alternate direction as repreſented in the figure. I chooſe rather to diſperſe the ſquares that come out of the ſame whetſtone to a diſtance from one another, than to place them together. They muſt then be reduced to one common convex ſurface to fit the braſs plate; and if the cement happen to riſe any where between them, ſo as to come up even with their ſurface, it muſt be dug out; and ſo from time to time as often as the hones wear down to it. Upon theſe ſquare pieces of whetſtone the laſt figure is to be given to the ſpeculum.

Fig. 565.

And a convex glaſs tool, or a marble one.

792. Beſide theſe there will be wanted for the laſt poliſh either a very thick round glaſs plate, (its diameter being about the middle ſize between that of the braſs tool and that of the ſpeculum it ſelf,) or if that cannot be procured of near $\frac{1}{2}$ an inch in thickneſs, I imagine a piece of true black marble of the eveneſt grain and freeſt from white veins or threads may do in its ſtead. This glaſs or marble muſt be figured on one ſide to the braſs tool likewiſe, and is to ſerve for the finiſhing the poliſh of the ſpeculum, when covered with ſarcenet as ſhall be directed.

And a concave bruiſer.

793. A ſmaller braſs or metal plate, of the ſame concavity with the larger will be uſeful, as well to help to reduce the figure of the hones when-

whenever it appears to be too convex, as to ſerve for a bruiſer to rub down any gritty matter happening to be amongſt your putty, before you put the ſpeculum on the poliſher, when you renew the powder. Any of the ſpeculums which prove bad in caſting will ſerve for this purpoſe.

Of grinding the braſs tool on the whetſtones.

794. When all is thus far ready, let the marble with the blue hones be fixt in ſuch manner, that it may be often waſhed during your work, by throwing on it about $\frac{1}{8}$ of a pint of water at a time, without inconvenience. Then place the braſs tool on the hone pavement, and rub it backwards and forwards with almoſt a direct motion; yet carrying it by turns a little to the right and left, ſo as to go a little over the edges of the pavement every way, regularly turning the tool on its own axis, and alſo changing the direction of the ſtroke on the hones. This continue, keeping them always very wet, till you have got out all the rings remaining in the plate from the turning, and the blackneſs from grinding the marble or glaſs in it; and towards the latter end often waſhing away the mud which comes from the whetſtones.

Of grinding the glaſs tool on the braſs one.

795. When this is done lay the braſs tool down, and in it grind again with fine emery the glaſs or marble deſigned for the laſt poliſher, giving it as true a figure as you can; in order to which you may obſerve the directions of Mr. *Huygens* in his Comm. de form. & pol. vitr*. But give it no farther poliſh.

* Art. 758. &c.

Of covering the glaſs tool with pitch and ſarcenet, for a poliſher.

796. Chooſe a piece of fine ſarcenet as free from rows and great threads as you can. Let it be 3 or 4 inches broader than the glaſs or marble; and turn down the edges of the ſarcenet round the ſides of the glaſs &c. and ſtrain it by lacing it on the backſide as tight and ſmooth as you can, having firſt cleared it of all wrinkles and folds with a ſmooth-iron, and drawn out the knots and gouty threads. Then wet it all over as evenly as you can with a pretty ſtrong ſolution of common pitch in ſpirit of wine; and when the ſpirit is dried out repeat the ſame, and if any bubbles or bliſters appear under the ſarcenet, endeavour to let them out with the point of a needle. This muſt be repeated till the ſilk is not only ſtuck every where firmly down to the glaſs or marble, but is quite filled with the pitch. A large painters pencil, made of ſquirrels hair, is of uſe for the ſpreading this varniſh equally on the ſilk, eſpecially when it begins to be full. It muſt then be ſet by for ſome days, for the ſpirit to dry well out of it, and the pitch to harden, before any thing more be done to it. If you do not care to wait ſo long, the pitch may be melted into the ſilk without diſſolving it in ſpirits. In order to this, ſtrain a ſecond thin ſilk over the firſt, but you need not be curious in the choice of it, and having heated all together as hot as you think the ſilk or glaſs will ſafely bear, pour on it a little melted pitch (firſt ſtrained through a rag) ſo much as you judge ſufficient to fill both ſilks; it muſt be kept hot ſome time till the pitch ſeems to have ſpread it ſelf evenly all over. If you find you cannot

get it to ſink all into the upper ſilk, but it ſtands above it any where, it is a ſign there was too much pitch layed on, which ſhould be taken away in thoſe places while it remains liquid, with a hot rag preſſed down on it. When all is cold again, ſtrip off the outward ſilk and cut away the uſeleſs looſe edges of the inward. To take off the ſuperfluous pitch where it lyes too thick, and reduce the whole to a regular ſurface, it muſt be rubbed in the braſs tool with a little ſoap and water, till they are coloured of a pretty deep brown with the pitch: then waſh them away, and repeat the ſame with more ſoap and water till the weaving of the ſilk appears every where as equally as you can make it. As this work takes up ſome time, you may expedite it by putting a few drops of ſpirit of wine to the ſoap and water, (which will help them to diſſolve and wear away the pitch ſomething faſter) till it comes towards a concluſion; and if there are any places where the pitch lies very thick, you may ſcrape it away with a ſharp knife.

Of preſerving and repairing the poliſher.

797. This poliſher muſt be carefully kept from all duſt and grit, but particularly from emery and filings of hard metals; and therefore ſhould not be uſed in the place where the others come. After they have ſerved a good while they are more apt to ſleek the metals than at firſt; to prevent which their ſurfaces may be taken off by rubbing them with ſoap and water in the tool as before, and then ſtriking them once or twice over with the beforementioned ſolution of pitch with a pencil, proceeding as before, only that you muſt not now put any ſpirit to your ſoap and water, nor will you need to change them above once or twice.

Of grinding the metal ſmooth.

798. You may now begin to give the figure to your ſpeculum on the hones, rubbing it and the braſs tool on them by turns, till both are all over equally bright; having firſt fixt on to the middle of the back of your ſpeculum a ſmall and low handle with only pitch ſtrained through a rag. For of all cements that ſeems the leaſt apt to bend the metals in ſticking theſe handles &c. on to them.

Of figuring the metal to the curvity of the poliſher.

799. The poliſher being fixt likewiſe in a proper manner for your work, rub either the metal it ſelf, or rather the beforementioned bruiſer, being firſt alſo figured on the hones, with a little putty waſhed very fine and fair water; till it begins to ſhew ſome poliſh. Then if you find it takes the poliſh unequally, that is more or leſs about the edges than in the middle, tis a ſign the braſs tool and metal &c. are more or leſs concave than to anſwer the convexity of the poliſher; and muſt be reduced to the curvature of this, rather than to attempt an alteration in the figure of the poliſher; which would be a much more difficult as well as laborious work. If the ſpeculum appears too flat, the larger braſs tool muſt be workt on the hones for ſome time, keeping its center near their circumference, with a circular motion; but concluding for 4 or 5 minutes with ſuch a motion as was before deſcribed[a]. Then figure the metal anew on the

[a] Art. 794.

the hones and try it again on the poliſher as before. If the metal be too concave, the ſurface of the hones may be flatted by rubbing the ſmaller braſs plate, or the beforementioned ill caſt metal, on the middle of them; with a direct but ſhort ſtroke, ſo as but juſt to reach over their circumference with the edge of it. Then the larger braſs is to be workt on them in the ſame manner; and laſt of all the metal to be poliſhed. When you find the braſs tool and hones &c. anſwer the curvature of the poliſher, you may then examine the truth of the figure of the ſpeculum more ſtrictly, to avoid the loſs of time and labour in finiſhing its poliſh while the figure is imperfect.

To find the radius of the ſphere of the metal. Fig. 566.

800. Place the ſpeculum in a vertical poſture on a table about 3 $\frac{1}{2}$ or 4 feet from the floor. On another table ſet a candle whoſe flame ſhould be about the level of the middle of the ſpeculum, and very near the center of its concavity. About $\frac{1}{2}$ an inch before the flame place a flat tin, or thin braſs, plate about 3 inches broad but 4 or 5 high, having ſeveral holes about the middle of different ſhapes and ſizes; ſome of them as ſmall as the point of the ſharpeſt needle will make them, the biggeſt about the ſize of a large muſtard ſeed: darken the room and move this candle and plate about on the table, till the light from the brighteſt part of the flame, paſſing through ſome of the larger holes to the ſpeculum, is reflected back ſo as to form the images of thoſe holes, cloſe without one of the ſide edges of that thin plate. Thoſe largeſt images in this caſe will be viſible, (although the ſpeculum have no other poliſh than what the hones give it,) when received on a thick white card held cloſe to that edge of the plate, if the back of the card be either blacked or ſo ſhaded that the candle may not ſhine through it; and the eye be alſo ſkreened from the candle's direct light. If any difficulty happens in diſcerning them, the plate may be removed and the image of the whole flame will be eaſily ſeen. Have ready an eye-glaſs whoſe focal diſtance may be ſomething greater than the double of that of the eye-glaſs you intend for the inſtrument when finiſhed: you may try ſeveral at your diſcretion. Let this be ſupported by a ſmall ſtand moveable on the table, and capable of raiſing and ſinking it, as the height of the flame requires, and of turning it into any direction. By means of this ſtand bring the eye-glaſs into ſuch a poſition, that the light from ſome of the holes, after its reflection from the ſpeculum, may be received perpendicularly on its ſurface; and that its diſtance from the ſpeculum be ſuch, that the reflected images of the holes may be ſeen diſtinctly through it, near the edge of the thin plate, by the light coming immediately from the ſpeculum: guide the candle and thin plate with one hand, and the ſtand carrying the eye-glaſs with the other, till you have got them into ſuch ſituation, that you ſee diſtinctly at the ſame time, through the eye-glaſs, the edge of the thin plate, and the image of one of the holes cloſe to it. Meaſure the exact diſtance of the

middle

middle of the ſpeculum from the thin plate directly againſt the flame, and alſo from the edge cloſe to which you ſee the image of the hole. If theſe meaſures are the ſame, ſet it down as the exact radius of concavity of your ſpeculum, and proper curvature for any that are to be poliſhed on your poliſher, though that will allow ſome latitude: If the meaſures aforeſaid differ, take the mean between them.

To examine the figure of the metal.

801. You will now alſo judge of the perfection of the ſpherical figure of your metal by the diſtinctneſs with which you ſee the repreſentations of the holes, with their raggedneſs, duſts and ſmall hairs ſticking in them: and you will be able to judge of this more exactly and likewiſe to diſcover the particular defects of your ſpeculum, by placing the eye-glaſs ſo as to ſee one of the ſmalleſt holes in or near its axis; and then by turns ſhoving the eye-glaſs a very little forward towards the ſpeculum and pulling it away from it by turns, letting the candle and plate ſtand ſtill in the mean time. By this means you will obſerve in what manner the light from the metal comes to a point, to form the images, and opens again after it has paſt it. If the area of the light, juſt as it comes to or parts from the point, appears not round but oval, ſquariſh, or triangular &c. it is a ſign that the ſections of the ſpecular ſurface, through ſeveral diameters of it, have not the ſame curvature. If the light, juſt before it comes to a point, have a brighter circle round the circumference, and a greater darkneſs near the center, than after it has croſſed and is parting again; the ſurface is more curve towards the circumference and flatter about the center, like that of a prolate ſpheroid round the extremities of its axis; and the ill effects of this figure will be more ſenſible when it comes to be uſed in the teleſcope. But if the light appears more hazy and undefined near the edges, and brighter in the middle before its meeting than afterwards, the metal is then more curve at its center and leſs towards the circumference; and if it be in a proper degree, may probably come near the true parabolick figure. But the ſkill to judge well of this, muſt be acquired by obſervation.

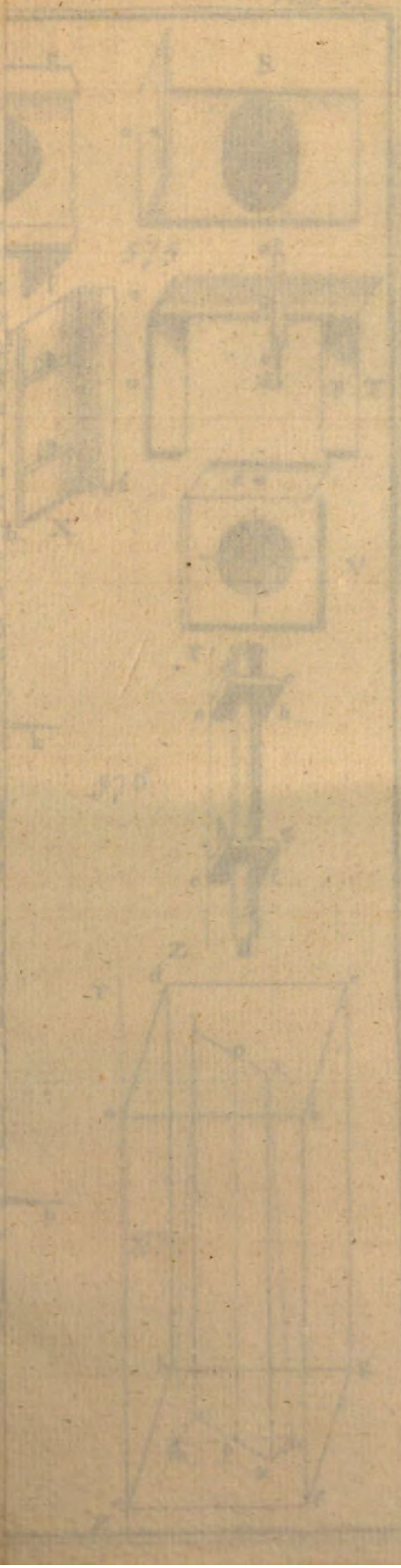
A caution to be obſerved.

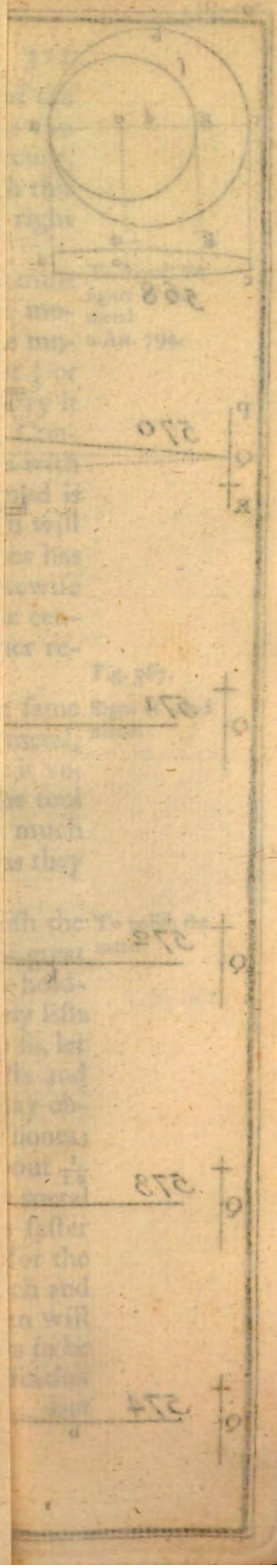
802. In performing the foregoing examination, the image muſt be reflected back as near the hole it ſelf as the eye's approach to the candle will admit of; that the obliquity of the reflection may not occaſion any ſenſible errors: in order to which the eye ſhould be ſkreened from the candle; and the glaring light which may diſturb the obſervation may be ſtill more effectually ſhut out, by placing a plate, with a ſmall hole in it, in that focus of the eye-glaſs which is next the eye. *A* is the ſpeculum, *B* the candle and plate with the ſmall holes, *C* the cell with the eye-glaſs and plate behind it.

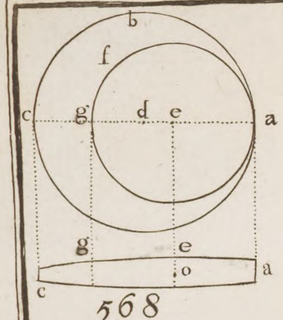
Fig. 566.

A circumſtance varied.

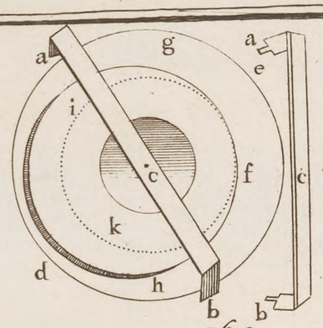
803. Inſtead of the flame of the candle and plate with ſmall holes, I ſometimes made uſe of a piece of glaſs thick ſtuck with globules of quickſilver, ſtrained through a leather and let to fall on it in a dew; placing this



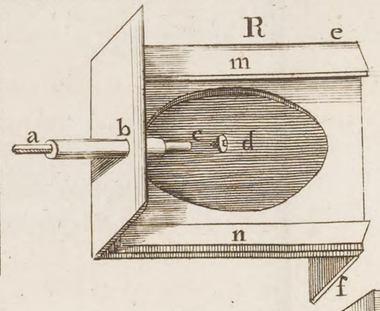


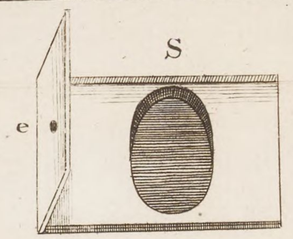


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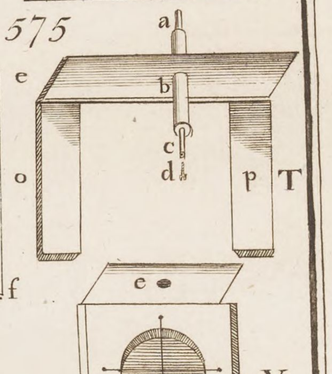


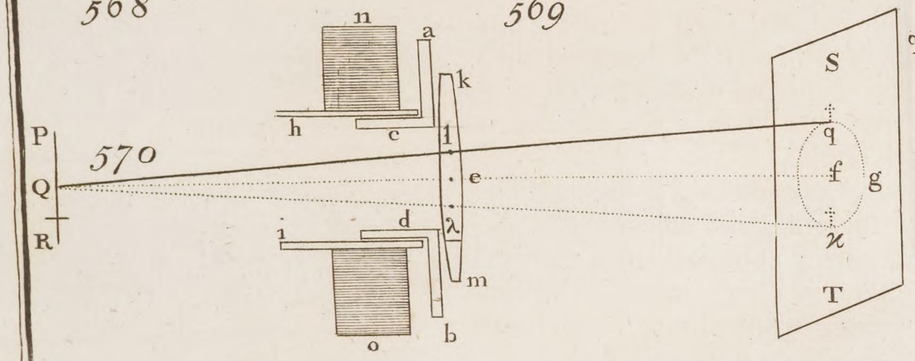
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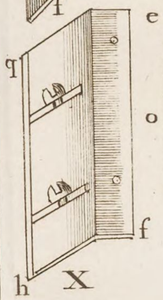


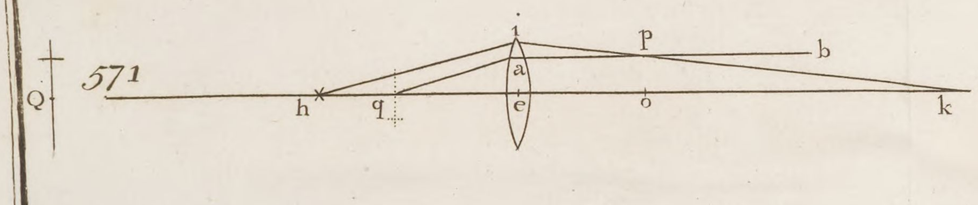
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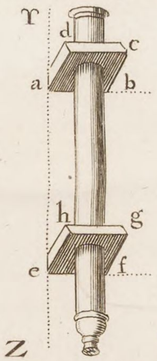


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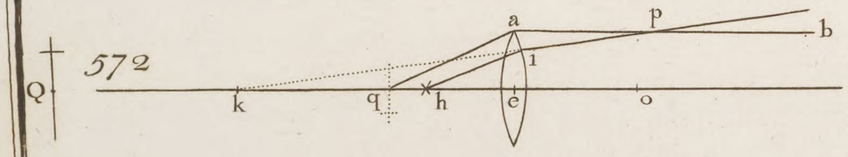




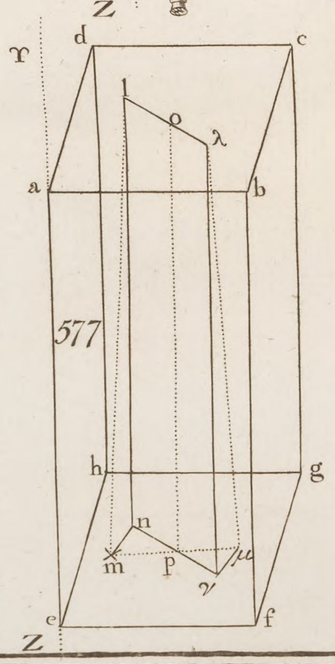
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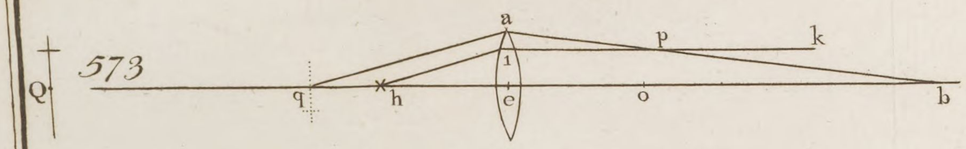
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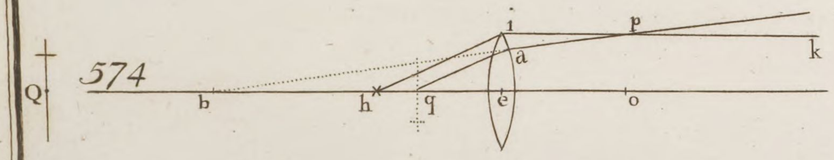
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this glaſs near a window and the ſpeculum at a diſtance on the ſide of the room, being it ſelf and every thing about it as much in the dark as can be. The light of the window reflected from the globules of mercury, appearing as ſo many ſtars, ſerves inſtead of the ſmall holes, with this advantage, that the reflection from the metal may be very near at right angles.

To correct the figure of the metal

804. If the figure of the metal appears not ſatisfactory, the hones muſt be workt with the braſs tool and water for 2 or 3 minutes with the motion &c. firſt directed[a]; then work the metal on them with the like motion, and ſuch length of the ſtroke as may carry the edge of it about $\frac{1}{6}$ or $\frac{1}{4}$ of its diameter beyond that of the hone pavement each way; carry it likewiſe by turns to the right and left to about the ſame diſtance. Continue this about 5 minutes, not preſſing the metal down to the hones with any more than its own weight, and obſerve that the oftener the mud is waſhed away the more truly ſpherical the figure of the ſpeculum will generally be: but the leaving a little more of this mud on the ſtones has ſometimes ſeemed to give the metal a parabolick figure. I have likewiſe given it the ſame by concluding with a kind of ſpiral motion of the center of the metal, near the circumference of the hones, in the manner repreſented in the figure, for about half a minute.

a Art. 794.

Fig. 567.

Signs of a bad metal.

805. If after ſeveral tryals the metal appears to have always the ſame kind of defect, and anſwering to the ſame particular part of the metal, it is a ſign of a different hardneſs in its ſeveral parts, which will make it very difficult to bring that ſpeculum to perfection. In working the tool or metals on the hones, there will often appear little ſpots in them, much blacker and harder than the reſt; theſe muſt be dug out as faſt as they appear.

To poliſh the metal.

806. When the figure is to your mind, you may proceed to finiſh the poliſh on the ſarcenet with very little putty, and that diluted with a great deal of water. Before you put the putty and water on it, obſerve by holding it very obliquely between your eyes and the light, if it have any liſts or ſtripes acroſs it, which appear more gloſſy than the reſt. If it be ſo, let the motion of the metal in poliſhing be directly athwart theſe liſts and not along with them, nor ever circular. In other reſpects you may obſerve the ſame directions as were before given for its motion on the hones; not forgetting, after every 15 or 20 ſtrokes, to turn it on its axis about $\frac{1}{12}$ or $\frac{1}{16}$ of a revolution. As the poliſher grows dry you will find the metal ſtick to it more and move ſtiffer; at which time it both poliſhes faſter and with a better gloſs: only take care that it grows not ſo dry as for the metal to take hold of the ſarcenet and cut it up; or for the pitch and putty to fix in little knobs here and there on it: which if it happen will preſently ſpoil the figure. As faſt therefore as the ſarcenet appears to be growing dry at any of its edges, touch the place with the end of a feather

dipt

dipt in clean water: you may uſe the ſame putty at leaſt half an hour. As often as you change it waſh the old clean away, and rub the new about firſt with your bruiſer, to ſee if there be any gritty or groſs particles in it, and rub them away for fear of ſcratching the metal; then laying down the edge of the ſpeculum a little way on the edge of the poliſher, where it is well covered with water, ſlide it on to the middle, and then proceed. The leſs putty you uſe at a time the ſlower the work will advance, but if you uſe too much, it will ſpoil a little of the figure round the edges. It will not want any conſiderable force to preſs it down; but if it be of 5 or 6 inches diameter or more it will be very laborious to go through the poliſh without ſome kind of machine. One ſomething like that deſcribed by Mr. *Huygens* in the aforenamed treatiſe*, may do very well; eſpecially if there be added to it ſome contrivance to hinder the metal's turning irregularly on its axis, and to give the workman a command of it in regard to the lateral motion.

* Art. 774.

CHAPTER III.

How to center an object-glaſs.

807. A CIRCULAR object-glaſs is ſaid to be truly centered, when the center of its circumference is ſituated in the axis of the glaſs[a], and to be ill centered when the center of its circumference lies beſide the axis. Thus let *d* be the center of the circumference of an object-glaſs *abc*; and ſuppoſe *e* to be the point where its axis cuts its upper ſurface. If the points *d* and *e* do not coincide, the glaſs is ill centered. Let *afg* be the greateſt circle that can be deſcribed about the center *e*, and by grinding away all the margin without this circle, the glaſs will become truly centered. Now the center *e* which lyes in the axis of the glaſs, may be found by ſeveral methods, but I prefer this that follows.

a Art. 38. Fig. 568.

Fig. 569.

808. Let a couple of ſhort cylindrical tubes be turned in wood or braſs, and let the convexity of the narrower be ſo exactly fitted to the concavity of the wider as juſt to turn round in it, with eaſe but without waddling; and let the planes of the baſes of the tubes be exactly perpendicular to their ſides. Place the baſe of the narrower tube upon a ſmooth braſs plate or a wooden board of an equal thickneſs, and with any ſharp pointed tool, deſcribe a true circle upon the board round the outward circumference of the baſe; and upon the center of this circle, to be found when the tube is removed, deſcribe a larger circle upon the board. Theſe two circles ſhould be ſo proportioned, that the one may be ſomewhat greater and the other ſomewhat ſmaller than any of the glaſſes intended to be centered by them. Then having cleared out all the wood within the inner circle, put the end of the tube into this hole, and there faſten it with glew,

glew, ſo that the baſe of the tube may lye in the ſurface of the board: then having fixt the wider tube very firmly in a hole made in a window-ſhutter, and having darkened the room, lay the glaſs to be centered upon the board fixt to the narrower tube; and having placed the center of it as near as you can gueſs over the center of the hole, fix it to the board with two or three lumps of pitch, or ſoft cement, placed at its circumference. Then put the narrower tube into the wider as far as it can go, and fix up a ſmooth ſkreen of white paper to receive the pictures of objects that lye before the window; and when they appear diſtinct upon the ſkreen, let the inner tube be turned round about its axis; and if the center of the glaſs happens to be in this axis, the picture will be perfectly at reſt upon the ſkreen; if not, every point of it will deſcribe a circle. With a pencil mark the higheſt and the loweſt places of any one circle, deſcribed by ſome remarkable point in that part of the picture which appears moſt diſtinct; and when this point of the picture is brought to the higheſt mark, ſtop the circular motion of the tube, and keeping it in that poſition depreſs the object-glaſs till the point aforeſaid falls exactly in the middle between the two marks. Then turn the tube round again, and the point of the picture will either reſt there or will deſcribe a much ſmaller circle than before, which muſt be reduced to a quieſcent point by repeating the ſame operation. Then I ſay the center (of refraction [a]) of the glaſs will lye in the axis of the tube, and by conſequence will be equidiſtant from the circumference of the large circle deſcribed upon the board fixt to it. Now to deſcribe a circle upon the glaſs *fgh* about its center of refraction, let a long ſlender braſs plate *acb* be bent ſquare at each end, as repreſented in the figure, leaving a piece in the middle equal in length to the diameter of the large circle *adbe* that was deſcribed upon the board; and let the ſquare ends of the plate be filed away ſo as to leave a little round pin in the middle of each. Then having laid it over the glaſs, along any diameter of the large circle *adbe*, make two holes in the board to receive the pins *a*, *b*; and find the center of this circle upon the long plate; and with this center *c* deſcribe as large a circle as you can, upon the glaſs underneath, with a diamond-pointed compaſs; and grind away all the margine as far as this circle *fik*, in a deep tool for grinding eye-glaſſes; and then the glaſs will be truly centered. If the pitch or cement be too ſoft to keep the glaſs from ſlipping, while the circle is deſcribing, it may be fixt firmer with wax or harder cement.

[a] Art. 228.

Fig. 569.

809. To ſhew the reaſon of this practice, the 570th figure repreſents a ſection of the object-glaſs *klm*, of the board *ab*, and of the tubes *cd* and *hi*, and of the window-ſhutter *no*. Imagine the plane of this ſection, or of the ſcheme, to paſs through *e*, a point in the glaſs which keeps its place while the reſt are turning round it, by the motion of the tube; let it alſo paſs through *l* the center of refraction in the glaſs, and cut an object in

The reaſon of this practice. Fig. 570.

the line PQR; then let a pencil of rays flowing from any Q be collected to the focus q upon the ſkreen ST; and the points Q, l, q will be in a ſtraight line deſcribed by the axis, or principal ray, of the pencil [a]. Draw Qef cutting the ſkreen in f; and while the tube is turning round, the line Qlq will deſcribe a conical ſurface whoſe axis is the fixt line Qef; and therefore the focus q or image of the point Q will deſcribe a circle qgx about f, to be found upon the ſkreen by biſecting the interval qx between the higheſt and loweſt points of the circle. Now as f is the center of this circle, ſo e is the center of another circle deſcribed by l; therefore by depreſſing the glaſs kl along the ſurface of the board ab till the image q falls upon the mark f, the point l will be depreſſed to e the center of motion, and then it will be in the axis of the tube, and conſequently equidiſtant from the circumference of the circle deſcribed upon the board ab; and here it is plain that the image q will be at reſt in the point f.

a Art. 228.

810. It is not neceſſary to the accuracy of the practice that the point Q ſhould be in the axis of the glaſs. For in fig. 181, if the glaſs KLM be turned about its axis QLq, the image p of any collateral point P will remain at reſt; becauſe the points P, L are at reſt, and the axis PLp of the oblique pencil is a ſtraight line.

811. The chief advantage of having a glaſs well centered is this, that the rays coming through any given hole or aperture whoſe center coincides with the axis of the glaſs, will form a diſtincter image, than if that center lay beſide the axis; becauſe the aberrations of the rays from the geometical focus of the pencil, are as the diſtances of their points of incidence from the center of refractions in the glaſs [b].

b Art. 325.

To examine how well an object glaſs is centered.

812. If the picture be received upon the unpoliſhed ſide of a piece of plane glaſs, inſtead of the paper ST, its motion may be diſcerned more accurately by viewing it from behind through a convex eye-glaſs; as in a teleſcope where croſs-hairs are uſually ſtrained over a hole put into the place of the rough glaſs. Therefore as object-glaſſes are commonly included in cells that ſkrew upon the end of the tube, one may examine whether they be pretty well centered, by fixing the tube, and by obſerving while the cell is unſkrewed whether the hairs keep fixt upon the ſame lines of an object ſeen through the teleſcope.

To place the croſs hairs in the focus of a teleſcope.

813. In the application of teleſcopes to aſtronomical inſtruments and many other purpoſes, it is abſolutely neceſſary to fix the plane of the croſs hairs exactly upon the plane of the picture of an object; which may eaſily be done from a knowledge of the following properties. Firſt let the interval between the two convex glaſſes of the teleſcope be adjuſted to ſhew an object diſtinctly; and if the hairs appear confuſed, they will ſeem to dance upon the object while the eye moves ſideways; and in dancing if they ſeem to move the ſame way as the eye does, they lye behind the picture of the object, but if they move the contrary way, they

lye

lye before it; and muſt be removed accordingly till they appear diſtinct; and then they will alſo ſeem fixt upon the object notwithſtanding the motion of the eye. Secondly let the interval between the hairs and the eye-glaſs be firſt adjuſted, till the hairs appear diſtinct; then if the object appears confuſed, it will alſo appear to dance while the eye moves ſideways; and in dancing if it moves the ſame way as the eye does, its picture is behind the hairs; if the contrary way, its picture is before them: and to bring it to the hairs, either the object-glaſs muſt be moved, or elſe the hairs and eye-glaſs both together. In both theſe caſes it is the confuſed object (for the hairs may alſo be called ſo) that ſeems to move, and the diſtinct one to ſtand ſtill; as in viſion with the naked eye. For to a perſon in motion, ſuppoſe he be walking, any object appears fixt that he fixes his eyes upon and ſees diſtinctly, while the reſt that are nearer or farther off, appear confuſed and in motion; the reaſon of it is too obvious to need an explanation. But to ſhew it in the teleſcope, let h be the interſection of the croſs-hairs, and hik a pencil of rays flowing from it; which after refraction through the eye-glaſs eai, belong to the focus k, either at a finite or an infinite diſtance. Draw he, the axis of this pencil, cutting the object in Q and its picture in q; and let the emergent rays of the pencil qab, flowing from q, cut the emergent rays of the former pencil in the points p, and belong to the focus b, either at a finite or an infinite diſtance. Now if the eye be placed at any point o in the common axis of theſe pencils, the points h, Q will both appear in the ſame direction oe; but if the eye be moved ſideways from o to p, the point Q will appear in the direction pa*, and the point b in the direction pi. And from hence the reaſon of the foregoing caſes will be ſufficiently manifeſt, by attending to the figures. Laſtly while the focuſes h, q are disjoined, the mutual inclination of the emergent rays in one pencil, muſt be different from the mutual inclination of the emergent rays in the other; and ſo the humours of the eye cannot be adapted to collect the rays in both pencils to two diſtinct points. If one be diſtinct the other will be confuſed and in a different part of the retina; (except when the eye is in the axis;) but when the focuſes h, q are united, the focuſes k, b of the emergent rays will alſo be united; and conſequently the coinciding rays of both pencils will be united in the ſame point of the retina, wherever the pupil of the eye be placed; and therefore the correſponding points of the object and croſs-hairs will appear fixt together without any parallax.

Fig. 571 to 574.

* Art. 102.

814. When the place of the hairs is thus determined, it may be of uſe to meaſure their diſtance from the object-glaſs; which is the exacteſt way of finding its focal diſtance if the object be very remote. And to keep this diſtance always the ſame whenever the teleſcope is uſed, it is convenient to have marks or ſtops at the end of each joint of the tube. For then whatever eye-glaſs be applyed, the object and hairs will both appear diſtinct

The focal diſtance thereby determined.

ſtinct at the ſame time, and without parallax. Inſtead of hairs the fineſt ſilver wires are now made uſe of, but are ſtill called hairs.

The line of ſight what. a Art. 228.

815. A line drawn from the interſection of the hairs through the center of refractions [a] in the object-glaſs, whether it coincides with the axis of the glaſs or is inclined to it, is called the line of Collimation or line of ſight. Becauſe this line produced falls upon the object in that point, whoſe image falls upon the interſection of the hairs: and therefore the ſtraight ray that deſcribes this line, anſwers to the viſual ray by which we take aim at an object with plain ſights. Hence when the object-glaſs and hairs are firmly fixed in a ſtrong tube or to a ſtraight ruler, it is manifeſt that the line of ſight is as immutable with reſpect to the tube, as if two little holes or plain ſights were ſubſtituted in the places of the interſection of the hairs and of the center of refractions in the object-glaſs.

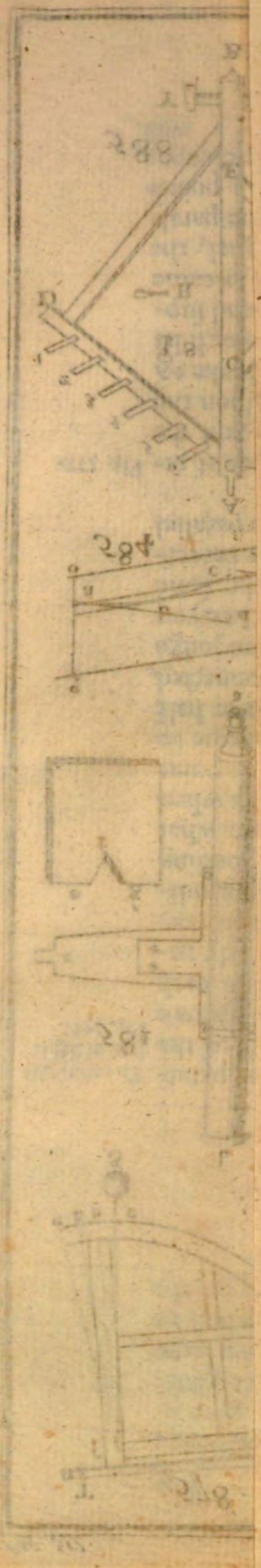
The mechaniſm for moving the croſs-hairs.

816. In order to ſet the line of ſight parallel to a given line upon the plane of an inſtrument, the object-glaſs muſt be firmly fixt, and the ring or plate that carries the croſs hairs muſt have two gradual motions in its own plane by two ſkrews at right angles to each other; for by this means the interſection of the hairs may be moved to any given point in that plane. Theſe motions are effected by 3 braſs plates laid over one another. The uppermoſt, having a circular hole in it, over which the hairs are ſtrained, ſlides over the middlemoſt in the direction of an oblong hole cut in it, whoſe breadth is ſomewhat greater than that of the hole above it; and theſe two together ſlide ſideways over the undermoſt plate in which there is a larger oval hole. I ſhall deſcribe theſe plates more particularly in a contrary order. On each ſide of the oval hole in the middle of the

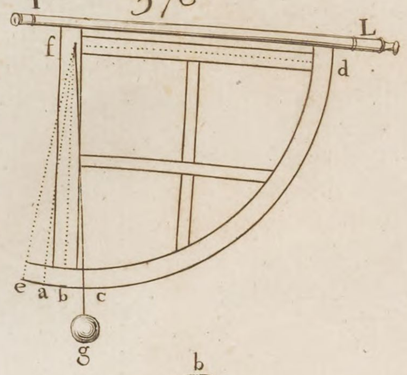
Fig. 575.

plate R laſt mentioned, two braſs ledges m, n are firmly riveted to receive the dove-tailed ſides of the plate S; and the contiguous ends of both theſe plates are turned up ſquare at b and e; and through a hole b, in the middle of the part turned up in the larger plate R, there works a pretty thick ſkrew abc, whoſe fore end c being filed to a neck, goes through a hole e in the lip of the other plate S; and in the end of the neck c there is made a ſmall ſkrew-hole to receive a ſkrew-pin d; ſo that by turning the ſkrew abc with a kind of a watch-key, the plate S is moved backwards or forwards between the ledges m, n. The figure T repreſents two more ledges o, p that are to be riveted upon the plate S; theſe ledges are part of the plate T turned up at right angles to them, in which part there is the like contrivance of a ſkrew $abcd$ to move a third plate V between the ledges o, p, at right angles to the former motion. The ſilver wires are ſtrained over the hole in the plate V by four ſmall peggs, that fix them in four little holes. The other end of the plate R, oppoſite to the part b that carries the ſkrew, is bent ſquare the contrary way to the part b; or which anſwers the ſame purpoſe, one ledge ef of the plate X bent ſquare, is riveted to the backſide of the plate R at the end oppoſite to the ſkrew b;

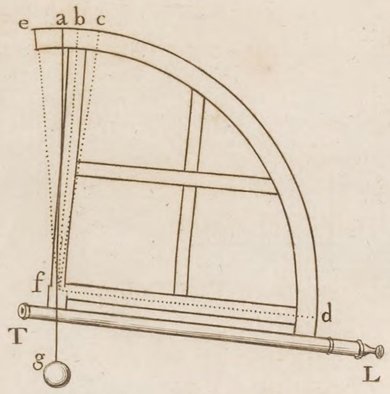
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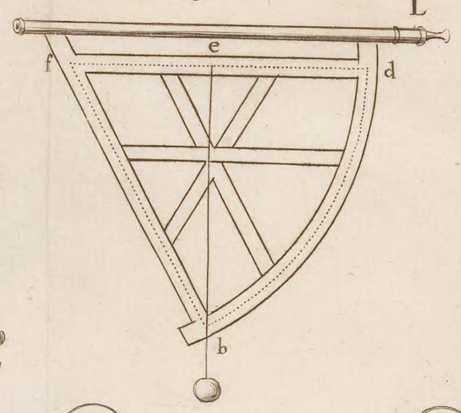
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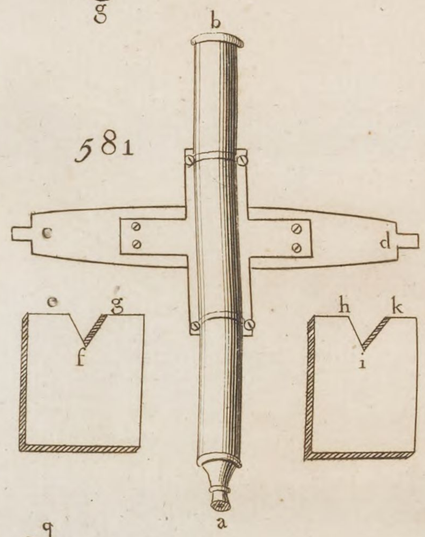
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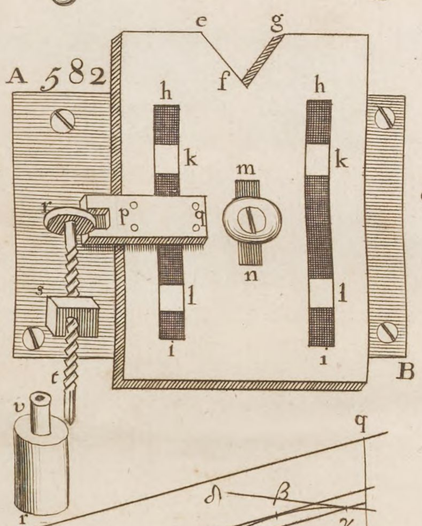
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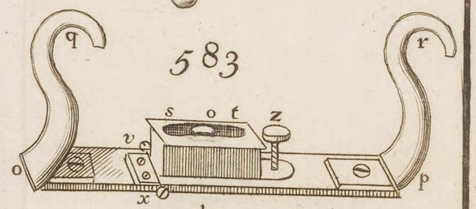
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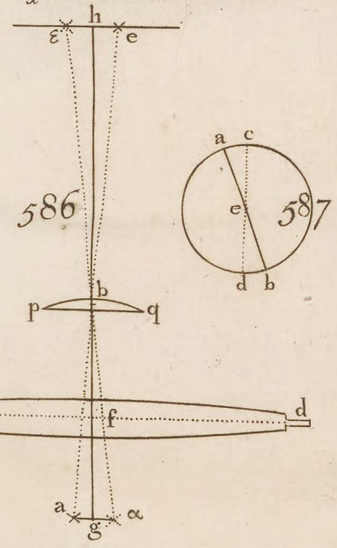
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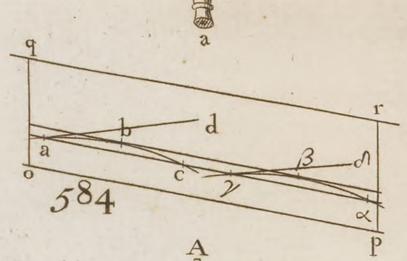


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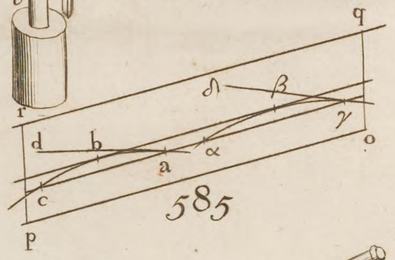


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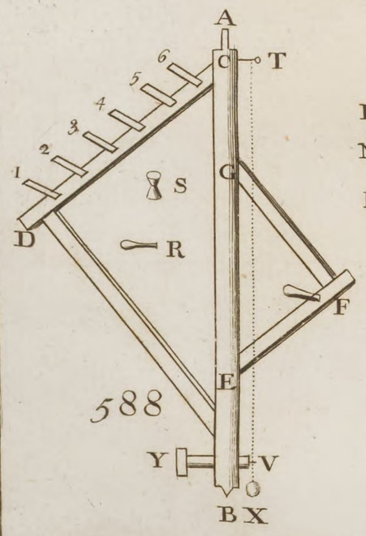
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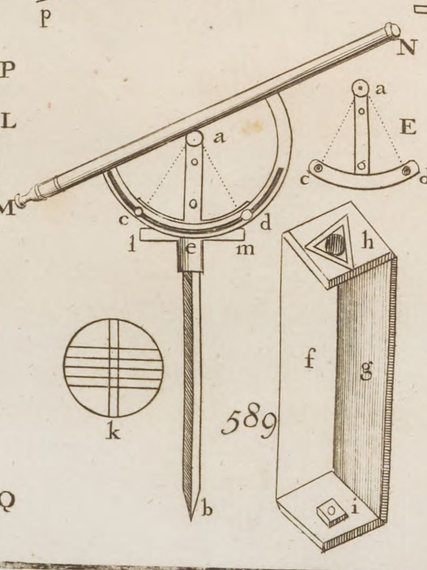


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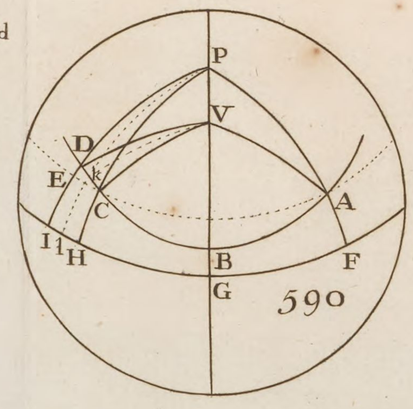




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and its other ledge *gh* is ſkrewed to the ſide of the tube of the teleſcope; and the necks of the ſkrews go through long ſlits in this ledge, to give liberty of placing it accurately at the due diſtance from the object-glaſs: and for the purpoſe of letting this braſs-work into the tube, two large ſlits muſt be cut in two contiguous ſides of it: one of which may beſt be covered with a thin piece of horn, to admit the light of a candle upon the hairs in obſerving ſmall ſtars in the night time.

CHAPTER IV.

Rectification of teleſcopick ſights.

817. To make the line of ſight through a moveable teleſcope parallel to a given line ΥZ upon a fixt plane; let the ends of the tube of the teleſcope, whether ſquare or cylindrical, be put through two holes in two ſquare plates *abcd*, and *efgh*, made exactly equal to each other, and ſo fixt to the tube that the ſides of the one may be exactly parallel to the ſides of the other; which is eaſily done by applying their corners *a*, *e* to the given line ΥZ and by drawing two lines *ai*, *ek* perpendicular to it, upon the given plane, and by making all the correſponding ſides as *ab*, *ef* coincide with theſe perpendiculars. Then obſerve what point of a remote object is covered by the interſection of the hairs when the corners *a*, *e* touch the given line ΥZ, and likewiſe what other point is covered by them when the oppoſite corners *c*, *g* touch the ſame line in the ſame places, that is when the teleſcope is turned upſide down or half round. Then conceiving theſe two points of the object to be connected by a ſtraight line, move the croſs-hairs by the two ſkrews till you judge their interſection biſects that line; and by repeating the ſame practice you may ſoon bring the interſection of the hairs to cover one and the ſame point of the object, when the oppoſite corners of the ſquares are ſucceſſively applyed to the line ΥZ; and then the line of ſight will be parallel to it.

To rectify the line of ſight. Fig. 576.

To ſhew the reaſon of this practice, we may ſuppoſe the center of refraction in the object-glaſs to be any point *l* of the ſquare *abcd*, and the interſection of the hairs to be any point *m* of the ſquare *efgh*. Upon the plane of the firſt ſquare, and through its center *o*, draw $lo\lambda$, and take $o\lambda$ equal to *ol*; alſo upon the plane of the ſecond ſquare, and through its center *p*, draw $mp\mu$, and take $p\mu$ equal to *pm*. Joyn *lm* and $\lambda\mu$, and ſuppoſing *ln* and $\lambda\nu$ parallel to the axis *op*, join *mn*, $n\nu$, $\nu\mu$. Then becauſe the reſpective ſides about the equal angles *mpn*, $\mu p\nu$ are made equal, the lines *mn*, $\mu\nu$ oppoſite to them are alſo equal and parallel. Now the parallel lines *nl*, *po*, $\nu\lambda$ produced will fall upon a remote object in 3 points ſo cloſe together as to appear like a ſingle point through the teleſcope; Fig. 577.

and

and conſequently the planes of the parallel triangles *l m n*, λ μ ν produced will cut the ſame object in two parallel lines ſo cloſe together as to appear but one line through the teleſcope; and ſince the angles *m l n*, μ λ ν are equal, the interſection of the hairs, now at *m* and then turned half round to μ, will cover two points in that line equidiſtant from the point abovementioned, and on oppoſite ſides of it; therefore by removing the interſection from *m* to *n*, it will appear to biſect the interval between thoſe two points; and then the line of ſight *n l* will be parallel to the axis *p o*, and to the ſides of the parallelopiped, and alſo to the given line *Y Z*.

The application.

818. A teleſcope thus prepared may be uſeful upon ſeveral occaſions; as if it be required to rectify the hairs in a teleſcope fixt to any inſtrument, ſo as to ſet the line of ſight parallel to a given line upon the plane of the inſtrument. Apply the corners of the ſquares of the teleſcope abovementioned to the given line, and obſerving what point of a remote object is covered by its croſs hairs, move the croſs hairs of the fixt teleſcope till they cover the ſame point of the object, and the buſineſs is done.

Rectification of a quadrant.

819. But the teleſcopick ſights of quadrants and ſectants, whoſe planes may be readily placed in any given poſture, may be rectified by a plumb-line. I will here tranſcribe an account of theſe rectifications from Mr. *Molyneux*'s Dioptricks p. 238. "I come now to the rectification of theſe ſights on quadrants and ſextants, for taking angles. This may be done either before or after the diviſion into degrees, &c. are made on the limb of the

Fig. 578, 579.

quadrant. If it be done before, then we ſuppoſe the teleſcope *T L* fixt to the quadrant, which we ſuppoſe continued a little farther than the fourth part of a circle. Chooſing then an object pretty near the horizon; let us look through the teleſcope, in the uſual poſture of obſervation, and obſerve the point in the object marked by the croſs-hairs; and at the ſame time we are to note moſt nicely the point *c*, which the plumb-line *f c g*, hung from the center *f* of the quadrant, cuts on the limb. Then we are to invert the quadrant into the poſture of fig. 579, (which is eaſily done by the uſual contrivances for managing great quadrants, by toothed ſemicircles and endleſs ſkrews) keeping ſtill the teleſcope *T L* nighly upon the ſame height from the ground as before, unleſs the object we look at, be ſo far diſtant, that the breadth of the quadrant ſubtends but an inſenſible angle. But yet for certainty, it is better to keep the teleſcope, as it is ſaid, upon the ſame height from the floor; then direct the teleſcope *T L*, that the croſs-hairs may cover exactly the ſame point in the object, as before in the poſture of fig. 578. And hanging now the plumb-line *a f g* on the limb of the quadrant; let us remove it to rnd fro, till we find out the exact point *a*, from which the plumb-line being hung, ſhall moſt nicely hang over the center of the quadrant *f*. Then carefully marking the point *a*, let us divide the arch *c a* into two equal parts in *b*; and drawing *b f*, the point *b* is the point from which we are to begin the diviſions

of

of the quadrant: and the line of collimation through the teleſcopick ſight, ſtands exactly at right angles to the line *bf*. So that the quadrant *bfd* being compleated and divided, the ſaid line of ſight through the teleſcope runs exquiſitely parallel to the line *fd*.

820. In the next place, ſuppoſing the quadrant *bfd* truly compleated and divided; and that we deſigned to fix thereto the teleſcopick ſight *TL*; ſo that the line of ſight may run exactly at right angles to the line *bf*, or parallel to the line *df*. We are to do as in the foregoing praxis. And if in dividing the arch *ac*, we find its half exactly coincident with the point *b*, we have our deſire. But if it differs from the point *b*, and falls between *b* and *d*, then the line of collimation through the teleſcope ſtands at an obtuſe angle with the line *bf*; and the inſtrument errs in exceſs: if this half arch fall without *b* and *d*, then the line of collimation makes an acute angle with the line *bf*; and the inſtrument errs in defect. And by often tryals, we are to remove the croſs-hairs within the tube, ſo much, as is requiſite to correct this error. And when we have thus rectified them to their due place, there they are to be ſtrongly fixt. Or elſe, in obſervations taken by this inſtrument, we are to make allowance for this error; by ſubtracting from (if it be in exceſs) or by adding to (if it be in defect) each obſervation ſo much, as we find the error to be.

821. The reaſon of this rectification is moſt plain; for it is manifeſt, that *cfd* wants of a full quadrant, as much as *afd* exceeds a quadrant. So the difference of the two arches in the two poſtures being *ac*; half this difference *bc* added in fig. 578, or *ab* ſubtracted in fig. 579, makes *bd* a compleat quadrant.

822. If we find our inſtrument errs in taking angles, and we deſire to know the error more nicely, than perhaps the diviſions of the inſtrument it ſelf will ſhew it, we are to do thus. Suppoſing the quadrant *bfd* already accurately divided, and that the plumb-line plays over the point *c*; and upon the inverſion of the inſtrument, we find that before we can get it to play exactly over the center *f*, we muſt hang it over the point *e*; ſo that the arch *eb* exceeds *bc* by the arch *ea*; it is plain that the angle *efa* is the error of the inſtrument: for had the plumb-line hung over *a*, and over the center *f* in this latter poſture, the inſtrument had been exact; becauſe *a* is as much on one ſide *b*, as *c* is on the other ſide *b*. Wherefore *efa* being the angle, by which our inſtrument errs in obſervation, let us turn the inſtrument into the uſual poſture of obſervation, as in fig. 578 and hanging the plumb-line on the center *f*; let us bring it to play nicely on the point *e*, and obſerve what diſtant object is covered by the croſs-hairs: then let us bring it to play exactly on the point *a*, and obſerve likewiſe what diſtant object is pointed at by the teleſcope-hairs. Laſtly, by a large teleſcope and micrometer, let us meaſure the angle between theſe two objects, and we ſhall have the angle of error much more nicely, than

it is poſſible the angle *efa* ſhould be given by the diviſions on the limb of the quadrant *ea*. And thus much for adjuſting a quadrant.

Rectification of a ſextant. Fig. 580.

823. A ſextant is rectified in like manner; if we conſider that if from the center *f* to the beginning of the diviſions *d* there be drawn the radius *fd*; and it be divided equally in *c*; and from *c* there be ſupended the plumb-line *cb*: when the plumb-line hangs over the 60th degree at *b*; then the line *fd* lies horizontal: and conſequently, if the line of collimation through the tube be parallel to *fd*, this line alſo lies horizontal. To try which, whilſt the ſextant ſtands in this poſture, obſerve the object marked by the croſs hairs; then invert the ſextant; and over the point *b* hang the plumb-line; and when from the point *b* the plumb-line hangs over the middle point *c*, then again is the line *fd* horizontal in this poſture. Mark then, whether the croſs hairs cover the ſame object as before; if they do, then the line of collimation is parallel to *fd*; if they do not, but the point in the object marked in this latter poſture be higher than the point marked in the firſt poſture, the inſtrument errs in exceſs; if it be lower, the inſtrument errs in defect. And either we are to remove the croſs hairs, till we bring all to rights, and there fix them; or by the methods before laid down in the rectification of the quadrant, we are to find the quantity of this erroneous angle, and to allow for it in obſervation.

Rectification of a moveable ſight.

824. In inſtruments furniſhed with two pair of teleſcopick ſights, one on a fixt arm, and the other on a moveable arm (by the ancients termed an *Alidade*); it is eaſy rectifying the ſights on the moveable arm thus. After the ſights on the fixt arm are rectified by what foregoes; bring the index of the moveable arm to the beginning of the diviſions on the limb of the inſtrument, be it quadrant or ſextant, &c. it is then manifeſt, that the line of collimation through the moveable teleſcope (if it be right) ſhould lye parallel to the line of collimation through the fixt teleſcope. Obſerve therefore, whether the croſs hairs in both teleſcopes do at the ſame time cut the ſame ſtar, or fall on the ſame point in an object diſtant three or four miles. If they do, then the moveable teleſcope agreeing with the fixt, and the fixt being ſuppoſed rectified to the diviſions on the inſtrument, the moveable is right likewiſe. But if the hairs in the moveable teleſcope do not agree in marking the ſame point with the croſs hairs in the fixt teleſcope; then the hairs in this moveable teleſcope are to be removed (by whatever contrivance there is for that purpoſe) and brought to rights, and there fixt.

825. There are other methods propounded for rectifying teleſcopick-ſights on other ſorts of inſtruments, by means of obſervations towards the zenith, as our former methods have been imployed towards the horizon. But it is ſufficient here to lay down only what foregoes, as being of the greateſt and moſt frequent uſe: referring for the others to M. *Picard*'s Trea-

Treatiſe of the *meaſure of a degree of a great circle of the Earth*, publiſhed at the end of the *Memoirs of a Natural Hiſtory of Animals* &c. by the Royal Academy at *Paris*; tranſlated into Engliſh and printed at *London* 1688. fol.

CHAPTER V.

A meridian teleſcope and its uſes.

826. A teleſcope *ab* fixt at right angles to an axis *cd*, and turned about it in the plane of the meridian, is called a meridian teleſcope. The vulgar uſe of it is to correct the motion of a clock or watch, by obſerving day after day the exact time when the ſun or a ſtar comes to the meridian. It ſerves alſo for other uſes to be mentioned hereafter. A meridian teleſcope what. Fig. 581.

827. The axis *cd* conſiſts of a ſtrong braſs plate, broader in the middle than towards the ends, having another long plate placed edgeways and ſoldered along the middle of the backſide of it from one end to the other. This plate is alſo broader in the middle than towards the ends to prevent its ſpringing. To the ends of theſe plates two ſolid pieces of braſs are firmly jointed and ſoldered; and the whole being placed in a turning lathe, the ſolid ends are turned truly cylindrical, to an equal thickneſs, and are alſo well ſmoothed and poliſhed with oyl and proper powders. Flat upon the broader plate of the axis, there lyes another braſs plate, in the ſhape of a croſs; one bar of it being parallel, and firmly ſkrewed, to the axis. The ends of the other bar, being turned up at right angles, have ſemicircular notches filed in them, to receive the cylindrical tube of the teleſcope; which is made of braſs to prevent its warping; and is firmly fixt in theſe notches by two half collars of braſs, which go over the tube and are ſkrewed to the ends of the ſemicircular notches; as repreſented in the figure. The cylindrical ends of the tranſverſe axis *cd*, are to be placed upon two angular notches, *efg* and *hik*, filed in two thick braſs plates, and well planed and poliſhed, for the axis to turn very ſmoothly upon them. In order to bring the axis *cd* exactly perpendicular to the plane of the meridian, each of the notched plates are contrived to be moveable by a ſkrew, one upwards or downwards and the other ſideways; by ſliding them upon the ſurfaces of two other braſs plates immoveably fixt to two poſts or freeſtone pillars. The teleſcope and tranſverſe axis deſcribed.

828. In order to move the notched plate *efg* up or down, two long upright ſlits *hi* are cut through it; in each of which, at *k* and *l*, are two ſquare ſtuds of braſs, firmly riveted to the fixt back-plate *AB*; being exactly as broad as the ſlits that ſlide by them. In the middle between them there is alſo a ſhorter parallel ſlit *mn*, through which a broad headed ſkrew- How the ſupporters are moved. Fig. 582.

pin is ſkrewed into the back-plate, having a round ſpringing plate under the head of it; thus the two plates are preſſed together or relaxed, according as the notched plate is to be fixt or moved. To perform this motion gradually, *p q* repreſents a ſmall piece of another braſs plate, laid upon the fore-plate and riveted to the middle of one of its upright ſides. In the end *p*, which projects over this ſide, there is a ſmall ſquare notch, which receives the circumference of a ſmall circular plate *r*, like a ſkrew-head, fixt perpendicular to a long ſkrew *r s t*, placed parallel to this upright ſide of the notched plate. This ſkrew works round in a concave ſkrew in the middle of a thick braſs ſtud at *s*, firmly riveted to the back-plate; and being turned by a hollow key *v*, put upon the ſquare end of it, it gradually raiſes or depreſſes the notched plate *e f g*. Which is fixt more firmly at its proper height by turning the middle ſkrew at *o* abovementioned.

The angular notch that ſupports the other end of the tranſverſe axis is moveable ſideways by the ſame contrivance: as will appear by ſuppoſing the notch *e f g* to be cut in the ſide oppoſite to the ſkrew *r s t*, and this ſide to be turned upwards.

Spirit level. Fig. 581, 583.

829. The tranſverſe axis is to be placed horizontal upon the angular notches by a ſpirit level; conſiſting of a hollow glaſs tube *s t*, 5 or 6 inches long; not quite ſtraight, but as little incurvated as poſſible; and filled with ſpirit of wine (or any liquor that will not freeze or grow foul) all but a bubble of air; which by the laws of hydroſtaticks will always move to the higheſt place in the cavity of the tube. The tube with its convexity upwards, is placed in a long ſlender braſs box, made fit to receive it, and is faſtened in it by very hard cement. This box is laid lengthways upon a long ruler *o p*; and through the bottom plate of the box, a little produced, there paſſes a fine ſkrew-pin *z*, to be turned by the fingers; whoſe end, by working againſt the ſurfaces of the ruler below, gradually raiſes or depreſſes this end of the box, while it turns upon a ſmall axis *v x* near the bottom of the other end of it.

Upon the ends of the ruler *o p* two braſs plates *o q*, *p r* are fixed, whoſe planes are perpendicular to the length of the ruler; theſe plates being equal in length have their upper parts formed into circular hooks *q*, *r*, to be hung upon the cylindrical ends of the tranſverſe axis *a b*; the interval between the hooks being of a proper length to interpoſe them between the ſhoulders of the cylinders at the end of the axis and the braſs notches they reſt upon.

How to level the tranſverſe axis.

830. The frame *p o q r* being thus ſuſpended upon the tranſverſe axis, raiſe or depreſs the tube of the level by twiſting the knob of the ſkrew *z*, till you bring either end of the air bubble to reſt at any point *o*, towards the middle of the tube, and obſerve any mark upon the ſide of the tube that is oppoſite to it. Then take off the frame *p o q r*, and turning the ends of it contrary ways, hang it again upon the cylinders. And if the air bubble reſts exactly

exactly at the ſame mark as before, the tranſverſe axis is truly horizontal: if not, depreſs that end of the axis which lyes on the ſame ſide of the mark as the bubble does, or raiſe the oppoſite end, till the bubble returns about half way towards the mark. Then having made a new mark where it reſts, tranſpoſe the frame again, and repeat the ſame practice till the bubble reſts exactly at the ſame point in both poſitions of the frame. Inſtead of making marks, over againſt one end of the bubble, it is better to have a fine pointer, ſliding upon a wire along the top of the tube.

The reaſon of this practice. Fig. 584, 585.

831. Let a plane paſſing through the axis *qr* and the mark *a*, where the bubble reſteth, cut the tube of the level in the curve *abc*; and let a line *ac* drawn parallel to the axis *qr*, cut off the arch *abc*, and after the frame of the level has been tranſpoſed, the chord *ac*, of the arch *abc* will ſtill be ſituated ſome where in the ſame parallel to *qr* as before, ſuppoſe at $\alpha\gamma$. Now ſince the ends of the chords *ac*, $\alpha\gamma$ are turned contrary ways in the ſame line, it is plain that the mark *a*, which was the higheſt point in the arch *acb*, being now tranſpoſed to α, is become the loweſt point of the ſame arch in the ſituation $\alpha\beta\gamma$. Therefore the bubble cannot now remain at the mark α, but will aſcend to the higheſt point of the arch $\alpha\beta\gamma$; which will be γ, if the arch be circular. For then the tangents *ad*, $\gamma\delta$ to the points *a* and γ will be parallel to each other; and *ad* being horizontal, $\gamma\delta$ will be ſo too; and conſequently γ will be the higheſt point. Let $b\beta$, drawn parallel to the axis or to $a\gamma$, touch the arch $\alpha\beta\gamma$ in β; and it is evident the point β will be lower than γ. Therefore by depreſſing that end of the axis *qr*, which lyes on the ſame ſide of the mark α as the bubble does at γ, the point γ will alſo be depreſſed; and ſo the bubble will recede from γ towards the mark α till it comes to β, when β becomes the higheſt point; that is when the tangent $b\beta$ and the axis *qr* are parallel to the horizon: and then if the frame *qopr* be tranſpoſed back again, the bubble will reſt at the ſame point β as before. Now if the arches *abc*, $\alpha\beta\gamma$ be circular, *b* and β are exactly the middle points of the arches. But if the arches be not circular, the points *b* and β will not be in the middle, yet being at the greateſt diſtance from the chords *ac*, $\alpha\gamma$, they will be the points where the bubble will reſt when the axis is horizontal.

A caution to be obſerved.

832. It muſt be obſerved of theſe levels ſo ſuſpended, that if you gently turn the frame about the axis of ſuſpenſion, the air bubble will frequently change its place though the ſupporters of the axis keep fixt; either becauſe the plane of the curvity of the tube is not parallel to the axis, or becauſe the cavity of it is not cylindrical, or upon both accounts. It is therefore neceſſary to reduce the plane of the frame as nearly to the ſame vertical poſition as poſſible, which may be done either by a plumbet hanging down by the ſide of a perpendicular ruler fixt to the horizontal one *op* in fig. 583.; or by another tube of a ſpirit level fixt upon the ruler *op* at

right angles to it. But if the frame *poqr* vibrates freely upon the axis, this additional apparatus will be needleſs.

How to ſtay the teleſcope.

833. If the far end of the teleſcope be juſt made to preponderate, the elevation of it, for obſerving the appulſes of ſtars to the croſs hairs, may be gradually altered by a ſtring, tied to its near end, and wound about a pegg fixt below the teleſcope. The pegg may be made to turn with a proper degree of ſtifneſs between two ſemicircular notches preſſed gently together by a ſkrew-pin paſſing through both the pieces. Or the ſame deſign may be executed ſeveral other ways.

How to rectify the croſs hairs.

834. If after the teleſcope is turned upſide down, and the contrary ends of the tranſverſe axis are placed upon the ſame notches, you perceive that the ſame point of the object is covered by the interſection of the hairs in the focus of the teleſcope; it is certain that the line of ſight [a] is perpendicular to the tranſverſe axis. But if the interſection covers a different point of the object, the hairs muſt be moved by the key abovementioned, till they appear to biſect the line that joined thoſe two points of the object as near as you can judge; then by reverting the teleſcope to its former poſition you will find whether the hairs be exactly adjuſted.

a Art. 815.

The reaſon of it. Fig. 586.

835. For let the line *c d* be the tranſverſe axis which the teleſcope turns about, *a* the interſection of the croſs hairs, *b* the center of the refractions in the object-glaſs, and *a b* the line of ſight. Let *a b* produced meet a remote object in *e*; and the point *e* will appear in the teleſcope to be covered by the interſection *a*, becauſe the image of it falls upon *a*. Imagine a line *g h* drawn through the center *b* to cut the axis *c d* at right angles in *f*, the hair *a α* in *g* and the object *e ε* in *h*; and when the ends of the axis *c d* are tranſpoſed by reverting the teleſcope, the perpendicular *b g* will keep its poſition, (or be parallel to it,) but *b a* will be tranſpoſed into the poſition *b α* equally inclined to the contrary ſide of the perpendicular *b f g*. Produce *α b* till it meets the remote object in *ε*, and the croſs hairs at *α* will now appear to cover the point *ε*. But the perpendicular *g b h* biſects the interval *e ε* in *h*. Therefore by moving the interſection of the croſs hairs from *α* towards *g*, till you perceive the middle point *h* is covered by it, it is manifeſt that the line of ſight is now perpendicular to the tranſverſe axis *c d*; and conſequently upon reverting the teleſcope the ſame point *h* will be covered as before. Hitherto I have ſuppoſed the teleſcope to be turned about an axis *c d* lying in the ſame plane with the line of ſight; but if it turns about any other line parallel to *c d*, it is eaſy to underſtand that the motion of the line *g b h* will ſtill deſcribe a plane perpendicular to this tranſverſe axis. It is ſcarce neceſſary to be obſerved that the perpendicularity of the line of ſight to the tranſverſe axis, has no dependence upon its paſſing through the middle of the aperture of the object-glaſs, nor of being parallel to the ſides of the tube; nor is it required to coincide with the axis of the object-glaſs [b].

b Art. 810.

836.

836. To adjuſt the period of a clock, it is not neceſſary that the line of ſight through the teleſcope ſhould deſcribe the meridian. If it always deſcribes the ſame vertical circle at every tranſit of a ſtar, it is ſufficient. For the intervals of time between the ſucceſſive appulſes of a fixt ſtar to the ſame vertical, are equal to the intervals between its appulſes to the meridian. But leaſt the uncertainty of the air's refraction ſhould cauſe any difference in the times, it is better to have the vertical circle near the meridian; and to obſerve a ſtar near the equator, becauſe its tranſit is quicker. Now the line of ſight may be kept in the ſame vertical circle at every obſervation, by examining the level of the tranſverſe axis every time; and by obſerving alſo whether the croſs hairs always cover the ſame mark upon a diſtant object, which in the night time muſt be illuminated by a lantern. For then the line of ſight will deſcribe the ſame vertical, notwithſtanding any warping of the materials or even of the building to which it is fixt. Now we learn from aſtronomy that the period of the clock will be exactly equal to a ſolar day of a mean length, if the ſame fixt ſtar comes to the croſs hairs 3 minutes 56 ſeconds ſooner every ſubſequent night than the night before it. For the interval of time between the apparent appulſes of a fixt ſtar to the ſame point of the heavens, is the ſame as the period of the earth's motion about its axis, which is called a ſidereal day and is $3'.56''$ ſhorter than a ſolar day of a mean length; I ſay of a mean length, becauſe ſolar days, or the intervals between the ſun's appulſes to the meridian, are unequal. But the error in the clock's period will be diſcovered more exactly by comparing two remote obſervations rather than two ſucceſſive ones. For example ſuppoſe a ſtar comes to the croſs hairs at $9^h.30'.18''$ by the clock, and ſeven days after at $8^h.50'.24''$ by the clock; the difference of theſe times is $39'.54''$, and a ſeventh part of it is $5'.42''$; which ſhews that the diurnal period of the clock is $5'.42''$ longer than a ſidereal day; whereas a ſolar day of a mean length is but $3'.56''$ longer; and by conſequence the diurnal period of the clock is $1'.46''$ too long.

To rectify the period of a clock.

837. Mr. *Huygens* preſcribes a way of obſerving theſe tranſits of ſtars even without a teleſcope; by chooſing a proper place from whence you may ſee ſeveral of them inſtantly diſappear in paſſing behind high buildings; and by fixing a plate with a hole in it, to look through, of the bigneſs of the pupil; in order to bring it always to the ſame point[a].

Without a teleſcope.

a Horol. Oſcil. p. 13.

838. If the meridian teleſcope be placed without doors, or ſo near the roof of a houſe, as to have a proſpect both to the north and ſouth part of the meridian, through a ſlit cut in the roof; it may be brought exactly into the plane of the meridian by obſerving the tranſits of the circumpolar ſtars both above and below the pole. For the tranſverſe axis being placed horizontal as before, if the vertical circle deſcribed by the line of ſight coincides with the meridian, it will biſect the polar circles deſcribed by the ſtars; if not, it will divide them into two unequal parts. Having then

To find the meridian.

then obſerved by the clock the times of a ſtar's appulſes to the under and upper parts of the vertical deſcribed by the line of ſight; according as the time which the ſtar takes in deſcribing the eaſtern part of its orbit, is found to be longer or ſhorter than the time of deſcribing the weſtern part, it is plain that the teleſcope points weſtward from the meridian in the former caſe, and eaſtward in the latter; and muſt be altered accordingly till thoſe times come out equal. When the meridian is thus found a mark muſt be made upon a point of a diſtant object then covered by the croſs hairs. And at all times after, when any obſervation is to be made of a tranſit over the meridian, the line of ſight muſt be examined by this mark, and the croſs hairs muſt be brought to cover it; the tranſverſe axis being firſt examined and corrected by the level.

To find the apparent time.

839. The meridian line being thus determined, the apparent time of the day is ſoon found, by obſerving the ſun's tranſit over the meridian; that is by noting the times by the clock when the precedent and the ſubſequent edges of his body come to the croſs hairs. For then the middle time between them is known by the clock; and its diſtance from XII ſhews how much the clock is faſter or ſlower than the apparent time for that day. The apparent time being found, the true time will alſo be known, by conſulting an Equation Table.

And differences of right aſcenſion and declination.

840. By the ſame inſtrument the aſcenſional differences of any objects in the heavens may be found moſt accurately, by taking the times of their tranſits over the croſs hairs, and by converting the intervals of time into the correſponding arches of the equator. The differences of declination of two ſuch objects as will paſs over the aperture of the teleſcope when fixt, may alſo be found by the oblique hairs or by the inclined plates hereafter mentioned: and for theſe purpoſes it is more expedtious to uſe a clock whoſe period is adjuſted to a ſidereal day.

For night-obſervations it is neceſſary to illuminate the croſs hairs by the light of a candle tranſmitted through a piece of horn fixt in a hole made in the ſide of the teleſcope. And to give the teleſcope a proper elevotion in the day time for taking a known ſtar into its aperture, it is convenient to have a ſmall ſemicircle, fixt in the plane of the meridian, to one of the pillars that ſupports the tranſverſe axis; having its center in a point of the axis, and a radius fixt to the axis, or ſome index fixt to the teleſcope, that ſhall ſhew the degrees of its elevation pretty nearly.

Dimenſions of the inſtrument,

841. Our great aſtronomer Dr. *Halley*, before he was provided with a Mural Arch, hereafter deſcribed, made all his obſervations of the moon's right aſcenſions with an inſtrument of this ſort. His teleſcope is $5\frac{1}{2}$ foot long; and the tranſverſe axis is about an ell long; being ſurrounded with light braces of braſs, to keep it from ſpringing, and ſupported upon two freeſtone pillars, erected upon a very large ſtone laid deep in the ground. Nevertheleſs a good teleſcope only $2\frac{1}{2}$ or 3 foot long is ſufficient for obſerv-

obſerving tranſits to the exactneſs of a ſecond of time; and the tranſverſe axis need not exceed 2 foot, eſpecially if the ſpirit level be accurate.

842. If any one deſires to know to what degree of exactneſs the tranſverſe axis is placed horizontal by the level, let him lay the bottom of the ruler *op*, which ſupports the level, along the top of the teleſcope; and tye them faſt together with ſtrings; then let him gradually move the teleſcope about the tranſverſe axis, by turning the pegg abovementioned[a], till one end of the bubble is ſituated exactly over againſt a fine point near the middle of the tube; then let him obſerve through the teleſcope what mark upon a remote object is covered by the croſs hairs. This being noted let him return to the air bubble, and turn the pegg again, but no more than is juſt neceſſary to cauſe the leaſt ſenſible remove of the bubble from the mark; then let him obſerve through the teleſcope what other mark on the object is now covered by the croſs hairs; and let the angle ſubtended at the object-glaſs, by the interval of the two marks, be computed; or be meaſured by a micrometer hereafter deſcribed; and this will be the leaſt ſenſible error of the level, and conſequently of the tranſverſe axis levelled by it.

To examine the accuracy of the level. Fig. 581, 583.

a Art. 833.

843. By the ſame method one may determine what part of a long tube is the beſt for making a ſpirit level; by fixing the tube along the ſide of the teleſcope, firſt with one ſide uppermoſt and then another; till you find in what ſide and in what part of that ſide the motion of the air bubble is the greateſt, with reſpect to a given motion of the teleſcope obſerved as above by the croſs hairs. If the air bubble has no adheſion to the tube, but moves as freely as the ball of a plumbet, it is eaſy to underſtand that a ſpirit level will diſtinguiſh a ſmall inclination to the ſame accuracy as a plumb line, whoſe length is equal to the radius of the curvity of the tube. For if *acbd* be a ſphere filled with water, and an air bubble be ſuppoſed at the upper end *a* of any diameter *ab*, and a round globule of lead be ſuppoſed at its lower end; the bubble and the globule will deſcribe equal arches about the center *e* in their paſſage from the oblique diameter *ab* to the vertical one *cd*: and for a given inclination of *ab* to *cd*, the arches *ac*, *bd* are proportionable to their radius *ae* or *eb*.

How to chooſe a tube for a level.

Fig. 587.

CHAPTER VI.

Teleſcopick Inſtruments for finding time, by obſerving when the ſun or any ſtar has equal altitudes on each ſide of the meridian.

844. WHEN Sir *Iſaac Newton* made a preſent, to our Obſervatory at *Trinity* College in *Cambridge*, of an excellent pendulum clock; that great mathematician Mr. *Roger Cotes*, my worthy predeceſſor, contrived an inſtrument of ſmall expence, but very accurate, to ſet

it by, and to adjuſt its motion; by finding the times by the clock it ſelf when the ſun or any ſtar has equal altitudes before and after its paſſage over the meridian. He alſo ſent the following deſcription of his inſtrument to Sir *Iſaac Newton*. " *AB* is a ſtrong wooden axis about ſix feet in length; *CD* and *DE* on one ſide, *EF* and *FG* on the other, are pieces framed to each other and to the axis as firmly as was poſſible. Into the piece *CD*, and at the angle *F*, were fixed ſtrong wooden pins nearly parallel to each other, and perpendicular to the plane *CDEFG*. *PQ* is the cylindrical braſs tube of a five foot teleſcope; (belonging to our ſextant;) this was well faſtened with iron ſtaples and ſkrews to the piece of wood *IKML*, whoſe under plane ſurface is here repreſented as objected to view. Into this ſurface there was perpendicularly fixed a ſtrong wooden pin *N*, which was deſigned to hang the upper end of the teleſcope upon any of the pins in *CD*, whilſt its lower end reſted upon the pin *F*. Now that the teleſcope might be taken off, and yet afterwards be again placed accurately in the ſame poſition, I ordered the edges *IK* and *CD* which touched each other, to be rounded like the ſurface of a cylinder, as alſo the edge *EF* into which the pin *F* was fixed, and againſt which the cylindrical tube of the teleſcope reſted; ſo that the contact in both places might be made in a point. Upon the ſame account the pins in *CD* were made a little hollow as is repreſented at *R*; and the pin *F* was a fruſtrum of a cone, that thereby the teleſcope might more ſurely touch the edges *CD* and *EF*. Into the two ends of the wooden axis were ſtrongly fixed two pieces of well tempered ſteel, that at the upper end *A* was a cylinder well turned, which moved in a collar whoſe cavity, repreſented by *S*, was figured like two equal hollow and inverted fruſtrums of cones joined together: the lower at *B* was a cone moving in a conical ſocket of a ſomewhat larger angle. This ſocket had liberty to be moved horizontally, and to be fixed in any poſition by two ſkrews, which preſſed againſt it ſideways at right angles to each other. The inſtrument being thus prepared, I fixed a needle *V*, at the lower end of the wooden axis, whoſe point ſtood out from it about an inch; then ſuſpending a fine plumb-line *TVX* from the upper end of the ſame axis, I altered the poſition of the inſtrument by the ſkrews, until the plumb-line came to beat againſt the point of the needle in the whole revolution of the inſtrument, and there I fixed it as prepared for uſe.

Mr. *Cotes*'s inſtrument. Fig. 588.

845. So far Mr. *Cotes*. The plumb-line or fine wire *TVX* was ſuſpended by a loop *T* upon a braſs pin that ſkrewed into the top of the axis *AB*; a nick being filed round the pin to ſtay the loop from ſliding out of it. Then by ſkrewing the pin in or out, the plumb-line was brought to the ſame diſtance from the axis *AB* as the point of the needle is at *V*: which was fixt in the end of a thick wooden pin *Y*, not in the axis of it, but towards one ſide; ſo that by turning the pin round it ſelf, in a hole bored

bored through the axis *AB*, the needle's point might deſcribe a ſmall circle and be brought to touch the plumb-line when parallel to the axis of motion of the wood *AB*.

The Earl of *Ilay*'s inſtrument. Fig. 589.

846. The Right Honourable the Earl of *Ilay*, in his noble collection of inſtruments, has a very excellent one of this kind, for taking equal altitudes, made all of braſs except a ſquare ſteel axis *ab* 30 inches in length. To one ſide of the upper part of this axis, there is fixt a ſmall ſextantal arch *cd* repreſented ſeparately at *E*; its center *a* being at the top of the axis *ab*. The teleſcope *MN* is alſo 30 inches long, and is fixt along the diameter of a ſemicircle of the ſame radius as the ſextant and concentrick to it. The teleſcope with the ſemicircle being moveable about this center upon the plane of the fixt ſextant *acd*, may be faſtened to it in any elevation by two nuts and ſkrews *c*, *d*, fixt in the ends of the ſextantal arch; a circular ſlit being made all along the limb of the ſemicircle for theſe ſkrew pins to ſlide in. Cloſe under theſe arches, the axis *ab* is ſurrounded by a ſhort cylinder *e*, about an inch in diameter, well turned and poliſhed. The lower end of the axis is formed into a fine conical point *b*. The frame in which the axis turns, is a long hollow parallelopiped wanting two ſides. Its other ſides *f*, *g* are two braſs plates, equal in length to the part *be* of the axis, and are ſkrewed together edgeways. It has for its baſes two equal plates *h*, *i*, 4 inches ſquare. In the middle of the upper ſquare there is a round hole, large enough to receive the cylinder *e* without touching it; and over this hole is fixt a triangular hole in another plate, one of whoſe ſides is moveable by a ſkrew, to make all the ſides of the triangle touch the cylinder. Upon the lower ſquare there lyes a ſmaller plate *i*, with a fine center hole in it, to receive the point *b* of the axis. This center plate is moveable ſideways by two ſkrews at right angles to each other, which, when the frame is firmly fixt into a nitch of a free-ſtone pillar, will bring the axis *ab* exactly perpendicular to the horizon. This poſition is known by a ſpirit level *lm* fixt at right angles to the axis, above the cylinder, upon the ſide oppoſite to the ſemicircle. Along the top of the level there is a ſliding pointer, to be ſet to the end of the air-bubble; and when the poſition of the axis is ſo adjuſted by the ſkrews below, that the air-bubble keeps to the pointer for a whole revolution of the inſtrument, the axis *ab* is certainly perpendicular to the horizon; and then the line of ſight through the teleſcope deſcribes a circle of equal altitudes in the heavens. There are ſeveral of theſe circles deſcribed in the heavens even when the teleſcope is fixt to the ſextantal arch. For the round hole in its focus, has five wires parallel to the horizon, at equal intervals from one another as at *k*; and they are croſſed at right angles in the middle by two other upright wires at a ſmall diſtance from each other. The deſign of ſo many wires, is to obſerve when the ſame ſtar is ſucceſſively covered by every one of the 5, both in the eaſt and

weſt; ſo that the time of its paſſage over the meridian may be had more accurately, by taking a medium among all the obſervations. The diſtances between the five wires need be no greater than to afford time enough to write down the ſeveral obſervations, which muſt be taken when the ſtar is between the perpendicular wires.

Uſes of theſe inſtruments.

847. Time ſhewn by a clock may be called mechanical, to diſtinguiſh it from ſolar and ſidereal time.

848. By obſerving when a ſtar has equal altitudes before and after its culmination or appulſe to the meridian, we have the mechanical time of its culmination. Then by ſubſtracting the ſun's right aſcenſion computed to this mechanical time, from the ſtars right aſcenſion determined for the ſame time, we have the ſolar time of the ſtars culmination, and conſequently the difference between the mechanical and ſolar times.

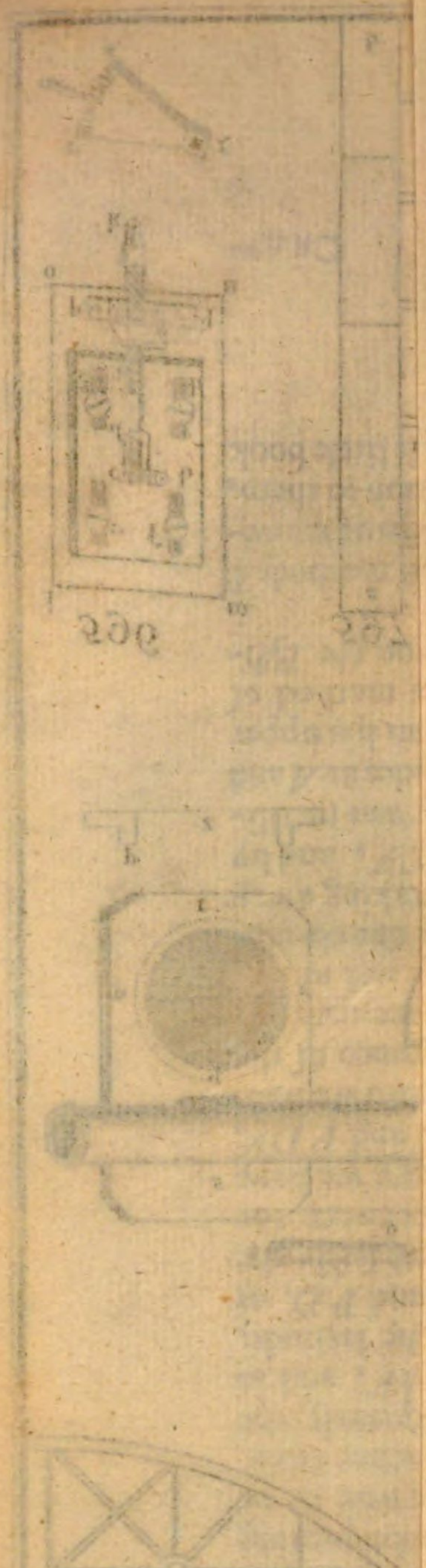
849. Thus by finding the mechanical times, when the ſame ſtar culminates any two nights, rather at a diſtance from each other than ſucceſſive, we have the difference between a ſidereal day and a mechanical day; and conſequently between a mechanical day and a ſolar day of a mean length.

Hence any number of mechanical minutes may be converted into ſolar, or into ſidereal minutes by the rule of three.

Theſe obſervations will anſwer the purpoſe the more exactly as the ſtar is nearer to the prime vertical; becauſe the variation of its altitude is here greater in a given time, than if it were ſituated in any other vertical oblique to the meridian. In the latitude of 50 degrees an error of one minute in altitude, at any point of the prime vertical, will cauſe an error of $6\frac{2}{9}$ ſeconds in time; and in the latitude of 55 degrees it will cauſe an error of near 7 ſeconds; as that excellent geometer Mr. *Cotes* has ſhewn, in his Treatiſe concerning the *Eſtimation and Limits of Errors* in mixt mathematicks, which I publiſhed at the end of his admirable Book called *Harmonia Menſurarum*. It is alſo the ſafeſt to chooſe a ſtar as high as poſſible, leaſt a different ſtate of the atmoſphere ſhould cauſe a different refraction of the viſual rays, and conſequently an error in the times of obſervation.

850. The ſolar time may alſo be found by obſerving when the ſun himſelf has equal altitudes in the morning and evening; if we correct the time of the latter obſervation by a juſt allowance for the variation of the ſun's declination, as follows. Upon a celeſtial globe let the pole be at P; the vertex of the obſerver's place at V; the complement of its latitude PV; its meridian $PVBG$; a circle of equal altitudes $ABCD$, deſcribed about its pole V, and paſſing through the ſun's center at A at the time of the morning obſervation, and through it at D at the time of the evening obſervation; two circles of declination, PAF, PDI, cutting the equator $FGHI$, in F and I; a parallel of declination ACE, cutting the

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the circle PDI in E, and ABD in C; three equal vertical arches VA, VC, VD; and laſtly a third circle of declination PCH cutting the equator in H. Now had not the ſun varied his declination, from A or E to D; in the evening he would have had the ſame altitude at C as in reality he has at D. And then as the angles VPC, VPA would have been equal, ſo the times of the evening and morning obſervations would alſo have been equidiſtant from noon; being meaſured by thoſe angles or by the arches GF, GH. Therefore the angle CPD or the arch IH, which meaſures it, is alſo the meaſure of a portion of time to be ſubſtracted from the evening obſervation, if the ſun's declination varies northwards, otherwiſe to be added, to give the time ſought equidiſtant from noon. Let another circle of declination Pkl biſect the ſmall angle HPI, and conſequently the ſmall arches CD and HI in k and l. Draw the vertical arch kV; and in the triangle kPV, we have given, PV the complement of the latitude, and Pk half the ſum of the given complements, PC or PA and PD, of the ſun's declinations at the times of the two obſervations, and laſtly the included angle kPV, by converting half the interval of time between the obſervations into degrees and minutes. Hence by trigonometry we have the angle PkV, of an intermediate magnitude between PCV and PDV, and therefore fitter to be uſed inſtead of either of them. Hence alſo we ſhall have the arch IH; by taking it in proportion to DE the difference of the declinations, as the co-tangent of the angle PCV, to the ſine of the arch PC or Pk. For IH is to DE in a ratio compounded of IH to CE and of CE to DE, that is of the radius to the ſine of the arch PC, and of the co-tangent of the angle DCE or PCV to the radius: as appears by taking away the common angle DCP from the right angles ECP and DCV; and by conſidering the ſmall triangle DCE, right angled at E, as if it was rectilinear. The calculation ſuppoſes the ſun's center has equal altitudes at A and D; which is agreeable to the obſervations that determine when his upper or his under limb has equal altitudes. I have drawn out this method of computation from Theorem. 23. of Mr. *Cotes*'s Treatiſe upon *the Eſtimation of Errors*.

851. Theſe are the beſt ſort of inſtruments and the beſt methods I know of for determining time; however thoſe that are deſirous of knowing a greater variety of them and of fuller inſtructions in relation to them, may receive ſatisfaction by conſulting Mr. *William Molyneux*'s little book upon his *Teleſcopick Dial*, printed at *London* in 4to. 1700.

CHAPTER VII.

The Mural Arch in the Royal Obſervatory at Greenwich *deſcribed.*

Preface. 852. SINCE the meaſure of time by pendulum-clocks, and conſequently of the apparent diurnal motion of the heavens, has of late been brought to the utmoſt perfection, Aſtronomers are agreed that a large quadrant (with a teleſcopick ſight) fixt in the plane of the meridian to a free-ſtone wall, and thence called a Mural Arch, is by far the moſt accurate, expeditious and commodious inſtrument of all others for the chief purpoſes in aſtronomy. For by obſerving the times by the clock of the appulſes of any objects in the heavens to the plane of the meridian, we have their right aſcenſions; and by their meridian altitudes obſerved at the ſame times, (the latitude of the place being once determined,) we have alſo their declinations; and by conſequence their places in the heavens. Thus by a good inſtrument of this kind, a Catalogue of the places of the fixt ſtars, may be made in leſs than a tenth part of the time, and with much greater certainty and exactneſs than by the beſt moveable quadrant or ſextant, as might eaſily be ſhewn; not to mention the ſaving an immenſe labour in trigonometrical calculations. I think the noble *Tycho Brahe* was the firſt Aſtronomer that uſed a mural arch, for taking meridian altitudes; but he could not determine right aſcenſions ſo accurately as now, for want of the late improvements in pendulum-clocks. After him, *Hevelius*, *Flamſteed* and others made uſe of the like inſtruments, whoſe deſcriptions may be ſeen in their printed works; but I paſs them all over as being far inferior to this at *Greenwich:* the expence of which was defrayed by the bounty of his late Majeſty King *GEORGE* the firſt; and the particular accuracy, whereby it excels all others, is owing to the extraordinary ſkill and contrivance of Mr. *George Graham* Watch-maker in *Fleet Street* and *F. R. S.*; who beſides the direction of the whole deſign, and inſpection of the inferior workmen, was pleaſed to perform the diviſions of the arch and all the nicer parts of the work with his own hands; and to him I am highly obliged for informing me in the methods he took to perform it. How far the tables of the moon's motion are corrected by an incredible number of obſervations made with this noble inſtrument by our Royal Aſtronomer Dr. *Halley*, and how near they are brought to ſufficient exactneſs for finding the Longitude at Sea, ſhall be related in the philoſophical diſcourſes of the next book. At preſent I proceed to a deſcription of the inſtrument.

Diſpoſition of the iron bars. Fig. 591, 592. 853. Excepting the circular limb, the quadrant is chiefly compoſed of ſtraight iron bars, joined together, as repreſented in the 591ſt and 592d

592d figures. The breadth of every bar is 2 inches and 9 tenths and its thickneſs 1 tenth and $\frac{3}{4}$ nearly. In ſpeaking of the diſpoſition of theſe bars, thoſe whoſe planes compoſe the plane of the quadrant, I ſhall call flat bars; and thoſe, whoſe planes are perpendicular to the former, I ſhall call perpendicular bars. The lines in fig. 591 repreſent the diſpoſition of all the flat bars, and thoſe in fig. 592 repreſent the diſpoſition of all the perpendicular bars, placed behind the flat ones; and are only to be ſeen on the back ſide of the quadrant. The chief deſign in this diſpoſition of both ſorts of bars, is to ſecure the figure and plane of the quadrant from any alterations that may be cauſed, either by the weight of the materials, or by their ſwelling or ſhrinking by the weather, or by the motion of the teleſcope about the center of the quadrant, or by any accident whatever. The whole fabrick is farther ſtrengthened by a great number of ſhort iron plates or pieces of the like bars, bent to a right angle, and placed behind the quadrant in the angles made by the flat and perpendicular bars, and riveted to them both. Their number and places, where they are riveted, are repreſented in the 592d figure, by the ſmall parallelograms adjoining to the lines: and to make more room for the rivets, the edge of each perpendicular bar does not divide the breadth of the flat bar in the middle, but in the ratio of 2 to 1; and the little plates are riveted on the broader ſide. The black thickenings of the lines at their interſections in the 592d figure, repreſent little plates of iron, bent in right angles, and riveted in the angles made by the interſections of the perpendicular bars. At the circumference of the quadrant there is alſo a perpendicular bar, bent circular, and faſtened all along the middle of the breadth of the limb or flat arch of the quadrant, by a ſufficient number of the little plates we have been ſpeaking of.

854. The limb of the quadrant is compoſed of two quadrantal arches, of the ſame length, breadth, and thickneſs; one of iron, the other of braſs laid over it. The breadth of each is 3 inches and 4 tenths, and the common part of their breadths, where they lye doubled one over the other and are riveted together, is 2 inches and 2 tenths, the braſs limb being remoter from the center than the iron one by an inch and 2 tenths. The limb was reduced to a true plane as follows. In the 593d figure, *abdo* repreſents the quadrant, placed very firm upon a level plane, with its braſs limb lying upwards; and *lm* repreſents an axis made of iron, placed perpendicular to the plane of the quadrant, and pointing to its center *o*; *mn* is an arm of iron, equal in length to the radius of the quadrant, and fixt at right angles to the bottom of the axis *lm*; to the end of this arm an iron ſcraper *np* was fixt directly over the braſs limb; and being firmly ſupported by the arm and its braces, was turned about the axis *lm*, till by ſcraping the braſs, it reduced its ſurface to a perfect plane; care

Structure of the limb.

Fig. 593.

care being taken that the edge of the ſcraper was exactly perpendicular to the axis of its motion.

Diviſion of the limb.

855. There are two arches ſtruck upon the braſs limb; one with a radius of 8 foot, or more exactly of 96, 85 inches; and the other, with a radius of 95, 8 inches. This inner arch is divided into degrees, and 12th parts of a degree; and the outward arch, into 96 equal parts, which are ſeverally ſubdivided into 16 equal parts. The beam of the compaſs which ſtruck theſe arches, was ſecured from bending by ſeveral braces faſtened to it; and when an arch was ſtruck, 60 degrees of it was determined by placing one point of the compaſs at *a* and by making a ſtroke with the other at *b*. This arch *a b*, was biſected in *c* by drawing two ſmall arches upon the centers *a* and *b*, with ſuch a radius as to croſs the arch *a c b*, in two points as near together as poſſible without touching each other; then the ſmall interval between them was biſected at *c*, by eſtimation of the eye, aſſiſted by a magnifying glaſs. After this, the interval between the points *a* and *c* or *c* and *b*, was taken with the beam-compaſs, and was transferred from *b* to *d*, which determined the length of the quadrantal arch *a c b d*. Every one of the three arches being biſected in the ſame manner, the quadrant became divided into 6 equal parts, containing 15 degrees apiece; and every one of theſe was divided into three equal parts as follows. To avoid making any falſe or ſuperfluous points in the quadrantal arch, with its radius unaltered, but upon any other center, there was ſtruck another faint arch; upon which the chord of 15 degrees already found was transferred from the quadrantal arch; and the third part of 15 degrees, being determined by trials upon the faint arch, was transferred back again upon the quadrantal arch; which then was divided into 18 equal parts containing 5 degrees apiece; and the 5th part of theſe was found by trials, as before in dividing a ſeparate arch, drawn upon a new center for this purpoſe only. The ſub-diviſions of the degrees into 12 equal parts were made by biſections and triſections as before. Thus was the whole quadrant divided without any falſe or ſuperfluous points.

856. The outward quadrantal arch was divided into 96 equal parts, by no other method than that of biſection, till 60 degrees or two thirds of the quadrant became divided into 64, and the remaining third into 32, equal parts; which make 96 in the whole. And every one of theſe were alſo divided into 16 equal parts by continual biſections. Theſe two ſorts of diviſions are a check upon each other, being in effect two different quadrants; and the diviſions in one being reduced into the diviſions of the other, by a table made for that purpoſe, they are never found to differ above five or ſix ſeconds in any place of the limb: and when they do, the preference ought to be given to the biſected diviſions, as being determined by a ſimpler operation.

857.

857. The diviſions hitherto mentioned being only very fine points in a fine arch *abd*, ſcarce diſcernible by the naked eye; it was neceſſary, as uſual, to ſtrike lines perpendicular to the arch, through every one of them. But ſince it is very difficult, and tedious too, to draw lines exactly through every point by the edge of a ruler, the following method was judged more accurate and expeditious. It was propoſed then to divide Fig. 593. any other concentrick arch, *fht*, by croſs ſtrokes, into ſimilar parts to thoſe in the given arch *acgebd*. Take a ſmall beam compaſs, and having once fixt its points at any convenient interval; upon the centers *e*, *g*, &c. being the given points of the divided arch, ſtrike the ſmall arches *fi*, *hk*, &c. cutting the undivided arch in *f*, *h*, &c: then will the intercepted arches as *fh*, &c. be ſimilar to the arches *eg*, &c. that is, they will ſubtend the ſame angles at their common center *o*. For joining *ef*, *gh*, and alſo *of*, *oh*, *oe*, *og*, the triangles *eof*, *goh*, will be ſimilar and equal to each other; every ſide in one being reſpectively equal to every ſide in the other. Therefore by taking away the common angle *eoh*, from the equal angles, *eof*, *goh*, the angles, *eog*, *foh*, that remain, will alſo be equal.

If the triangles *efo*, *gho*, &c, be right angled at *f* and *h*; the dividing ſtrokes *fi*, *hk*, &c. will cut the quadrantal arch *fht*, at right angles alſo, at *f* and *h*, &c.

858. In the 594th figure *abcd* repreſents a ſquare piece of braſs (with ſeveral ſteady pins in it) ſkrewed to the flat bars, at the center of the quadrant, the ſkrew holes being ſo large as not to touch the ſkrews; and *klmn* repreſents a thick circular plate of braſs, with a hollow pipe *fg* fixt perpendicular to the middle of it; this plate was turned exactly circular in a lathe upon a braſs arbor *oi*, turned tapering and a little hollow in the middle, ſo as to fit the cavity of the pipe *fg*, and to bear againſt it chiefly at both ends. When the hollow pipe *fg* is put through the hole (exactly fit for it) in the middle of the ſquare *abcd*, the braſs circle *klmn*, is fixt to the plane of the ſquare *abcd*, with ſkrews and ſteady pins. The point *o*, in the pole of the arbor *oi*, is not only the center of the circular neck *klmn*, about which the teleſcope muſt turn, but alſo the very center upon which the divided arches were ſtruck upon the limb of the quadrant. The end of the teleſcope that holds the object-glaſs lies croſs one end of an oblong plate of braſs *st*, at right angles to its ſides, and is held to the plate by a brace that may be widened and ſtraightened by a ſkrew. Towards the other end of the plate *st*, there is a round hole, lined with a ſteel collar *pqr* to be put over the braſs neck *klmn*, and to turn round upon it. The ſection of this collar, made perpendicular to the plane of it, is repreſented at *z*; the broader of the two rings being under the plate *st* and contiguous to the ſquare plate *abcd*. Over this neck and collar there is fixt a braſs ſpring repreſented at *v*, and ſkrewed to the neck *klmn*, to keep the collar from ſlipping from it; and over all theſe is ſkrewed a cap repreſented

The center work. Fig. 594.

ſented at *x*, to cover the center work, and to keep off the duſt; which is alſo prevented from coming between the plates *abcd* and *st* to the neck and collar, by means of a braſs hoop ſurrounding the broad rim or baſe of the collar *pr*, and ſkrewed to the backſide of the plate *st*; which hoop is received into a circular groove 1, 2, 3, 4 made in the ſquare plate *abcd*, without touching any part of it.

The chief excellency of the preſent center work conſiſts in preſerving the place of the central point of the quadrantal arch in the pole of the arbor *oi*. For whenever the neck-plate *klmn* ſhall be wore ſo much as to cauſe an unſteady motion of the teleſcope about the center of the quadrant; a new neck-plate and pipe may be caſt, well hammered and turned upon the poles of the ſame arbor *oi*, to fit the hole and collar, and then it will carry the teleſcope about the center of the limb, as exactly as when all was new.

How the quadrant is fixt to a free-ſtone wall. Fig. 595.

859. The 595th figure gives a view of the quadrant fixt to the eaſtern ſide of a free-ſtone wall, built for that purpoſe in the plane of the meridian. The whole weight of the quadrant is ſupported by two ſtrong iron pins fixt to the wall (as hereafter deſcribed) and projecting through two holes made in two ſquare plates of iron riveted to the quadrant at *a* and *b*, in the 591ſt figure. The pin at *a*, which bears the greateſt part of the weight, is immoveably fixt in the wall, but the pin at *b* is moveable up or down by a ſtrong ſkrew, in order to bring one ſide of the quadrant to an horizontal, and the other to a vertical poſition.

Fig. 596.

The contrivance for the motion of the pin *b* is this. In the 596th figure *lmno* repreſents an oblong plate of iron, let into the free-ſtone wall, and faſtened to it by bolts of iron, which paſs through the wall and through another plate let into the oppoſite ſide of it; the bottom of each plate being bent ſquare and bedded in the ſtone. And *e*, *f*, *g*, *h*, are the heads of 4 iron ſkrews, whoſe ſhanks going through 4 long ſlits, made in another iron plate, repreſented by the ſmaller parallelogram, are ſkrewed into the fixt plate *lmno*. The moveable pin *bc* is fixt to this leſſer plate, which is raiſed, or depreſſed by means of a long ſkrew *ki*, working againſt the bottom of the pin *bc* at *d*, being turned round in a ſtrong concave ſkrew, fixt to the bottom of the larger plate at *pq*. The key for turning the long ſkrew *ki*, is a ſector of a circular plate repreſented at *rst*; the ſquare hole in its center *t* being put upon the ſhank *k*. The radius of the key is juſt ſo big as to move in the ſpace between the wall and the bars of the quadrant; and a chiſel *y* is inſerted into the teeth upon the arch of the key, to give power to the hand that moves it.

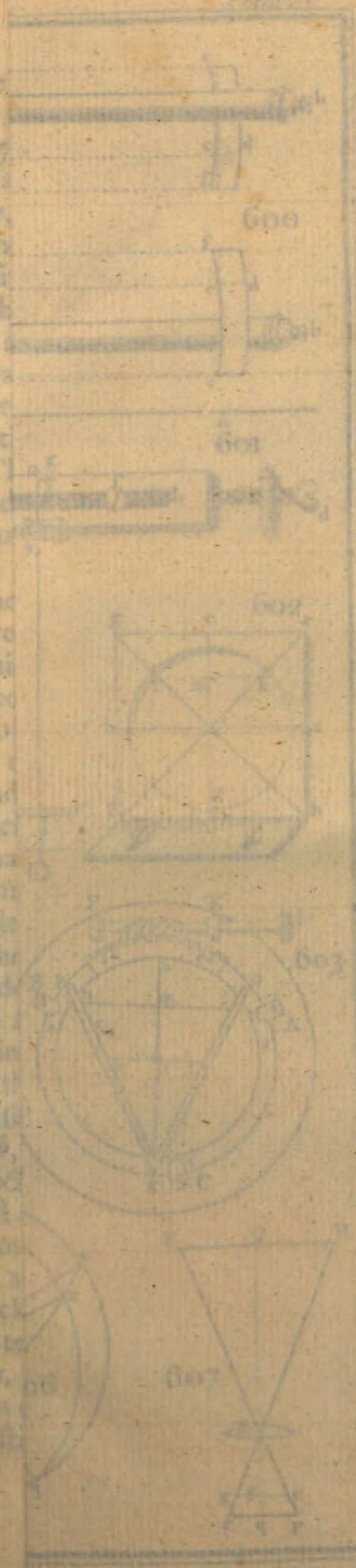
The weight of the quadrant being thus ſupported by the pins *a*, *b*, the plane of it is fixt to the wall and adjuſted in any poſition, by much the ſame number of hold-faſts as there are little ſquares round about the quadrant

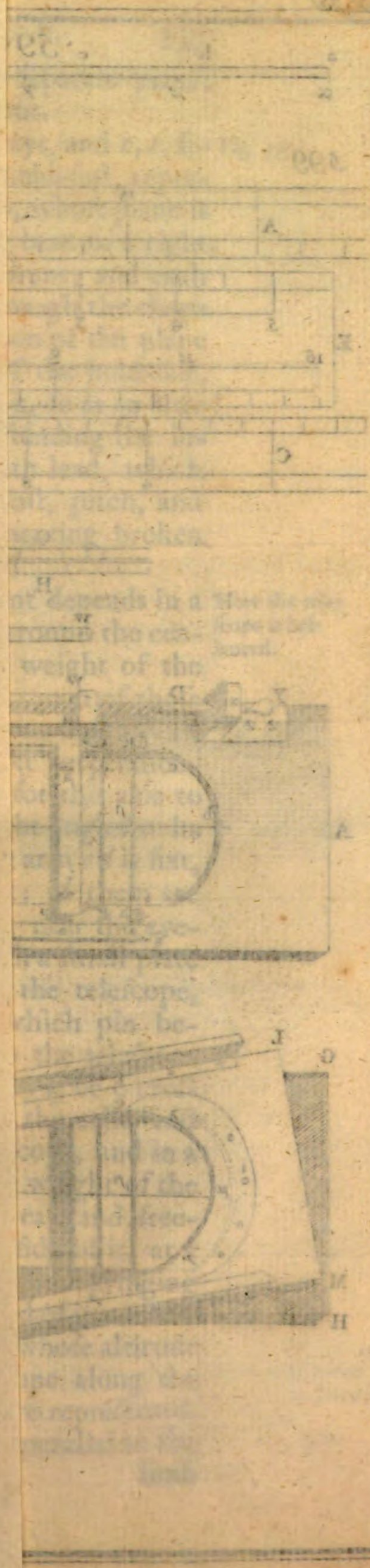
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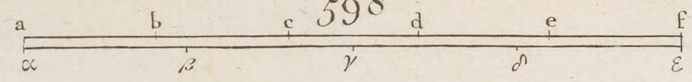
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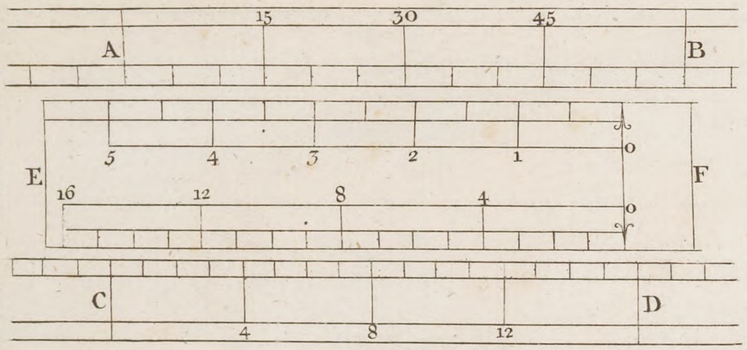


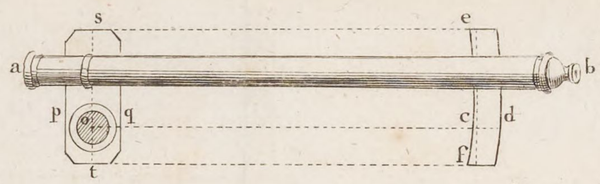


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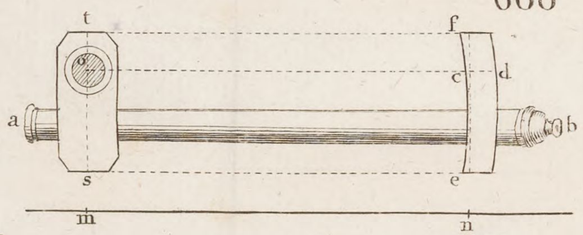


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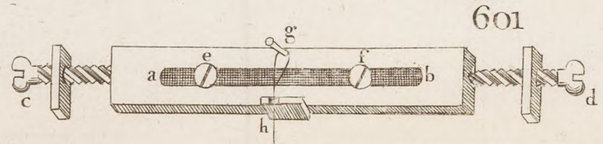


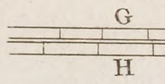


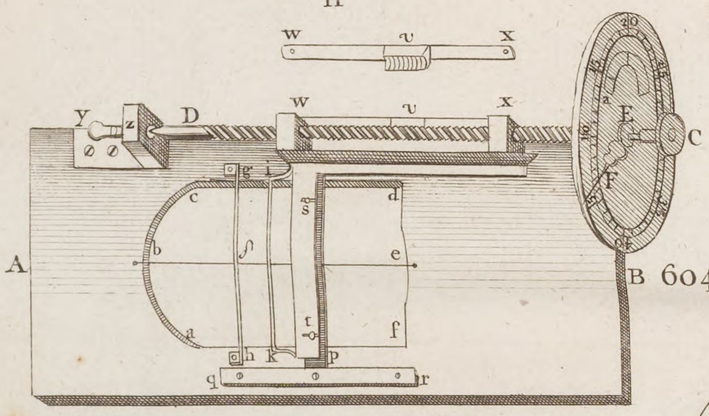
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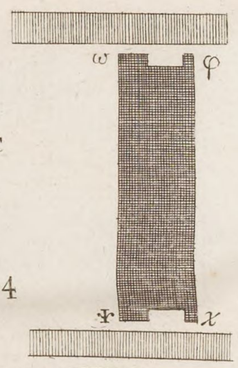


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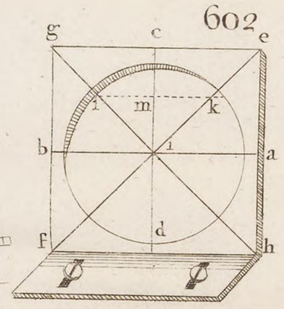


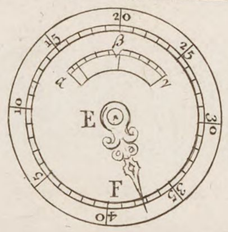




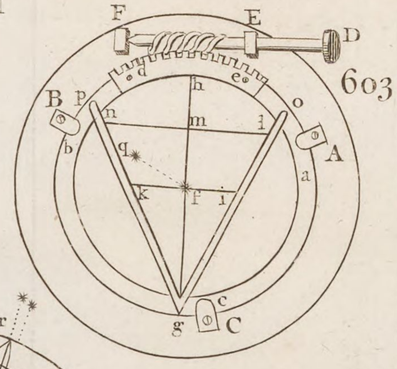


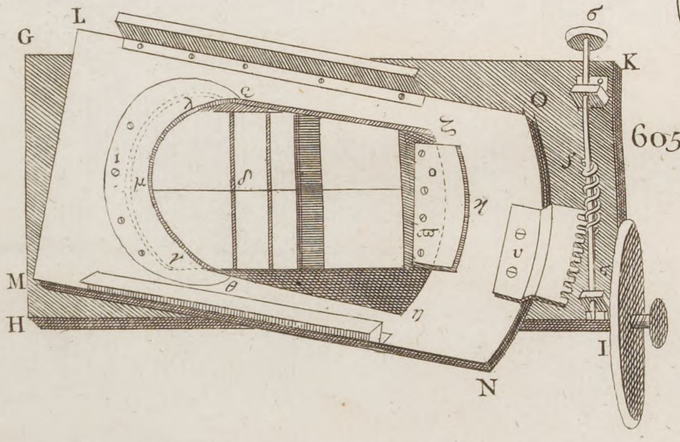
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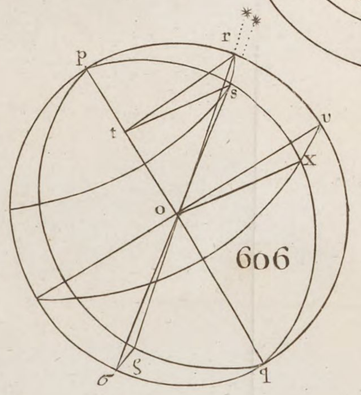
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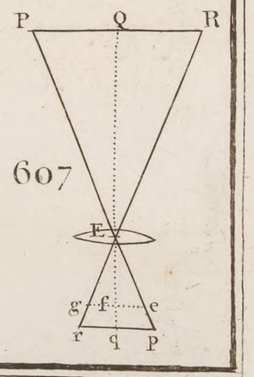


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drant in the 592d figure. Each hold-faſt conſiſts of two ſeparate parts; one of them is fixt to the wall, and the other to the quadrant.

In the 597th figure, *ab* repreſents the wall ſeen endways, and *c*, *c*, ſe- Fig. 597. veral hold-faſts fixt into it. Between the chaps of each hold-faſt, repreſented at *d*, *e*, there paſſes one end of a ſmall plate of braſs, whoſe plane is parallel to the plane of the quadrant, the other end being bent to a right angle and riveted to the perpendicular bars of the quadrant; and each plate is pinched by two oppoſite ſkrews *r*, *s*, that work through the chaps *d*, *e*; which are made pretty wide for adjuſting the poſition of the plane of the quadrant. The intent of the ſkrews in the chaps of the hold-faſt, was alſo that if the wall or quadrant ſhould ſwell or ſhrink, ſo as to alter their proportions, the braſs plates might ſlide without diſtending the inſtrument. The hold-faſts are not faſtened in the wall with lead, which is apt to yield, but with a compoſition made of ſtone-duſt, pitch, and brimſtone, or roſin; ſuch as ſtone-cutters uſe for cementing broken ſtones.

How the teleſcope is balanced.

860. The continuance of the exactneſs of the quadrant depends in a great meaſure upon a free and eaſy motion of the teleſcope round the center of it; which will be obtained by counterpoiſing the weight of the teleſcope, and by eaſing the center of the quadrant of as much of thoſe weights as poſſible. For this purpoſe in fig 595, *ab* repreſents an iron axis Fig. 595. laid croſs the top of the wall; having two braſs plates fixt perpendicular to the ends of it, with notches or holes cut in them for this axis to turn in, which points to the center of the quadrant at right angles to its plane. To that end of this axis next the quadrant, an iron arm *cd* is fixt, having two braſs plates *ce*, *df* almoſt perpendicular to it; to them are riveted two ſlender ſlips of fir, whoſe other ends meet at *g*, near the eye-glaſs; being held together in a braſs cap or ſocket. Through a ſmall plate fixt to one ſide of a collar, embracing this lower end of the teleſcope, there paſſes a ſkrew-pin at *g* parallel to the teleſcope; which pin being ſkrewed into the cap at the end of the ſlips, holds up the teleſcope tight againſt the center-work. The ſlips are ſtrengthened by 5 or 6 croſs braces of the ſame wood, as repreſented in the figure. To the other end of the axis *ab*, another arm *hi* is fixt parallel to the teleſcope, and in a contrary direction, carrying a weight *i* to counterpoiſe the weight of the teleſcope, and make it reſt in any poſition. And for greater eaſe and freedom of its motion, two ſmall braſs rollers are fixt to each ſide of it, at *k* and *l*, which are held tight to the plane of the limb by a plate ſpringing againſt its backſide, which plate has alſo a roller at each end of it.

When the teleſcope is pretty nearly directed to an object whoſe altitude is to be taken, a plate *mn*, which is carried by the teleſcope along the limb, and lies croſs it, may be fixt to it by a ſkrew, not here repreſented. Then by twiſting the head *o* of a long ſkrew *op*, which is parallel to the

limb and which works through a female ſkrew, annexed to the plate *m n*, and whoſe neck at *p* turns round in a collar annexed to the teleſcope; a very gradual motion is given to the teleſcope for bringing the croſs hairs exactly to cover the object.

The ſub-diviſions how made and numbered. Fig. 598.

861. To avoid the trouble of ſub-dividing the quadrantal arch into ſmaller parts, the teleſcope carries a ſmall braſs plate, which ſlides upon the limb, and is called a *Nonius*, from the name of its inventer. To underſtand the reaſon and uſe of this plate, it is convenient to premiſe the following Theorem. If a line *af*, be divided into any number of equal parts, *ab*, *bc*, *cd*, *de*; and an equal line $\alpha\varepsilon$ be divided into other equal parts, $\alpha\beta$, $\beta\gamma$, $\gamma\delta$, $\delta\varepsilon$, whoſe number is one leſs than the number of parts in *af*; I ſay that $\alpha\beta$, $\alpha\gamma$, $\alpha\delta$, $\alpha\varepsilon$, will exceed *ab*, *ac*, *ad*, *ae*, reſpectively by one, two, three, four parts of *ab*, whoſe denominator is the number of parts in *ae* or in $\alpha\varepsilon$. For let the lines *af*, $\alpha\varepsilon$ be coincident at both ends, and ſince any equimultiples of two quantities *ab*, $\alpha\beta$, are in the ſame ratio as the quantities themſelves [a], it will be as $ab : \alpha\beta :: ac : \alpha\gamma :: ad : \alpha\delta :: ae : \alpha\varepsilon$ or *af*, and disjointly as $ab : b\beta :: ac : c\gamma :: ad : d\delta :: ae : e\varepsilon$ or *ef*. The conſequents, $b\beta$, $c\gamma$, $d\delta$, $e\varepsilon$, are therefore in the ſame arithmetical progreſſion as the antecedents *ab*, *ac*, *ad*, *ae*; and the firſt of the conſequents $b\beta$ is the ſame part of its antecedent *ab*, as the laſt conſequent *ef* is of its antecedent *ae*, or as $\alpha\beta$ is of $\alpha\varepsilon$, the number of parts in *ae* and $\alpha\varepsilon$ being equal by the firſt ſuppoſition. And it is manifeſt that any two equal and coincident arches of a circle have the ſame property.

a Eucl. V. 15.

Fig. 599.

862. The upper arch *AB* repreſents a degree divided into 12 equal parts, containing 5 minutes in each; and the under arch *CD* a 96th part of the quadrant, divided into 16 equal parts; and *EF* the Nonius, or ſubdividing plate fixt to the teleſcope, and ſliding with it in the ſpace between the arches *AB*, *CD*. The degrees and minutes, and alſo thoſe 96 parts of the quadrant, are numbered from the left hand to the right, beginning from the interſections of the vertical radius, in order to meaſure the diſtances of objects from the zenith; but the parts upon the Nonius are numbered the contrary way, beginning from the line 00, called the index; which is drawn perpendicular to the ſides of the Nonius at the end next the right hand; and the line of ſight through the teleſcope is ſo adjuſted by the croſs hairs in its focus, as to be parallel to the index 00 produced through the center of the quadrant. In the ſcheme, the Nonius *EF* is ſo ſituated, that the upper end of the index 00 is not oppoſite to any one ſtroke upon the adjoining arch, but to ſome unknown point of a 12th part of a degree, intercepted between 50 and 55 minutes. To find the overplus above 50, I obſerve by looking back from the index, that a ſtroke of the Nonius, which lies between the numbers 3 and 4 is directly oppoſite to a ſtroke upon the adjoining arch; which ſhews that 3 minutes and a half is to be added to the 50 minutes aforeſaid.

For

For ſince a degree is divided into 12 equal parts, containing 5 minutes in each; and ſince the length of the Nonius is made equal to 11 of thoſe parts, and is divided into 10 equal parts; it appears by the theorem [a], in counting back again from the coincident ſtrokes to the index, that the firſt part of the Nonius exceeds the firſt upon the limb by $\frac{1}{10}$ of this latter part, that is by $\frac{1}{10}$ of 5 minutes, which is half a minute; and by conſequence that 7 parts of the Nonius, from the coincident ſtrokes to the index, exceed the 7 correſponding parts of the arch, by 7 half minutes, or $3'.30''$.

[a] Art. 861.

863. When it happens that no one ſtroke upon the limb is directly oppoſite to a ſtroke upon the Nonius, then look for that ſingle part of the limb, which is ſo oppoſed to a ſingle part upon the Nonius, as to be exceeded by it at both ends; as repreſented in the parts *G* and *H*. Then if by eſtimation of the eye, this part of the Nonius exceeds the part of the limb equally at each end, allow $15''$ more than if they had coincided at their ends next the index; and according as the exceſs next the index is judged to be one third, one half, double or treble of the other exceſs, allow $7\frac{1}{2}''$, $10''$, $20''$, $22\frac{1}{2}''$, reſpectively. For ſince the ſum of the two exceſſes is always the ſame, and anſwers to $30''$, (as is plain when one of them is diminiſhed to nothing,) the number of ſeconds to be added will always be to $30''$, as the exceſs next the index, is to the ſum of the two exceſſes. Fig. 599.

864. The lower arch of the Nonius is divided into 16 equal parts, and is equal in length to 17 equal parts upon the oppoſite arch, and conſequently will determine 16th parts of any one of them, by the theorem and the method abovementioned. In the preſent ſcheme the oppoſite ſtrokes of the Nonius and the lower arch, are ſuppoſed to coincide at the end of the 9th part upon the Nonius, which ſhews that the index cuts off 9 ſixteenths of the oppoſite part of the arch. And ſo the length of the arch, from the beginning of a 96th part of the quadrant, is thus denoted, 15, 9, the lower pointer being paſt the 15th ſtroke.

865. This way of ſub-dividing by a Nonius is preferable to the common method of drawing diagonals; both becauſe the trouble of drawing ſo many diagonals is intirely avoided; and alſo becauſe they cannot be drawn ſo exactly by the edge of a ruler, as the lines upon the Nonius; and laſtly becauſe the interſection of theſe diagonals with the index or fiducial edge, (as they call it,) by reaſon of their great obliquity to each other, cannot be determined ſo exactly by the eye, as the coincidence of two ſtrokes in the Nonius and the arch, which ſtand directly oppoſite to one another.

Rectification of the line of ſight.

866. The object-glaſs being firmly and immutably fixt in the teleſcope, the Nonius-plate *c d* and the collar-plate *s t*, were both ſkrewed faſt to the teleſcope when taken off from the quadrant; and then the Fig. 600.

line of ſight was brought to be parallel to the line *co*, drawn through *o* the center of the collar *pq*, to *c* the beginning of the diviſions on the Nonius, in this manner. The lines *sot* and *ecf* being drawn upon theſe plates both perpendicular to *oc*, any diſtances *ot* and *cf* were taken equal to each other on one ſide of *oc*; and any other diſtances *os* and *ce*, (long enough to go beyond the teleſcope) were alſo taken equal to each other on the oppoſite ſide of *oc*. Through the four points *e*, *s*, *t*, *f*, the ends of the two plates were filed exactly parallel to *oc*. Then placing the points *t*, *f* upon two points *m*, *n* of an horizontal line drawn upon a firm plane, a point of a remote object covered by the croſs hairs was marked. And the teleſcope being turned half round its axis *ab*, and the oppoſite points *e*, *s* of the plates being placed upon the ſame points *m*, *n*, another point of a remote object now covered by the croſs-hairs was alſo marked; and the teleſcope remaining fixt, the croſs-hairs were moved in its focus, till after ſeveral repetitions of this practice, the ſame point of the object was covered by them in both poſitions of the teleſcope; and then the line of ſight was exactly parallel to the line *oc* *, ſuppoſing the object was very remote. But becauſe ſmaller marks upon a nearer object are better diſcerned, the hairs were ſo adjuſted till in each poſition of the teleſcope they covered a ſeparate mark, the interval of the marks being taken equal to the difference of the heights of the axis of the teleſcope above the fixt line *mn*, as near as could be meaſured.

* Art. 834.

How the quadrant was adjuſted for uſe.

867. The object-glaſs being well centered by the method in art. 808, the line of ſight was firſt of all made parallel to the plane of the quadrant, as near as it need be, by the meaſures of the braſs-work annext to the teleſcope; and then the plane deſcribed by the line of ſight, turned about the center of the quadrant, was brought into the plane of the meridian, by obſerving whether the fixt ſtars paſſed over the croſs hairs at the ſame inſtant of time, as they paſſed over a meridian teleſcope, adjuſted as above deſcribed[a], and placed ſo near the quadrant, that the two obſervers could hear each other calling out at the times of the tranſits. And by the coincidence of theſe obſervations upon ſtars at various altitudes, it appeared that the plane of the quadrant was wrought very true. For it is certain that the meridian plane deſcribed by the meridian teleſcope, as turning upon a tranſverſe axis, muſt be truer than that deſcribed by the quadrantal teleſcope, as guided by the rollers upon the limb.

a Art. 838.

Fig. 594, 595.

868. When the quadrant was thus reduced into the plane of the meridian by the hold-faſts above deſcribed, that radius of it, which terminates 90 degrees, was placed exactly vertical (by the movement abovementioned) with a plumb-line of very fine ſilver wire; ſo ſuſpended as to play exactly over the middle of the central point *o* (in the pole of the arbor *oi*) and alſo over the ſtroke at 90 degrees upon the limb below. This poſition of the quadrant being once found, another plumb-line was

ſuſpended

ſuſpended by the ſide of the quadrant, quite clear of the center-work; ſo as to play exactly over the middle of a fine point made in the limb below; in order to examine afterwards with more expedition whether the quadrant has kept its place. For this purpoſe an oblong piece of braſs *ab*, laid flat upon the ſquare plate at the center of the quadrant, was gradually moved to the right or left, by two ſkrews *c*, *d*, working againſt the ends of it: a ſlit *ab* being cut lengthways through the plate to ſlide along two other ſkrew-pins *e*, *f*, fixt in the back plate. The wire of the plumbet was hung by a loop upon a pin *g*, and lay in a very fine angular nick filed in the edge of a little plate *h*, which projected a little farther than the loop, for the wire to bear upon it. This plate *h* and the pin *g* were both fixt to the oblong plate *ab*; by whoſe gradual motion above deſcribed, the wire *hi* was brought to play exactly over the middle of the hole *i* in the limb; and then the plate *ab* was preſſed to the quadrant by the ſkrews *e*, *f*. Fig. 60.

869. When this quadrant was fixt upon the eaſtern ſide of the wall, ſome proviſion was made for fixing ſuch another upon the weſtern ſide; to obſerve the altitudes and tranſits of ſtars over the northern half of the meridian; and as ſoon as the Government ſhall think proper to allow the expence of ſo uſeful an inſtrument, we may expect in a few years time to be furniſhed with a more exact Catalogue of all the ſtars viſible in our hemiſpere, than can poſſibly be had from all the inſtruments and obſervations yet extant. And ſuch a Catalogue being the very foundation of all the deſired accuracy in Aſtronomy, Geography and Navigation, I need not dilate upon the uſefulneſs of it. Proviſion for another quadrant.

870. As to the Catalogue of the ſouthern conſtellations, partly inviſible in our latitude, no doubt it will ſoon receive an accurate correction and increaſe, by an excellent Mural Arch of 4 foot radius, made by Mr. *Siſſons* in the *Strand*, exactly after the model of this at *Greenwich*; excepting that the limb of it is continued about 12 degrees towards the ſouth of the plumb-line, for obſerving the places of the moon and planets in paſſing towards the north from the zenith of *Jamaica*; where this quadrant is to be fixt, for the uſe of *Colin Campbell* Eſquire; a young gentleman of whom the learned in the Sciences have great expectations: not only from his own genius and application, but alſo from the advantages of his ſituation, his fortune and education; in which laſt he has had the happineſs to be directed by no leſs a judge than the Right Honourable the Earl of *Ilay*; to whom he has alſo the honour of being related. Mr *Campbell's* Mural Arch at *Jamaica*.

CHAP-

CHAPTER VIII.

Of meaſuring ſmall angles with a teleſcope.

A micrometer what and how applyed.

871. A MICROMETER is a ſmall piece of mechaniſm, contrived for moving a fine wire parallel to it ſelf, in the plane of the picture of an object formed in the focus of a teleſcope, and with great exactneſs to meaſure its perpendicular diſtance from a fixt wire in the ſame plane: and the uſe of this inſtrument is to meaſure ſmall angles ſubtended by remote objects at the naked eye. Let a planet, for example, be viewed through the teleſcope; and when the parallel wires are opened to ſuch a diſtance as to appear exactly to touch two oppoſite points in the circumference of the planet, it is evident that the perpendicular diſtance between the wires is then equal to the diameter of the picture of the planet, formed in the focus of the object-glaſs Let this diſtance, whoſe meaſure is given by the mechaniſm of the micrometer hereafter deſcribed, be repreſented by the line pq in fig. 181; then ſince the meaſure of the focal diſtance qL is alſo known by the method abovementioned[a], the ratio of qL to qp, that is of the radius to the tangent of the angle qLp, will give the angle it ſelf, by a table of ſines and tangents; and this angle is equal to the oppoſite angle PLQ[*], which the real diameter of the planet ſubtends at L or at the naked eye. But the ſame angle may be meaſured ſtill more exactly and independently upon the abſolute meaſures of qL and qp, their proportion being ſufficient, as ſhall be ſhewn hereafter.

a Art. 814.

* Art. 43.

Mr. *Huygens*'s micrometer.

872. Mr. *Huygens* uſed to meaſure the apparent diameters of the planets or any ſmall angles by a very eaſy contrivance which he thus deſcribes. "There is a certain place, ſays he[b], within the tube of a teleſcope about as far before the convex eye-glaſs as the eye is behind it, wherein the fineſt and ſlendereſt things being placed, appear through the eye-glaſs exceeding plain and diſtinct. And by conſequence will intercept the view of a certain diſtinct part of an object ſeen through the teleſcope. Here if you place a round hole, (made in a braſs plate,) whoſe diameter is a little leſs than that of the eye-glaſs, it will moſt neatly and diſtinctly circumſcribe all the ſpace in the heavens viſible at once through a fixt teleſcope. We muſt firſt determine ſome way or other how many minutes of an arch are the meaſure of the apparent diameter of this ſpace. The beſt way is by meaſuring the time of a ſtar's paſſage over the diameter of it, either by a ſimple pendulum or by a pendulum-clock. For it is well known that a very ſmall matter above one degree of the heavens paſſes over a fixt hour circle in 4 minutes of time. So that for example if a fixt ſtar, ſituated in the equator, or very near it, appears to paſs over the diameter of the ſaid ſpace in 69 ſeconds, the teleſcope being fixt, it follows that the apparent

b Syſtema Saturn. p. 82.

diameter

diameter of that ſpace in the heavens ſubtends an angle of 17 $\frac{1}{4}$ minutes at the naked eye; and this is the meaſure of the ſpace taken in by my teleſcope whoſe length is 23 feet. This being determined, let two or three long and ſlender braſs plates be prepared of various breadths, whoſe ſides are very ſtraight and converge very gradually. Then while a planet is viewed through the teleſcope, ſlide one of them through two ſlits in the oppoſite ſides of the tube, ſo that the plane of the long plate may touch the plane of the round hole; then take notice in what place of the plate the breadth of it can juſt cover the whole planet; for by taking this breadth between the points of a fine pair of compaſſes, and by comparing it with the diameter of the hole, the apparent diameter of the planet may be eaſily collected. Thus far Mr. *Huygens*.

873. But the diameters of planets thus meaſured, as Sir *Iſaac Newton* has obſerved, are ſomewhat bigger than they ſhould be; as is evident by comparing Mr. *Huygens*'s meaſures with others taken by parallel hairs; and alſo by comparing the diameter of Mercury when obſerved in the ſun, with his diameter obſerved out of the ſun; which latter is ſenſibly bigger than the former. All lucid objects ſeen upon dark ones appear bigger, and dark ones ſeen upon bright ones appear leſs, than they would do if the lights of both objects were equally ſtrong. Becauſe the circumference of the brighter image upon the retina diffuſes it ſelf into the contiguous darker image; and the bright image of the planet being intercepted by Mr. *Huygens*'s plate, that faint diffuſed light becomes more ſenſible, and is miſtaken for the edge of the planet. But this error may be avoided by uſing long tapering ſlits, cut in braſs plates, inſtead of tapering plates; or elſe very fine converging wires inſtead of the ſlits.

Relative places of ſtars how obſerved by a teleſcope.

874. The place of any ſtar, ſo ſituated with reſpect to a known ſtar, as by its diurnal motion to follow it or go before it over the aperture of a fixt teleſcope, may be found very accurately without a micrometer of any ſort; only by obſerving the differences of their right aſcenſions and declinations; which differences being added to, or taken from, the given right aſcenſion and declination of one of the ſtars, give thoſe of the other, and conſequently its place. For this purpoſe it is requiſite to have four croſs hairs in the focus of the teleſcope; of which *ab* and *cd* croſs at right angles, as alſo *ef* and *gh*; which are inclined to the two former in half right angles; and all croſs one another at the point *i*. Then having directed the teleſcope to make the preceding ſtar appear upon the hair *ab*, and having turned the teleſcope about its axis till the ſtar moves along *ab*, let it remain fixt; and obſerve by a pendulum-clock the time of this ſtar's appulſe to the center *i*; and alſo the time of the ſubſequent ſtar's appulſe to the perpendicular hair *cd*. The interval of time between theſe appulſes to the hair *cd*, which coincides with part of an hour circle, being turned into degrees and minutes, gives the number of them contained in the correſpond-

Fig. 602.

reſponding arch of the equator that paſſed over the ſaid hour circle; which arch is the difference of the right aſcenſions of the ſtars. To find the difference of their declinations, the times of the ſubſequent ſtar's appulſes at *k* and *l* to the oblique hairs *ef*, *gh* muſt alſo be obſerved; half of which is the time it ſpends in deſcribing half the line *kl*, that is *mk* or *ml*. This half being converted into minutes of an arch, gives the number of them contained in the correſponding arch of the equator that paſſed by the fixt hour circle *cd* in that time; and this number being diminiſhed in the ratio of the radius to the ſine complement of the given ſtars declination, gives the difference of their declinations, or the number of minutes in the arch *mi*. This rule ſhall be demonſtrated by and by, and is ſufficiently accurate though the ſine complement of the given ſtar's declination is uſed inſtead of the ſine complement of the unknown ſtar's declination, becauſe their difference is but ſmall in compariſon to the whole.

875. Signor *Caſſini* who firſt introduced this uſeful and accurate method of obſerving by 4 hairs, has ſhewn us how he applies it to determine all the particulars obſervable in Eclipſes. See Phil. Tranſ. N°. 236. It has one great advantage, that the obſervations are not liable to the uncertainty of the air's refraction. For the teleſcope being fixt, all objects that appear in it, having nearly the ſame altitudes, are equally refracted very nearly; it is therefore the beſt method of obſerving the places of Mercury and Venus or any of the planets and comets when near the ſun, and by conſequence but little elevated in the night time, as Dr. *Halley* has obſerved, Phil. Tranſ. N°. 366. Where ſpeaking of the accuracy of this way of obſerving, he tells us, " that he himſelf was preſent when Dr. *Pound* and his Nephew Mr. *Bradley* did this way demonſtrate the extream minuteneſs of the ſun's parallax ſo exactly, that upon many repeated trials it was not more than 12″ nor leſs than 9″. Theſe obſervations may be ſeen in the Phil. Tranſ. N°. 363; from which Dr. *Halley* calculated the parallax by *Caſſini*'s method (deſcribed in his book upon the comet in 1680.) which is very well explained in the 7th of Mr. *Whiſton*'s Aſtronomical Lectures.

This method made eaſier.

876, The only trouble in this way of obſerving, lyes in turning the teleſcope about its axis into ſuch a poſition, that the ſtars ſhall move parallel to one of the hairs. But this difficulty is intirely removed, though the teleſcope be never ſo long, by a very pretty contrivance, invented by that ingenious and accurate Aſtronomer the Reverend Mr. *James Bradley*, *Savilian* Profeſſor at *Oxford*; which he has alſo applyed to the micrometer hereafter deſcribed. As near as I can remember it is this. Let *ABC* repreſent a flat ring of braſs fixt in the focus of the teleſcope; and *abc* a ſmaller concentrick ring lodged in a circular groove turned within the larger, and kept in the groove by three ſmall plates of braſs fixt to the

Fig. 603.

the outward ring and extended over the edge of the inner one. Upon the inner ring is fixt a concentrick arch of a wheel *de*, having teeth cut in its convexity, which are driven round by the threads of an endleſs ſkrew, whoſe axis *DEF* turns in a collar at *E* and upon a point at *F*, both fixt to the outward ring. The hairs *gh*, *ik* croſs at right angles in *f* the center of the rings; and when the teleſcope is ſo fixt that the image of a ſtar falls upon *f*, let it move along any line *fq*, and by turning the ſkrew *DEF* and by conſequence the hair *fk* about the fixt point *f* till it touches the ſtar at *q*, it will then coincide with the tract of the ſtar's motion; and then all other ſtars will move parallel to it as was required.

To find the difference of declination of two ſtars, he obſerves the times of their appulſes to the edges of two ſlender braſs bars *gio*, *gkp*, fixt to the inner ring, and equally inclined to its diameter *gh* in ſuch angles that the perpendiculars *fi*, *fk*, on each ſide of *fg*, ſhall be ſeverally equal to half *fg*; and conſequently that the whole baſe *ik*, of the equicrural triangle *igk*, ſhall be equal to its perpendicular height *fg*; and by conſequence that the difference of any two baſes *ifk*, *lmn* ſhall be equal to *fm* the difference of their heights; ſo that the difference of the times of the tranſits of two ſtars over theſe baſes, may give the difference of their declinations as before explained.

877. A micrometer of the beſt ſort is made in this manner. In the middle of an oblong plate of braſs *AB*, there is cut an oblong hole *abcdef*, (to be placed in the focus of a teleſcope,) having a fine wire *be* extended length ways over the middle of it, at right angles to two ſlender braſs bars or ſights *gh*, *ik* lying croſs the hole; of which *gh* is fixt to the plate *AB* by ſkrews at *g* and *h*, but *ik* is moved parallel to *gh* by twiſting a round knob *C* fixt upon one end of a long iron ſkrew *DE*; which turns upon a tapering point at its end *D*, while its other end turns round in a hole at *E* in the center of an index-plate *EF* fixt at right angles to the main plate *AB*. The long ſkrew *DE* works through two hollow ſkrews in two cubical blocks of braſs fixt behind the plate *lm*, bent ſquare to the plate *no*, that ſlides upon the main plate *AB*, either backward or forward, and carries a perpendicular arm *op* extended over the hole *be*; while *p* the extremity of the arm *op* ſlides under a braſs ledge *qr* ſkrewed to the main plate *AB* along the ſide of the hole. One ſide *st*, of the moveable ſight-plate *ikst*, lyes over the arm *op*, being fixt to it by flat headed ſkrews at *s* and *t*, the holes in the plate *st* being oblong or larger than the ſhanks of the ſkrews, to give liberty for placing the edge *ik* coincident with *gh*, when carried up to it by turning the ſkrew *DE* by the knob *C*; the part *ik*, which projects over the arm *op*, being hammered down to lye flat upon the main plate *AB*. The edge *ih*, after this adjuſtment, will always move parallel to the edge *gh*; its inclination to the ſkrew *DE* being every where the ſame, provided the ſkrew be

Deſcription of a micrometer. Fig. 604.

ſtraight, and the interval of the concave ſkrews behind *l m* be ſufficiently greatand their motion ſteady. For this purpoſe about a quarter-round of a third concave ſkrew preſſes upon the long ſkrew *D E* at *v*, the block of it being fixt to the middle of a ſpringing plate *w v x*; whoſe extremities lying behind the blocks at *l*, *m*, not ſo near as to touch them, are preſſed towards them with ſkrews at *w* and *x*; which occaſions the block at *v* to ſpring upon the ſkrew *D E*, and to hold it tight to the oppoſite ſides of the concave ſkrews at *l*, *m*. To prevent any motion lengthways in the ſkrew *D E*, its tapering point *D* turns in a hollow point at the end of an oppoſite ſkrew *y*, which working through a fixt block at *z*, holds up the ſhoulder of the long ſkrew *D E* againſt the back of the index-plate, where its neck is inſerted.

The two indexes upon the plate *E F* ſhew the number of revolutions and parts of a revolution of the ſkrew *D E*, anſwering to the interval of the ſights *g h*, *i k*. In the outward plate there is a circular ſlit *α β γ*, which diſcovers part of the diviſions upon the circumference of an inner plate, turned about a center by two wheels and pinions within: ſo that for every revolution of the ſkrew and index *E F*, which ſhews the parts of it, one diviſion upon the plate *α β γ*, paſſes by a fixt pointer at *β*; which ſhews the number of revolutions anſwering to the interval of the ſights *g h*, *i k*.

An improvement of it.

878. This micrometer has lately received a very great improvement by an ingenious contrivance of the Reverend Mr. *James Bradley* Profeſſor of Aſtronomy at *Oxford*, for turning it in its own plane about the interſection *δ* of the fixt ſight *g h* and of the tranſverſe wire *b δ e*, without ſtirring the teleſcope; which is thus executed. Upon the backſide of the main-plate turned upwards, and here repreſented by the parallelogram *G H I K*, there is laid ſuch another plate *L M N O*, of the ſame breadth and thickneſs but ſomewhat ſhorter; in the middle of which there is an oblong hole, anſwering to that other in the lower plate, but ſomewhat larger; being terminated at its ſides by the ſtraight lines *ε ζ*, *ϰ θ*, and at its ends by the concave arches *θ ι ε*, *ζ ϰ η*, whoſe common center is the point *δ* abovementioned. The concave arch *ε ι θ* ſlides round this center againſt a concentrick convex arch of an annular plate *λ μ ν*; ſomewhat longer than the concave arch, of the ſame thickneſs as the upper parallelogram, and ſtrongly ſkrewed to the under one, round that end of the hole which is neareſt to the center *δ*: and at the ſame time the other concave arch *ζ ϰ η* ſlides alſo againſt another concentrick convex arch *ο ϖ*, of another annular plate juſt as thick as the upper parallelogram, and ſtrongly ſkrewed to the under one. This convex arch *ο ϖ* is ſhorter than the contiguous concave one *ζ ϰ η*, to give room for the circular motion of the plates; which are held together by two annular plates ſimilar to *λ μ ν* and *ο ϖ*, but ſomewhat broader, to cover the coincident arches, when laid over them, and ſkrewed down to the reſpective annular plates underneath. The cir-

Fig. 605.

cular

cular motion upon theſe arches about their center δ, is gradually given to the upper parallelogram by an endleſs ſkrew at ς, having an axis $\sigma\tau$ laid croſs the end of the under parallelogram, and turning upon a point at one end and in a collar at the other, both fixt to the under plate; while the ſpiral thread ς moves the teeth of a braſs arch fixt at v to the end of the upper parallelogram.

To hold the micrometer in the tube of a teleſcope, along each ſide of the upper parallelogram there is fixt a long braſs plate about an inch broad; having its oppoſite ſides bent contrary ways, ſo as to form two oppoſite ledges, about $\frac{1}{8}$ of an inch broad, at right angles to the intermediate part of the plate, as repreſented in the figure. One of the ledges of each plate is placed inwards along the ſides of the upper parallelogram, and is firmly fixt to it by ſeveral ſkrews. The figure $\varphi\chi\psi\omega$ repreſents one of the equal and oppoſite holes cut in the ſides of a ſquare tube, through which the micrometer is put; the notches $\varphi\chi$ being made to receive the ledges of the ſide-plates, to keep the plane of the micrometer perpendicular to the tube at a juſt diſtance from the object-glaſs. Which diſtance being once determined by trials, as above explained, muſt be kept invariable in all obſervations, by ſtops or pins, if the tube conſiſts of two or more joints that draw in and out.

The meaſures of the Micrometer.

	Inches
The length of the plate AB	8, 0
Its breadth MN	3, 6
Its thickneſs	0, $\frac{1}{2}$
Length of the hole be	3, 5
Its breadth $gh = \delta e$	2, 2
Breadth of the hole in the other plate at $\zeta\eta$	2, 6
Length of the ſkrew DE	5, 5
Its thickneſs	0, 3
The line Ab	1, 6
The interval $lm = wx$	8, 0
Length of the ſide cheeks	4, 5
Their breadth	0, 8
Their ledges	0, 2
Diameter of the index-plate	3, 1
Its thickneſs (being double with two wheels within)	0, 3
The greateſt opening of the ſights gh, $ik = \delta e$	2, 2

Threads of the ſkrew in an inch, 40

The inch is divided by the index-plates into 40 times 40 or 1600 equal parts. Inſtead of the braſs ſights gh, ik, two others with parallel wires may be ſkrewed on at pleaſure.

To find the angles answering to the revolutions of the skrew.

879. When the sight-plates are made to coincide, the two indexes of the revolutions and their parts must be set to the beginning of the numerations upon the index-plates. Then as the sights are opened, it is evident from the make of a skrew, that the numbers of revolutions will be as the intervals of the sights, and consequently as the angles subtended by them at the center of the object-glass; the intervals being insensibly different from the arches that measure these small angles. Therefore when any one angle corresponding to a given number of revolutions, is determined by experiment, an angle corresponding to any other number of revolutions may be found by the rule of three. And thus may tables be made to shew by inspection the number of minutes and seconds in an angle answering to any given number of revolutions and parts.

880. To determine some one angle, the larger the better, because the same error in the determination will be proportionably smaller in a given angle deduced from it; fix the telescope upon any known star in the equator or very near it, and open the sights to their utmost limit and note the number of revolutions of the skrew. Then by a pendulum-clock observe the interval of time in the star's transit over the given interval of the sights, and having turned it into minutes and seconds of an arch, they are the measure of the angle required. But if the star be remote from the equator, the number of minutes and seconds thus found, must be diminished in the ratio of the radius to the sine of the star's distance from the pole.

Fig. 606.

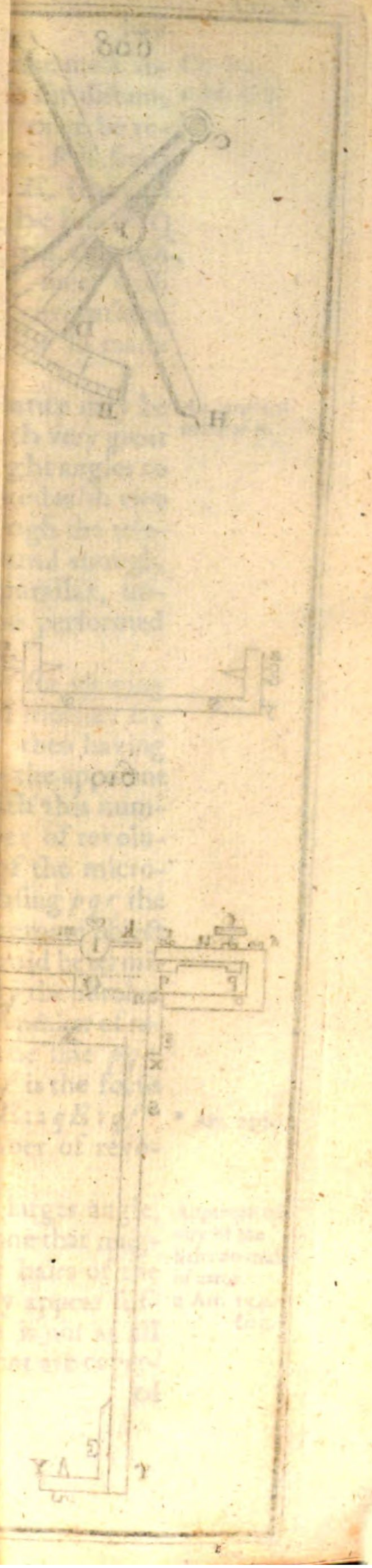
881. For let o be the center and poq the axis of a sphere, parallel to the earth's axis; $prvq$ and $psxq$ two hour circles cutting the equator of the sphere in v and x, and the circle parallel to it, described by the star's diurnal motion, in r and s. Join ov and ox, and from t the center of the parallel draw tr and ts. Then if you please we may suppose o to be the center of the object-glass, and the diameter $roϱ$ to be equal to twice the length of the telescope, and the little arch rs, or the equal and opposite arch $ϱσ$, to be the interval of the sights; and then the angle ros or $ϱoσ$ is the angle required. Now since the lines $roϱ$, $soσ$ are the visual rays at the times of the star's appulses to the sights, the interval of time, turned into minutes and seconds of an arch or angle, gives the number of them contained in the angle vox or rts, in which the planes of the hour circles are inclined to each other. But the angle rts is to the angle ros, subtended by the same perpendicular rs, as ro to rt*, that is as the radius to the sine of the star's distance from the pole p. Or thus, the triangles or sectors vox, rts being similar, the radius vo is to the sine tr (as the arch vx to the arch rs or) as the angle vox to the angle ros, taking rs for an arch of a great circle.

* Art. 60.

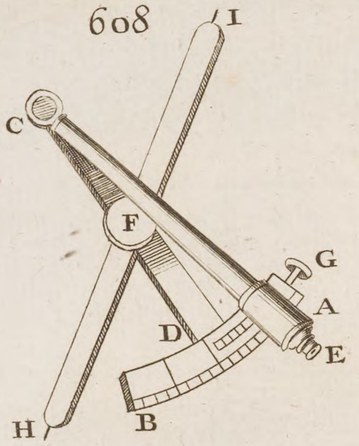
By *Molyneux's* method. Dioptr. p. 249.

882. Another way of finding the angle answering to one of the greatest openings of a micrometer, is this. Having fixt the micrometer at its

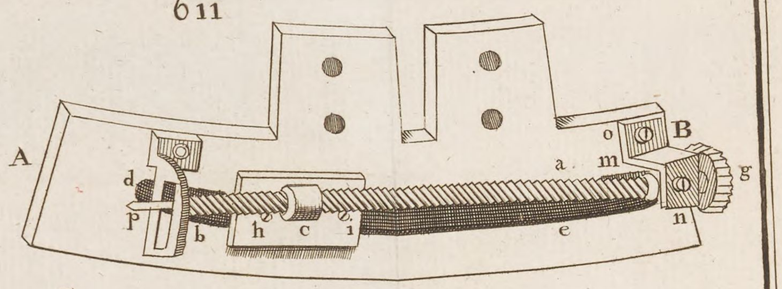
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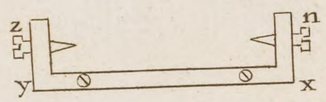
611





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610



612

due distance *Ef* from the object-glass, requisite for viewing the most remote object without parallax[a]; on the side of a wall or house far distant, mark out two conspicuous objects as *P*, *R* that may both at once be received into the fixt telescope. Measure nicely their distance *PR* from each other, and also their distance *EQ* from the object-glass *E*, (the axis of the telescope *QEq* being perpendicular to the middle of the line *PR*) and by trigonometry calculate the angle *PER*. Then looking through the telescope open the micrometer, till its two edges exactly meet with and embrace the two objects *P*, *R*; and observe how many revolutions, and parts of a revolution are performed in this opening. For so many compleat the angle before calculated. So far Mr. *Molyneux*. Fig. 607. a Art. 815.

An improvement of it.

883. Upon an even piece of ground where a long distance may be measured exactly by a pole, this experiment may be made with very great accuracy, only by fixing up a rail or horizontal board, at right angles to *QE*, with a black line drawn lengthways upon it, and marked with two cross strokes *P*, *R* of a sufficient thickness to be visible through the telescope. And although it should happen for want of even ground enough, that the object is too near to be seen distinctly and without parallax, unless the telescope be lengthened, yet the experiment may be performed with sufficient accuracy by making the following correction.

Draw back the micrometer to the distance *Eq* requisite for viewing the present object most distinctly, and without parallax; and measure *Eq* as exactly as you can, for great exactness is not necessary; then having observed the number of revolutions and parts, answering to the apparent diameter of the object or the computed angle *PEQ*; diminish this number in the given ratio of *QE* to *Qq*; and you have the number of revolutions answering to the same angle *QER* when the place of the micrometer is adapted for viewing the remotest objects. For supposing *pqr* the image of the near object *PQR* and *efg* the image of the most remote object subtending the same angle as *PQR* does; these two images would be terminated by the same lines if both were distinct. And consequently the number of revolutions that would answer to the image *efg*, is to the number of revolutions answering to the image *pqr*, as the line *efg* to the line *pqr*. But supposing the rays to go back again from *q* to *Q*; since *f* is the focus of parallel rays coming the contrary way, we have *qQ*: *qE*:: *qE*: *qf** and disjointly *QE*: *Qq* :: (*Ef*: *Eq* :: *eg*: *pr* ::) as the number of revolutions in *eg*, to the number in *pr*. * Art. 239.

Apparent curvity of the hairs no cause of error.

884. A micrometer is the more useful for taking in a larger angle, and for this purpose the broadest and thickest eye-glass and one that magnifies the most is the best. Through such an eye-glass the hairs of the micrometer will appear crooked indeed[b], but provided they appear sufficiently distinct, the accuracy of the measure of an angle is not at all disturbed thereby. Because the right lines upon the object that are covered

b Art. 154. 605.

ed

ed by the hairs, appear to be juſt as much bent as the hairs themſelves. If a hair be ſtretched upon a page of a book, and be viewed through the edge of a very convex glaſs, the hair though diſtorted will appear to cover the very ſame letters as when viewed by the naked eye. And the caſe is the ſame in the focus of a teleſcope; becauſe the rays that come from the points of the object, diverge upon the eye-glaſs from points contiguous to the hairs.

CHAPTER IX.

Mr. Graham's *Aſtronomical ſector.*

Deſign of this inſtrument.

885. IT is allowed that a micrometer is the moſt accurate and convenient inſtrument for obſerving the place of a planet or comet, when it happens to be near enough to any known ſtar; by taking the differences of its right aſcenſion and declination from thoſe of the ſtar, as explained above. But this being frequently impracticable, by reaſon that many large places in the heavens are void of ſtars whoſe places are known; it is neceſſary to have recourſe to moveable quadrants or ſextants, furniſhed with teleſcopick ſights for taking larger diſtances. But beſides the difficulty and charge of procuring good inſtruments of this ſort, the great trouble and uncertainties in obſerving with them are very notorious, ariſing chiefly from the difficulty the obſervers find in making their obſervations, at each teleſcope, correſpond together at the ſame inſtant, while the inſtrument is following the diurnal motion of the heavens. The lovers of Aſtronomy are therefore much obliged to the ingenious Mr. *George Graham* F. R. S. not only for many uſeful improvements in the mechaniſm of ſeveral aſtronomical inſtruments, but alſo for contriving a very commodious and accurate one for the purpoſe aforeſaid; that is for taking ſuch differences of right aſcenſion and declination as are too large to be obſerved through a fixt teleſcope; and yet with equal facility, and exactneſs too in proportion to the radius of the inſtrument. I will firſt give an idea of it and then deſcribe the particulars of the mechaniſm.

An idea of the inſtrument. Fig. 608.

886. Let *AB* repreſent an arch of a circle containing 10 or 12 degrees well divided, having a long plate *CD* for its radius, fixt to the middle of the arch at *D*. Let this radius be applied to the ſide of an axis *HFI*, and be moveable about a joint fixt to it at *F*, ſo that the plane of the ſector may be always parallel to the axis *HI*; which being parallel to the axis of the earth, the plane of the ſector will always be parallel to the plane of ſome hour circle. Let a teleſcope *CE* be moveable about the center *C* of the arch *AB*, from one end of it to the other, by turning a ſkrew at *G*; and let the line of ſight be parallel to the plane of the ſector.

Now

Now by turning the whole inſtrument about the axis *HI*, till the plane of it be ſucceſſively directed firſt to one of the ſtars and then to another, it is eaſy to move the ſector about the joint *F* into ſuch a poſition, that the arch *AB* when fixt, ſhall take in both the ſtars in their paſſage by the plane of it; provided the difference of their declinations does not exceed the arch *AB*. Then having fixt the plane of the ſector a little to the weſtward of both the ſtars, move the teleſcope *CE* by the ſkrew *G*, and obſerve by a clock the time of each tranſit over the croſs hairs and alſo the degrees and minutes upon the arch *AB* cut by the index at each tranſit: then is the difference of the arches the difference of the declinations, and by the difference of the times we have the difference of the right aſcenſions of the ſtars.

887. The mechaniſm of the principal parts of the inſtrument is this. Upon the ſide of an iron axis *HIF* wrought ſquare, and near the top of it, there is fixt a broad circular plate *abc* of ſolid braſs; upon which there lies a braſs croſs *KLMN*, which turns about a joint at the center *F*. At the ends of the croſs plate *MN*, are erected two equal perpendiculat arms *O* and *P*, whoſe extremities are fixt by the ſkrews *d*, *e* to the backſide of the radius *CD*; which is ſtrengthened by a long braſs rib on its backſide, placed edgeways from one end to the other as repreſented in fig. 610. The arms *O*, *P* are no longer than is neceſſary for the ſector *ABC* to turn about the joint *F* quite clear of another circular plate *QR*; which is fixt to the upper baſis of a braſs cylinder *I*, the iron axis *HF* being put through a ſquare hole in the middle of them both, and immoveably fixt in it. *ST* repreſents a long ſubſtantial plate of braſs, having two ſhort plates *VX* and *YT* fixt perpendicular to the ends of it. Let us ſuppoſe the length of the plate *ST* to be parallel to the earth's axis, and to be firmly fixt in this poſition upon a pedeſtal, or otherwiſe, with its flat ſides facing the north and ſouth. Remove the axis *HI* and place the conical hole made in the end *H* upon a conical point of a ſkrew-pin at *Y*; and the cylinder *I* into the ſlit *VZX*; whoſe parallel ſides *V*, *X* embrace it, while it reſts againſt two points of an angular notch in the bottom of the ſlit at *Z*. By this means the whole inſtrument will always turn true about one and the ſame imaginary line. When the ſector is turned about the joint *F* till the radius *CD* becomes parallel to the axis *HI*, the 610th figure repreſents a ſection of the whole inſtrument, made by a plane paſſing at right angles through the radius *CD*, and through the rib on the backſide of it, and through the axis *HI* and through the ſupporter *ST*. The ſeveral parts of the inſtrument are here denoted by the ſame letters in both figures. The arms *O* and *P* have two ſlits through the middle of their ends to receive the edge of the rib *CD*. The circular plate *ac* is fixt to the axis by the ſkrews *h*, *i*. The braſs rod *gk* is ſkrewed into the axis *HI*, and carries a braſs ball *lm* that ſlides along it, and is fixt by a ſkrew *m* at a proper

Deſcription of the parts. Fig. 609.

Fig. 609, 610.

place

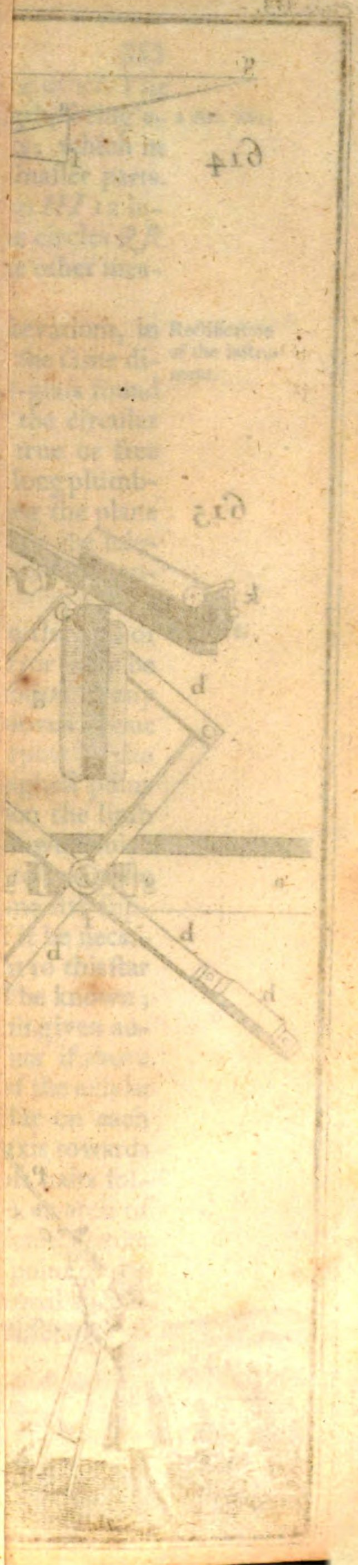
place for ballancing the weight of the ſector and teleſcope placed on the oppoſite ſide of the axis. At the top of the ſupporter *ST* there is a hold-faſt *nopqrstv*, whoſe cavity *nopq* receives the circular plate *QR*. The end *q* of a ſpringing plate *pq* is fixt by a ſkrew *r* to the inſide of the upper plate *rs*, while its other end *p* may be preſſed down upon the circle *Q*, by twiſting the knob of a ſkrew *t* which works in a ſocket *v*. And to prevent this preſſure from diſlocating the plane of the circle *QR* and conſequently the poſition of the axis *HI*, the hold-faſt *nopq* has liberty of yielding or turning upon the ends of two ſkrew-pins, that go into two conical holes in the oppoſite edges of the under plate *no*; one of theſe ſkrews is ſeen at *n*; and the fixt piece that they ſkrew into, is repreſented ſeparately in full view at *nxyz*; *n*, *z* being the points that the hold-faſt turns upon. By this means the ſame ſkrew at *t* cauſes the upper and under plates of the hold-faſt *nopq* to compreſs the circle *Q* with equal forces. To the arm *O* there is faſtened ſuch another hold-faſt, which ſo compreſſes the circle *ac* and the croſs plate *MN*, as to ſtay the ſector and teleſcope in any given poſition, from turning about the joint *F*. This joint is nothing but a cylindrical pin paſſing through the plates *MN*, *ac*. The flat head of the pin is fixt by 3 ſmall ſkrews to the plate *MN*; and to the oppoſite end of this pin a circular ſpringing plate is fixt by a ſkrew, that ſkrews into the end of the pin. And the joint *C* at the center of the ſector *ABC* is made in the ſame manner.

Fig. 611.

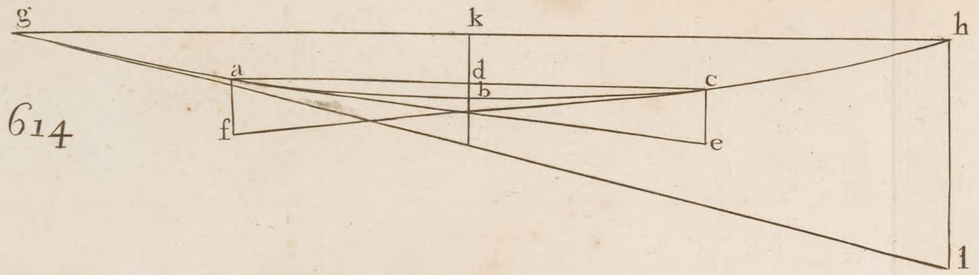
888. The 611th figure repreſents the ſkrew-work upon the backſide of the limb *AB*; contrived for moving the teleſcope by twiſting the knob *g*. Here *gab* is a long ſtraight ſkrew, which works through a ſkrew-hole in a braſs head *c*; whoſe neck is moved in a long circular ſlit *de* cut through the limb; the other end of this neck turns round in a hole in the Nonius-plate, (one end of which is fixt to the teleſcope,) and draws it along the limb: *h* and *i* are the heads of two ſkrews, whoſe ſhanks go through a ſpringing plate, to make the motion ſteady, and through the ſlit *de*, and are ſkrewed into the Nonius-plate on the other ſide of the limb. Since the length of the ſkrew *ab* muſt have a ſmall angular motion while it carries the head *c* along the arch *de*; the ſhank of it near the end *a* turns round in a hole made in a ſhort flat axis *mn*, placed perpendicular to the limb and held ſo by an arm *no*, while the other end *b* moves in a ſlit *p* parallel to the limb, being cut in a ſmall plate fixt perpendicular to the limb. The long ſkrew *gab* is kept from ſlipping backwards and forwards through the hole in the axis *mn*, by ſhoulders or nuts fixt on each ſide.

Dimenſions of the inſtrument.

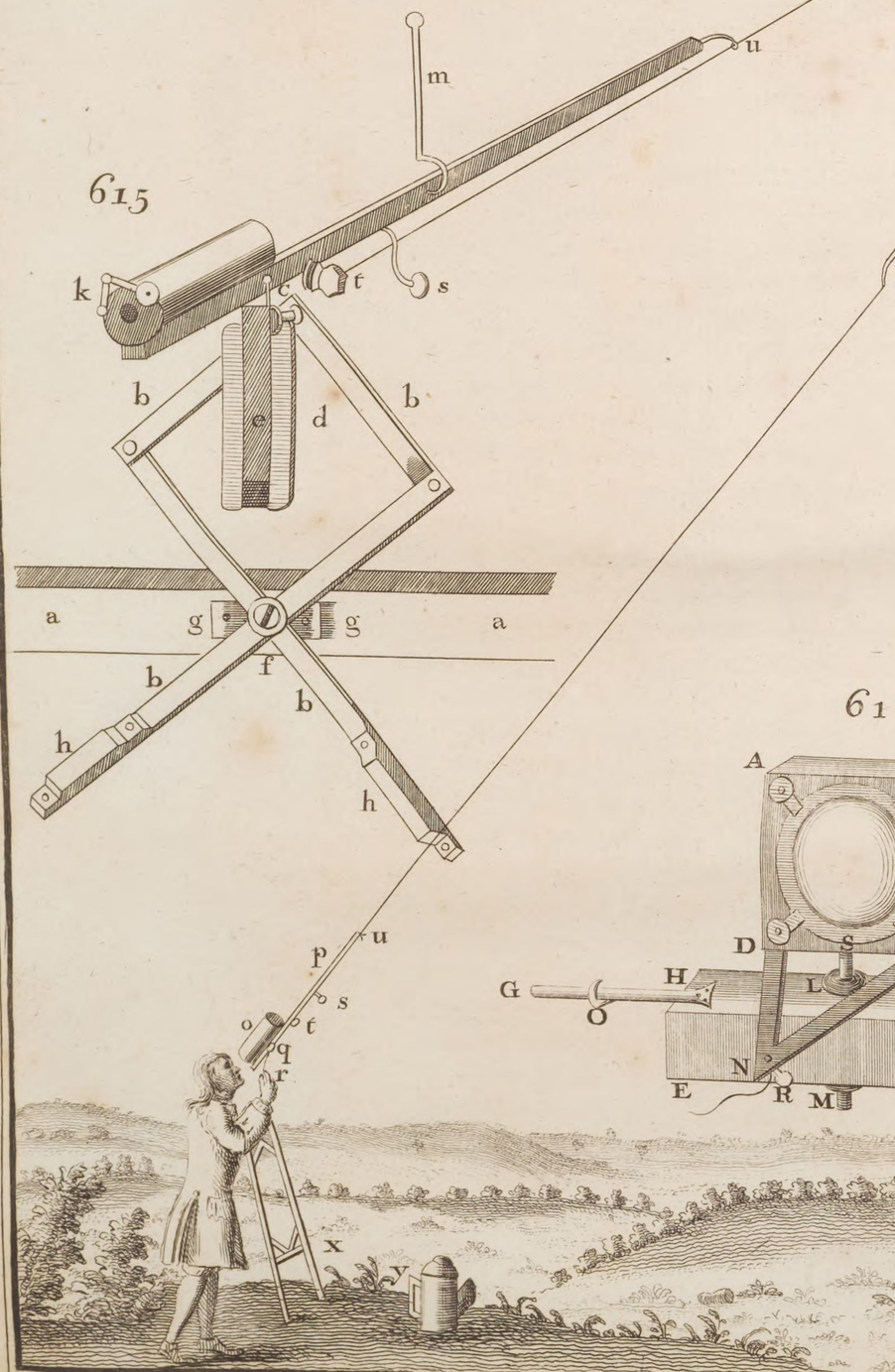
889. The dimenſions of this inſtrument are theſe. The length of the teleſcope, or the radius of the ſector is 2 $\frac{1}{2}$ feet; the breadth of the radius near the end *C* is 1 $\frac{1}{2}$ inch; and at the end *D* 2 inches. The breadth of the limb *AB* is 1 $\frac{1}{2}$ inch, and its length 6 inches, containing 10 degrees


P
p
k
b
d
d
d
a

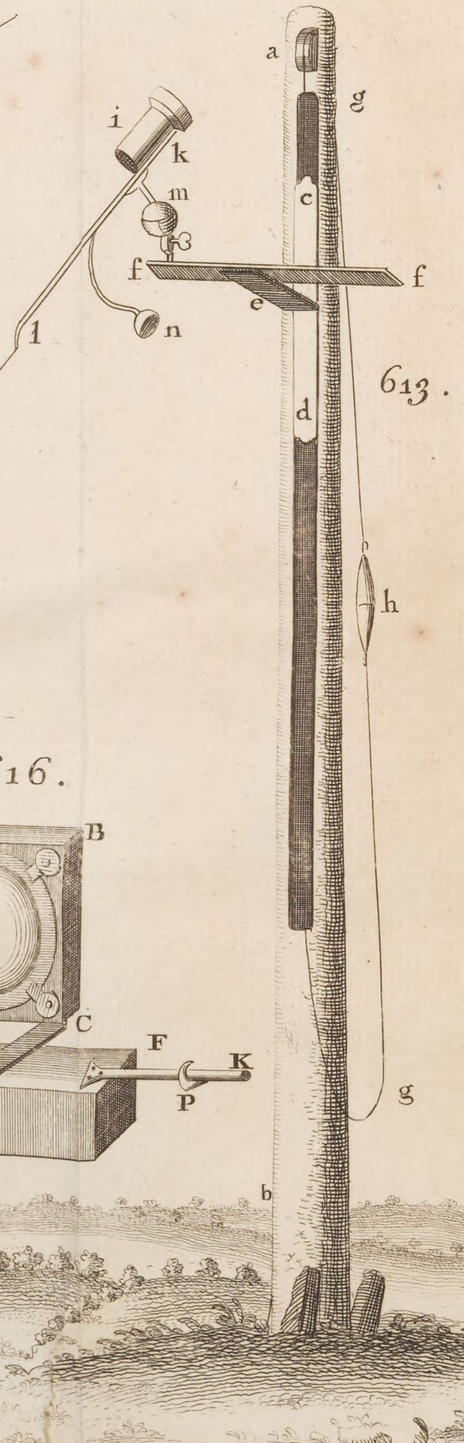
614



615



613.



616.

grees divided into quarters and numbered from either end to the other. The telescope carries a *Nonius* or subdividing plate [a]; whose length, being equal to 16 quarters of a degree, is divided into 15 equal parts; which in effect divides the limb into minutes, and by estimation into smaller parts. The length of the square axis *HIF* is 18 inches, and of the part *HI* 12 inches, and its thickness is about $\frac{1}{4}$ inch; the diameters of the circles *QR* and *abc* are each 5 inches. The thickness of the plates and the other measures may be taken at the discretion of a workman.

[a] Art. 861.

890. This instrument may be rectified, for making observations, in this manner. By placing the intersection of the cross hairs at the same distance from the plane of the sector, as the center of the object-glass found by art. 808, the plane described by the line of sight, during the circular motion of the telescope upon the limb, will be sufficiently true or free from conical curvity: which may be examined by suspending a long plumb-line at a convenient distance from the instrument, and by fixing the plane of the sector in a vertical position, and then by observing, while the telescope is moved by the skrew along the limb, whether the cross hairs appear to move along the plumb-line.

Rectification of the instrument.

891. The iron axis *hfo* may be elevated nearly parallel to the axis of the earth by means of a small common quadrant; and its error may be corrected by making the line of sight follow the circular motion of any of the circum-polar stars, while the whole instrument is moved about its axis *hfo*, the telescope being fixt to the limb. For this purpose let the telescope *kl* be directed to the star *a* when it passes over the highest point of its diurnal circle, and let the division cut by the Nonius upon the limb be then noted. Then after 12 hours, when the star comes to the lowest point of its circle, having turned the instrument half round its axis, to bring the telescope into the position *mn*; if the cross hairs cover the same star supposed at *b*, the elevation of the axis *hfo* is exactly right: but if it be necessary to move the telescope into the position $\mu\nu$ in order to point to this star at *c*, the arch $m\mu$ which measures the angle $mf\mu$ or *bfc*, will be known; and then the axis *hfo* must be depressed half the quantity of this given angle if the star passed below *b*; or must be raised so much higher if above it: and then the trial must be repeated till the true elevation of the axis be obtained. By making the like observations upon the same star on each side the pole, in the six a clock hour-circle, the error of the axis towards the east or west may also be found and corrected, till the cross hairs follow the star quite round the pole. For supposing *aopbc* to be an arch of the meridian, (or in the 2d practice, of the six a clock hour-circle,) make the angle *afp* equal to half the angle *afc*; and the line *fp* will point to the pole, and the angle *ofp*, which is the error of the axis, will be equal to half the angle *bfc* or $mf\mu$, found by the observation; because the difference of

Fig. 612.

the two angles *afb*, *afc* is double the difference of their halves *afo* and *afp*. Unleſs the ſtar be very near the pole, allowance muſt be made for refractions by the Table in the next book.

CHAPTER X.

To manage long teleſcopes without tubes.

Preface. 892. IT has has been ſhewn in the ſeventh chapter of the ſecond book, that the only way to improve dioptrick teleſcopes is to increaſe their lengths, in no leſs a ratio of the propoſed increaſe of the apparent diameter of an object, than what is duplicate of it. So that to magnify twice as much as before with the ſame light and diſtinctneſs, the teleſcope muſt be lengthened four times; and to magnify thrice as much, nine times; and ſo on. It was ſhewn at the ſame time, that the neceſſity of uſing ſuch great lengths proceeds from the imperfections of the picture of the object formed by the object-glaſs; which imperfections mathematicians formerly attributed to the unfitneſs of ſpherical ſurfaces to produce an exact union of the rays in each pencil; and therefore they preſcribed to give their glaſſes the figures of conical ſections turned about their axes. But the great Sir *Iſaac Newton* diſcovered their miſtake, by ſhewing that conical figures would only remove an inconſiderable cauſe of theſe imperfections of the picture, while the different refrangibility of the rays, which is the principal cauſe, would unavoidably remain as before. This led him to the invention of his reflecting teleſcope about the year 1670, but before it was lately brought into common uſe by the ingenious Mr. *Hadley*, mathematicians ſtill deſpaired of any other improvements of teleſcopes but by lengthening their tubes, and contriving proper machines to move and manage them. A great variety of theſe machines may be ſeen in *Hevelius*'s *Machina Cæleſtis*; but aſtronomy at laſt was very happily freed from this expenſive lumber by the ingenuity of the great *Hugenius*: who placing the object-glaſs upon a long upright pole, contrived to direct its axis towards any object by a fine ſilk line, coming down from the glaſs above to the eye-glaſs below. This invention was ſucceſsfully practiced both by himſelf and others; particularly with us by the late Dr. *Pound* and by his nephew Mr. *James Bradley* Profeſſor of Aſtronomy at *Oxford*, with an object-glaſs of 123 feet focal diſtance, and an apparatus belonging to it, made and preſented by *Hugenius* to the Royal Society, and deſcribed in his *Aſtroſcopia Compendiaria tubi optici molimine liberata*, printed at the *Hague* in 1684; which treatiſe I have here tranſlated. For although Sir *Iſaac Newton*'s reflecting teleſcopes are now made in ſo great perfection, that one of five feet is equivalent to the *Hugenian* glaſs of 123 feet; yet there is one conſiderable advantage in theſe dioptrick teleſcopes

which

which the catoptrick ones have not, to wit in applying a micrometer. For the diameters of the pictures of objects in both teleſcopes being as their lengths, the ſame micrometer will meaſure the larger image more exactly than the ſmaller, in the ſame proportion of the lengths of the teleſcopes. It is chiefly for this reaſon that I give Mr. *Huygens*'s account of his aerial teleſcopes; and alſo of a ſmall variation in their contrivance by Mr. *de la Hire*.

893. "In a large area every way open to the view of the heavens, let a long pole or maſt be fixt upright in the earth. That which we firſt uſed was 50 feet high; and was ſufficient for a teleſcope 70 feet long or more, though not for the greateſt altitudes of the ſtars, for which the pole ſhould be almoſt as long as the teleſcope. Before the pole is erected let one ſide of it be wrought plane, to which let two long ledges be nailed parallel to each other, at the diſtance of an inch and a half; ſo as to make a long channel a little broader within than without, reaching from the top of the maſt within three feet of the bottom. And through the top of the maſt, over againſt this long channel, let a ſlit or mortiſe be made to receive a pully, over which let a rope be put as long again as the maſt, and near half an inch thick. And for a perſon to aſcend to the top upon occaſion, let wooden triangles be fixt to the maſt at equal intervals, to ſerve for ſteps. Then let the bottom of it be daubed over with hot pitch, ſo far as it is to be fixt in the ground, and let ſand be put into the hole round about it to keep it from rotting. The uſe of the maſt is to raiſe the object-glaſs to a proper height as follows. Fig. 615.

894. Let a board of two foot long be ſloped on each ſide, ſo as to move freely up and down in the channel abovementioned; and to the middle of this board let a wooden arm be fixt and extended a foot in length from the maſt; and let the middle of another board, a foot and an half long, be laid horizontal and at right angles over the end of that arm, and be fixt to it. The object-glaſs muſt be placed upon one end of this tranſverſe board; and the whole muſt be lifted up and down by the rope abovementioned; the ends of it being tyed to the top and bottom of the upright board that ſlides in the channel; and the whole muſt be counterpoiſed by a leaden weight fixt to the rope on the other ſide of the pully, in ſuch a place that the weight may be at the top when the object-glaſs is at the bottom, and on the contrary. The ends of the weight muſt be made of a conical figure to prevent its catching and ſticking at the wooden triangles abovementioned.

895. The manner of fixing the object-glaſs is this. Firſt let it be fixt within a tube four inches long, made of ſheet-tin or braſs; and to the outſide of this tube (or rather to a hoop that ſurrounds it) let a ſtraight

ſtick be fixt, about an inch thick, and extended about 8 inches or a foot from the end of the tube. Then to ſupport the whole let a braſs-ball as big as a filbert-nut, be fixt to the ſaid ſtick by a ſhort neck, and be lodged in a hollow ſocket, in which it may play very freely without danger of dropping out. Let the ſocket and its cylindrical pedeſtal be ſlit in two halfs, and be held together by a ſkrew paſſing through both, but not ſo cloſe as to pinch the ball. By this means the object-glaſs and the ſtick annexed, are moveable every way; and to keep them in equilibrio, an equal counterpoiſe of lead is fixt to the under part of the ſtick by a ſtiff braſs wire; ſo that by bending this wire to and fro, the common center of gravity of the weight, the lens and parts annexed may be eaſily placed in the center of the braſs ball; and then the whole compound will be moveable with the leaſt touch, and will reſt in any given poſition: and in this lies the judgement of the whole invention. Having ſtuck the pedeſtal of the ball and ſocket into a hole in the end of the tranſverſe board above deſcribed; to the tail of the ſtick, annext to the object-glaſs, let a ſilk line be tied, and let the length of it rather exceed the length of the intended teleſcope, that the other end of it may be brought to the eye-glaſs. Hence when the object-glaſs is raiſed towards the top of the maſt, by gently drawing this thread while you are moving round the maſt, the object-glaſs will readily obey its motion, and be directly oppoſed to what ſtar you pleaſe; which could never be performed without placing it in the ſtate of libration above deſcribed. Now ſince it is abſolutely neceſſary that the ſtick, annexed to the object-glaſs, ſhould be parallel to the extended line; for this purpoſe a ſhort braſs wire is fixt into the tail of the ſtick, and is bent downwards ſo far till the end of it, where the ſtring is tied, be as much below the ſtick as the center of the ball and ſocket is. The reaſon for uſing a flexible ſpringing wire ſhall be given hereafter.

896. Let us now deſcribe the poſition of the eye-glaſs and its connection with the object-glaſs; which will take up but few words, ſince the mechaniſm below is almoſt ſimilar to that above. Here alſo the eye-glaſs is included in a ſhort tube, connected to a ſtick, which may alſo have a ball to reſt upon or rather a little tranſverſe axis, and a weight below the ſtick to ballance the tube and eye-glaſs. The obſerver takes hold of a handle fixt to the tranſverſe axis, and holds the lower ſtick directed towards the upper one, by means of the line that connects them and winds about a pegg fixt in the lower ſtick; ſo that by pulling gently, to extend the line, it is evident the two glaſſes will become parallel to each other. The lower part of the ſtring comes through a ſmall hole made with a wire at the far end of the lower ſtick; and the obſerver by turning the pegg, like that of a muſical inſtrument, ſhortens or lengthens the line at pleaſure; till he brings the interval between the glaſſes to a juſt length, requiſite for diſtinct viſion.

897.

897. But in order to keep the eye-glaſs ſteady, as it ought to be, it is convenient for the obſerver, whether ſtanding or ſitting, to ſupport his arms upon a reſt; conſiſting of a ſlender tranſverſe beam ſupported by two feet, made of any light ſtuff, while he holds the eye-glaſs in one of his hands: which is a readier and more commodious way than to fix the eye-glaſs upon a reſt ſupported by three feet.

898. In order to find a ſtar in the teleſcope, when the nights are dark, we make uſe of a lantern; which collects the light into a ſtream either by tranſmiſſion through a convex lens, or by reflection from a concave ſpeculum. For by directing this ſtream of light till it falls upon the object-glaſs and makes it viſible, it is eaſy for the obſerver to change his place, till he finds the ſtar is covered by the middle of the object-glaſs; and then to apply his eye-glaſs. Which is ſooner done than with a teleſcope conſiſting of a long tube. By moon-light the object-glaſs is viſible without the help of the lantern. But in viewing the moon through the teleſcope, it is neceſſary to fix an umbrella made of thin paſt-board about the object-glaſs, of ſuch a diameter as ſhall cover a ſpace in the ſky above twice as broad as the moon; to intercept that light from coming to the eye which would paſs by the ſides of the object-glaſs, and by mixing with the light that comes through the teleſcope, would dilate the appearance of the lights and ſhades in the face of the moon. But to underſtand this mechaniſm more clearly we have added a deſcription of the figure.

Fig. 613.

The pole or maſt is *a b*
The moveable board in the long channel, *c d*
The arm fixt perpendicular to it, *e*
The tranſverſe board that ſupports the object-glaſs, *f f*
The endleſs rope going round the pully, *g g*
The weight annext to it, *h*
The pully at the top of the pole, *a*
The tube that holds the object-glaſs, *i*
The ſtick faſtened to that tube, *k l*
The braſs ball fixt to the ſtick and lying in the ſocket, *m*
The lead at the end of the braſs wire, *n*
The ſhort wire at the end of the ſtick, *l*
The tube that holds the eye-glaſs, *o*
The ſtick fixt to the tube, *p*
The little moveable axis, *q*
The handle, *r*
The leaden bullet, *ſ*
The ſpool to wind the thread upon, *t*
The pins ſtuck croſs each other to make a hole for the line to paſs through, *u*

The ſlender ſilk-line, *lu*
The reſt for the obſerver to lean upon, *x*
The Lantern, *y*
The triangular ſteps for climbing up the maſt *a*, that would have embarraſſed the figure, are omitted.

899. I now proceed to remove ſome objections that may ariſe in peoples minds, who have not tryed this mechaniſm. And firſt they may fear that the ſmall deflection of the line, eſpecially when ſo long as 100 or 200 feet, may poſſibly deſtroy the parallelism of the glaſſes to which it is connected: and indeed there would be good reaſon for this apprehenſion, if it were neceſſary to uſe a ſtrong heavy line; for then it would require a very great force to reduce it to a tolerable degree of ſtraightneſs. But the object-glaſs being ſo nicely poiſed in the manner deſcribed, is capable of being directed by the leaſt touch of the lighteſt thread. Fifty foot of the ſilk-line we make uſe of, weighs but half a dram, and bears a tenſion by ſeven pound weight before it breaks. Therefore the ſmall deflection of a line of this length or even much longer can do no harm, though but moderately ſtretched by a force equivalent to 2 or 3 pound weight. Eſpecially as it is well known that a geometrical parallelism of the glaſſes is by no means neceſſary.

900. For it is certain that the forces requiſite to ſtretch two lines of equal lengths to equal curvities, are as the weights of the lines. For example 50 foot of a line weighing one ounce, would require a force equivalent to 48 pounds, to reduce it to the ſame curve as 50 foot of our line is reduced to by 3 pounds weight. For it is the ſame thing whether 16 ſeparate lines, each weighing half a dram, be ſtretched ſeverally by 3 pounds apiece, or whether they compoſe a cord of an ounce weight, to be ſtretched by 16 times 3 pounds or 48 pounds.

901. But this flexure of the line may be farther examined by the joint aſſiſtance of geometry and experiment. For the curvity of the line, when ſo very ſmall, may be conſidered as a portion of a parabola without ſenſible error; and when 150 feet of this line was ſtretched horizontally by a weight of 2 pounds and a half, we found by experiment that the depth of the loweſt point of it, below the level of the extreams, was about a quarter of a foot. Let *abc* repreſent this parabolick thread, *db* its depth below the level *adc*. Let the lines *ae*, *cf* be tangents to the parabola, meeting the lines *ce*, *af* drawn parallel to *db*. Now by looking from the point *a* along the direction of the thread to the point *e*, the line of viſion fell upon *e* about a foot below *c*. Whence it follows that *db* was a quarter of a foot. But *ce* and *af* are equal. Therefore the thread *cba* directs the axis of the object-glaſs placed at *c*, not to the point *a* but along the tangent *cf*; ſo that the eye at *a* is a foot too high; which does no harm at the diſtance of 150 feet. For the angle of deflection *cae* or *acf* is but two fifths

Fig. 614.

of

of a degree, which though it may ſafely be neglected, yet I ſhall ſhew how it may be corrected once for all. If we take the diſtance *gh* double of *ac*, or 300 feet, ſo that *gabch* may repreſent this bended line, the depth *kb* will be quadruple of *db*; but the angle of deflection only double of the former that is $\frac{4}{5}$ of a degree; as is eaſily underſtood by drawing the tangent *gl* meeting the perpendicular *hl*. For *hl* is quadruple of *kb* or *ee*; but *gh* was double of *ac*; therefore the angle of deflection *hgl* may be reckoned double of the former angle *cae*.

902. Now though this error of 48 minutes might be neglected without inconvenience, yet to take away all ſcruples I will ſhew how this deflection of the object-glaſs and all others may eaſily be corrected once for all. The object-glaſs being ballanced as before and drawn down to the level of the eye, ſtretch out the ſilk-line with one hand and apply it to your eye; and holding the lantern by the ſide of it with your other hand, while the line runs between your fingers in receding from the object-glaſs, take notice whether a double image of the candle appears in the middle of the lens; and if this ſhall happen when the whole line is run out (its length being equal to the focal diſtance of the lens) it is a certain ſign that the lens has a true ſituation. But if only one of the images of the candle appears, the lens is ill ſituated, and worſe if neither of them. By obſerving on which ſide of the ſtring the reflected light is caſt, the wire at the end of the ſtick, annext to the object-glaſs, muſt be bent a little towards the ſame ſide: and then the obſerver muſt examine the reflected light again, till he finds both the images of the candle to coincide at the end of the extended ſtring; which muſt be drawn by a gentle force equivalent to two or three pounds weight; to which he muſt endeavour to accuſtom his hand. The poſition of the lens being thus adjuſted will ſerve for any elevation of it. Too nice a judge may object indeed that when the line is obliquely ſituated to the horizon, as in making obſervations, the flexure of it will be leſs than when it was horizontal; but the difference will be exceeding ſmall in a ſtring ſo very light; and as we obſerved before, an exact paralleliſm of the glaſſes is not at all neceſſary.

903. If it be objected that the wind will occaſion a great diſturbance, by driving and curling the ſtring, eſpecially when ſo very long as we have mentioned; it muſt be remembered that very long tubes are much more expoſed to the like commotions; inſomuch that it is frequently impoſſible to make any obſervations while a moderate wind is ſtirring. But at theſe times it is well known that no obſervations can poſſibly be made for another reaſon; which is, that the tranſparency of the air, though it looks ſerene, is almoſt always vitiated by winds, and rendered unfit to obſerve in. Which happens alſo ſometimes in a very calm and ſerene atmoſphere, when the ſtars twinkle very much, by the interpoſition of ſome humid vapours; which occaſion the edges of the moon and planets

to tremble and fluctuate in the teleſcope, and utterly deſtroy the diſtinctneſs of their appearance. Inſomuch that the goodneſs of the glaſſes might often be ſuſpected, if they had not been approved in a more favourable atmoſphere. The ſame moiſture does alſo frequently adhere to the ſurface of the object-glaſs, and makes objects appear dull. But this inconvenience may be prevented by warming the glaſs at a fire.

904. If the lantern does not project a ſufficient light to a great diſtance, it may be improved both by uſing a larger wick to the candle, and alſo by a broader lens and of a leſs convexity in proportion as the object-glaſs is remoter.

905. There are ſeveral ways of preparing a pole of a proper height. A pole that is too weak may be ſtrengthened by another of half the height, placed by the ſide of it and braced to it by tranſverſe boards; or ſtill much firmer by uſing two ſhort poles placed at 2 or 3 feet diſtance from each other and from the long one in the form of a triangle and braced all together. By this means it is eaſy to elevate the object-glaſs to an hundred feet; and ſtill much higher, either by making a ſtronger preparation of poles and boards at the ground, or by placing the pole upon the top of a turret or the corner of a high building, or even upon the middle of the building; where a perſon may ſtand, whoſe buſineſs it is to elevate or depreſs the object-glaſs according to direction from below.

906. Becauſe unexperienced perſons cannot eaſily find out and follow an object, with this ſort of teleſcope, nor be ſhewn it by another without ſome method of fixing the eye-glaſs; we will now deſcribe how this may be done, by a ſmall machine placed upon a two legged reſt.

Fig. 615. Part of the top of this reſt is repreſented at *a a*, and a variable rhombus made of braſs plates at *b b*; two of whoſe ſides are produced through their interſection *f* till they equal the ſides of the rhombus. The length of each ſide is $5\frac{1}{2}$ inches and the breadth a little above an inch, and the thickneſs one tenth. This rhombus is fixt at *g g* to the ſide of the upper beam of the reſt by an iron ſkrew-pin, paſſing through one angle of it, and through a thin circular ſpringing plate, beaten concave a little; by which the motion of the rhombus about the pin may be equable and pleaſant with a proper degree of ſtiffneſs. From the oppoſite and ſuperior joint of the rhombus, a ſmall axis or pillar projects about half an inch; and from the end of it there hangs a moveable plate 4 inches long and half an inch broad; which lyes out of ſight, being covered by a wooden handle *d* of the ſame length and riveted to it. Into a dove-tailed channel made lengthways in the foreſide of the wood, is inſerted another plate *e*, which upon a very ſmall axis ſupports the ſtick and tube that carries the eye-glaſs. And the whole is ballanced or counterpoiſed upon the axis *f* by proper weights *h h* fixt to the ends of the two ſides of the rhombus that were produced.

907.

907. Things being thus ordered to whatever place the obſerver ſhall move the eye-glaſs by the handle *d*, while it always hangs downwards, there it will remain at reſt; and thus may another experienced perſon ſucceed the obſerver to view any object already found for him. For the reſt being placed a little inclining, the tenſion of the line that connects the glaſſes will keep it from falling, though it has but two feet; and the gravity of the reſt thus leaning towards the ſpectator, will keep the ſtring to its due tenſion; ſo that a more convenient contrivance cannot be wiſhed for. The height of our reſt is 4 feet 9 inches, and its weight is 2 pounds and $\frac{3}{4}$ths. The weight of the eye-glaſs, tube and ſtick annexed is $\frac{1}{2}$ a pound. That of the rhombus and its counterpoiſe $2\frac{1}{4}$. Which I mention for the uſe of ſuch perſons as may be deſirous of having a like apparatus to ours, which by experience we have found moſt convenient.

908. To perfect this method of obſerving I will mention one contrivance more, which in ſome kinds of obſervations is very uſeful. In ſearching very diligently for thoſe ſatellits of ſaturn which *Caſſini* diſcovered, I perceived a great difficulty in finding them out, unleſs the night was very dark; and ſoon underſtood that a faintiſh light in the air was the cauſe of it; not that which comes through the object-glaſs, but what paſſes by, round about the ſides of it. To intercept this troubleſome light I put up the paper-umbrella round about the object-glaſs, which I uſed for the moon; and upon farther conſideration I added another expedient of greater efficacy; and this in effect was to contract the pupil of the eye, (which in the dark is very much dilated,) by looking through a very ſmall hole made in a thin plate held cloſe to it. Through this I could preſently perceive three ſatellits of ſaturn very plainly, but when I removed the plate I could only perceive the middlemoſt of them, which I formerly diſcovered. But becauſe the object is not ſo eaſily found when the pupil is thus contracted as when it is wider, therefore I fixt that little perforated plate to a ſmall joint at *k* ſhaped like a greek Λ, which turned upon a pin in the bottom of the tube that carries the eye-glaſs; in which bottom there is a larger hole to look through for finding the object, before the little plate with the ſmaller hole is ſlipped over it.

909. It may poſſibly be thought that the object ſhould appear darker through the ſmaller hole than through the wider. But this is certain, if the diameter of the ſmaller hole, be to the diameter of the aperture of the object-glaſs, as the focal diſtance of the eye-glaſs, is to that of the object-glaſs, that every thing will appear juſt as bright through the teleſcope as if that hole was removed. Nevertheleſs it is better to double this breadth of the hole or rather more, to facilitate the finding of a ſtar and to keep it longer in view. The breadth of this hole applyed to our 34 foot teleſcope, is about the ſixteenth part of an inch, and its diſtance from the eye-glaſs is exactly $2\frac{1}{2}$ inches, which is alſo its preciſe focal diſtance.

ſtance. And this place of the hole muſt be accurately adjuſted; becauſe if it be ſituated any where elſe, you cannot ſee ſo much at one view through the teleſcope as uſual. And the place of it is eaſily adjuſted by bending the joint it is fixt to, either to you or from you; having placed it at firſt about half an inch from the end of the tube.

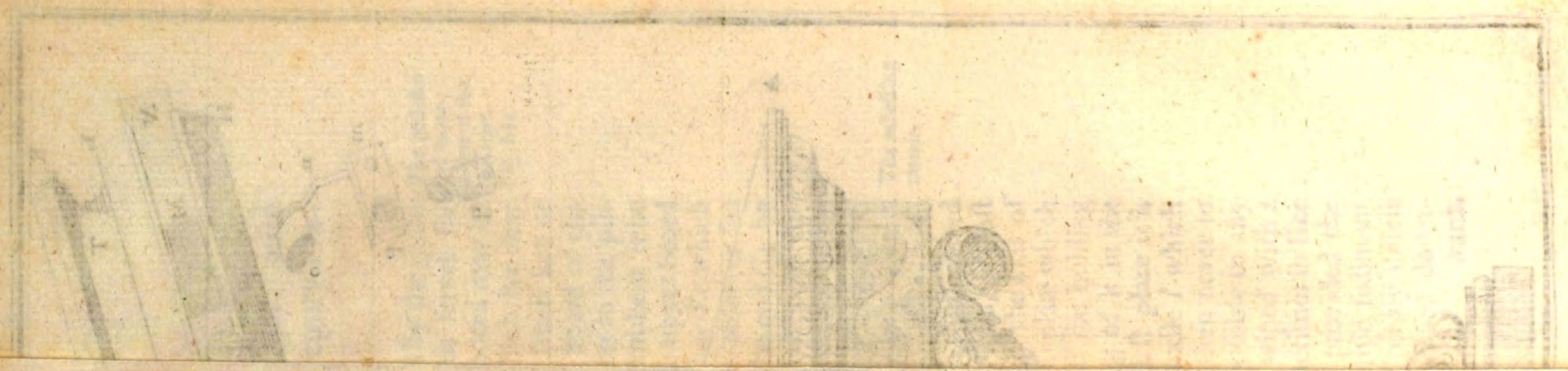
910. The diameter of the paper-umbrella ſhould be about the 45th part of the length of the teleſcope. And becauſe the interpoſition of it creates ſome difficulty in finding an object; to obviate this inconvenience, I erect a perpendicular ſight *m* upon the ſtick annexed to the eye-glaſs; the top of it being elevated as high above the axis of the teleſcope as the upper margin of the umbrella. Therefore by raiſing the eye till we can juſt ſee the ſtar over the upper edge of the umbrella, and then bringing the ſight or top of the ſtyle into the ſame line, when the eye is removed you will alſo find the ſtar in the teleſcope or very near it. By a little uſe and practice this method of obſerving becomes very eaſy. So far Mr. *Huygens*.

911. Signor *Blanchini* in his book upon the Rotation of Venus, pag. 59. refers to ſome farther improvements of this method by *Huygens* himſelf; which he ſays were printed in the Memoirs of the Royal Academy of Sciences at *Paris* about the year 1712, but I could not find them.

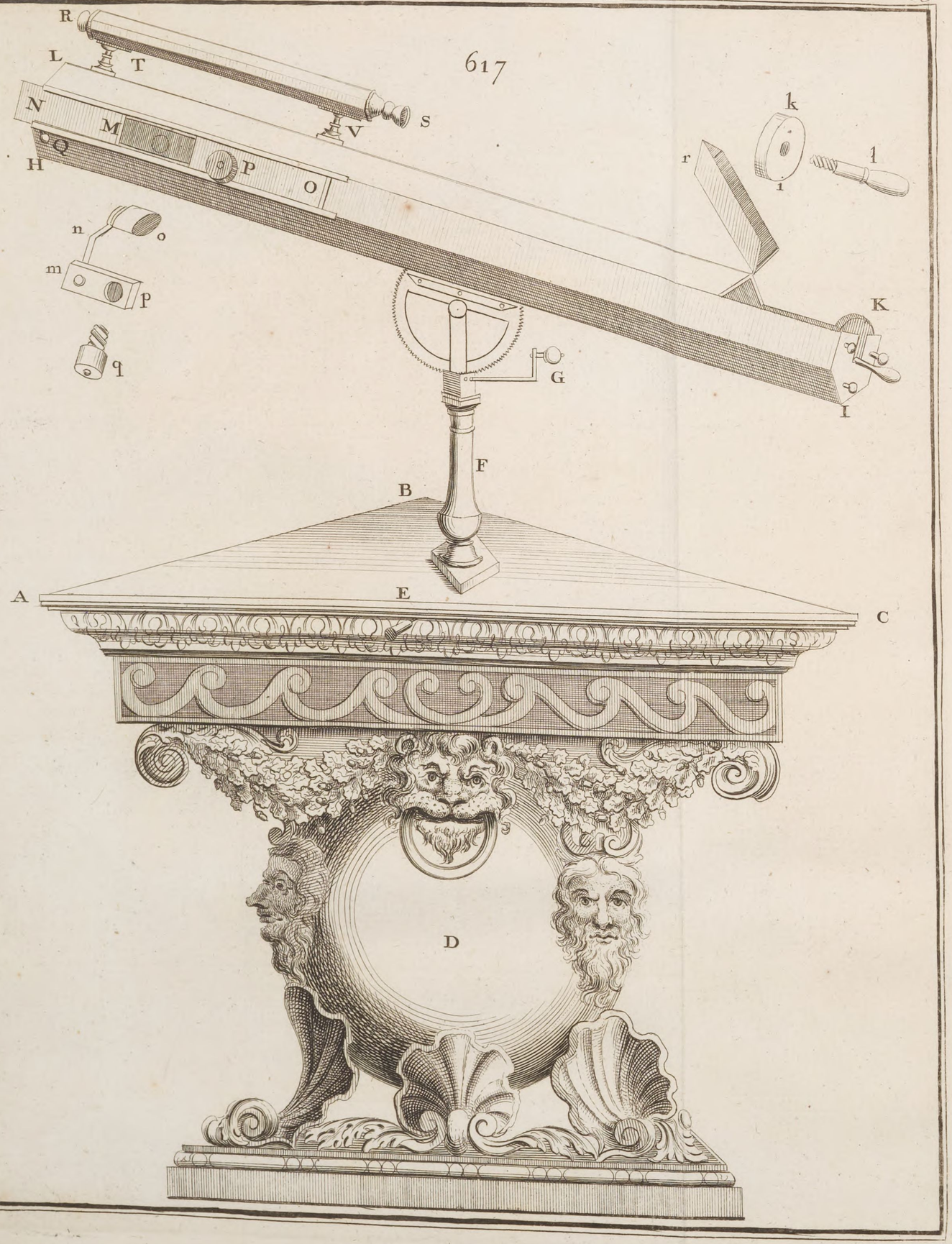
912. Mr. *de la Hire*'s little machine for managing the object-glaſs, is this [a], "I take, ſays he, an oblong piece of wood *EF* of a convenient ſhape and magnitude, and to the ends of the upper ſide of it I fix two cylindrical ſtaffs *GH*, *IK* to ſerve as an axis. Then I bore a hole through the middle of the piece *EF* at right angles to that axis: through this hole there paſſes another wooden axis *SLM*, two wooden nuts *L*, *M* being ſkrewed over each end of it, to keep it from dropping through the hole. To its upper end *S* there is fixt an upright ſquare board *AC* with a circular hole in it, to receive the object-glaſs. To the bottom corners of this board are fixt two equal wooden rulers, which meet at *N* and form an equicrural triangle, whoſe plane is perpendicular to the board *ABCD*. Under the corner *N* is ſkrewed in a wooden pin, to which the long ſtring for moving the object-glaſs is tied, as much below the line *CD* as the axis *GK*; which lyes upon two tenter-hooks *O*, *P*. To keep off the dew from the object-glaſs he preſcribes to include it in a paſteboard tube, made of ſpongy paper to ſuck up the humidity of the air. And to find an object more readily he preſcribes a broad annulus of white paſteboard to be put over the tube that carries the eye-glaſs, upon which the picture of the object being painted, an aſſiſtant that ſees it may direct the tube of the eye-glaſs into the place of it: or that the obſerver himſelf may ſee it, he preſcribes to receive it upon a ring of tranſparent oiled paper, paſted upon a circular frame inſtead of the paſteboard.

[a] Mem. de l' Acad. 1715. Fig. 616.

CHAP-



617



CHAPTER XI.

Sir Isaac Newton's *reflecting teleſcope made and deſcribed by the Honourable* Samuel Molyneux *Eſquire, and preſented by him to his Majeſty* John V. *King of* Portugal: *with other kinds of mechaniſm for this and for Mr.* Gregory's *reflecting teleſcope.*

913. *ABC* repreſents a triangular board or table ſupported by the globe *D* and by the annexed carvings and maſks; this ſerves for the baſis or pedeſtal of this inſtrument. Upon occaſion this board may be taken off by unſkrewing three iron ſkrews, the heads of which lye near the three volutes at the three corners. At *E* is repreſented a ſmall key or handle, which turns ſome wheel-work concealed under the board of the table, and which ſerve to give an horizontal circular motion to the pillar *F*, placed in the middle thereof, and to the ſuperincumbent tube *HIKL*. If this ſhould ever be out of order, by taking off the upper board as aforeſaid it may be rectified. At *G* is repreſented another handle which gives the tube its perpendicular motion; ſo that while the obſerver ſits with his right ſide applyed to the ſide of the table *AC* at the end *C*; by turning the two handles *E* and *G* he can give the tube any required elevation or azimuth, and thereby follow the motion of the heavenly bodies very commodiouſly.

The pedeſtal and movements of the teleſcope. Fig. 617.

914. The reflecting teleſcope it ſelf conſiſts of two metalline ſpecula and an eye-glaſs, which are to be duly placed in the tube *HIKL* left open at the end *HL*. The large concave ſpherical ſpeculum *ik* is to be placed within the tube at *IK*, in which are fixt 3 ſtops or bits of wood, againſt which the poliſhed ſurface of the ſpeculum being applyed, the axis of reflection will fall exactly in the axis of the tube. In the braſs-plate which cloſes this end of the tube, there are three ſkrews intended for holding the metal in this ſituation. As to this metal and the placing of it in the tube, many cautions are neceſſary to be obſerved. In the firſt place it is never to be touched but by ſkrewing into its backſide a handle *l* which fits the hole therein. In the next place great care muſt be taken never to breath in the leaſt upon it, and to expoſe it as little as poſſible to any damp air. If any thing of that kind happen, it ſhould be wiped with a thorough dry linnen cloth before a fire; and it may be ſometimes in like manner cleaned with a clean rag wetted in ſpirit of wine; provided the ſpirit be not left to evaporate; for that would leave an humid ſediment thereupon, ſufficient to hurt the fineneſs of its poliſh. Thirdly unleſs when in uſe it ſhould be conſtantly kept with the face applyed downwards

The reflecting metals.

wards upon the piece of plain glaſs, fixt for that purpoſe in the round cell in the red box.

Cautions in preſerving and placing them.

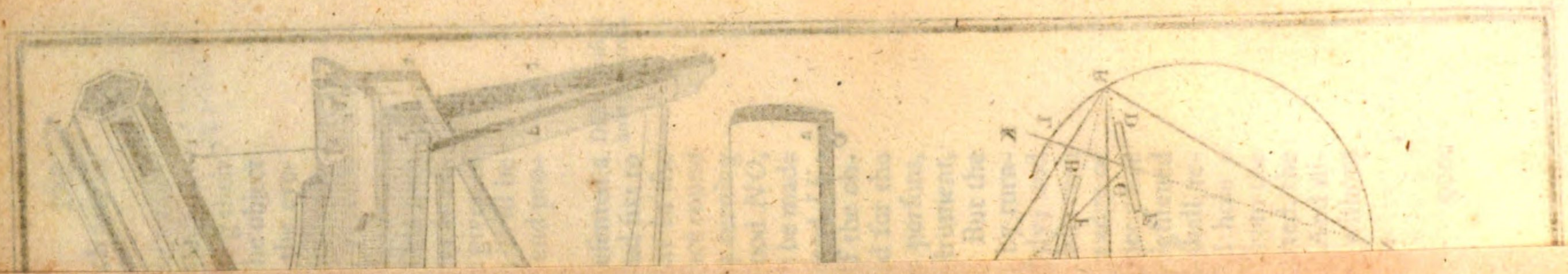
915. The ſpeculum is formed as a portion of a concave ſphere whoſe ſemiradius is about two foot 2 inches. The laws of reflection are ſuch that any error in its figure will produce about ſix times as great an irregularity in the picture formed in its focus, as the like irregularity would cauſe in a common refracting teleſcope. We have found by experience that an error of leſs than $\frac{1}{100}$ inch is capable of vitiating its figure; ſo that great care muſt be taken in placing the metal in the tube for uſe, againſt the three ſtops aforeſaid; that the three ſkrews at *IK* be gently and tenderly ſkrewed, only juſt ſufficient to bear the metal truly againſt the ſtops; for the ſmalleſt exceſs of ſtreſs in the ſkrews againſt the back of the metal, may diſtort and very much damage its figure. There is alſo a piece of wood *m* having a round hole *p* in it, and carrying a ſmall braſs arm *n* which holds the other ſmaller ſpeculum *o*, which is a plane. This ſpeculum muſt be always preſerv'd from the air, when out of uſe. When the teleſcope is to be uſed the cover of the ſmall ſpeculum muſt firſt be taken off; then place it in the tube at the hole *M* which exactly fits the aboveſaid ſquare piece. Preſs it in pretty tight and true, which if duly performed, the center of this ſmall ſpeculum will be placed in the axis of the large concave one, and will reflect all the parallel rays (which enter at the open end *HL*) to the round hole *p* in the ſaid ſquare piece, in which hole one of the two eye-glaſſes in its cell *q* is to be placed; and then the inſtrument is prepared for uſe.

The poſture of the obſerver.

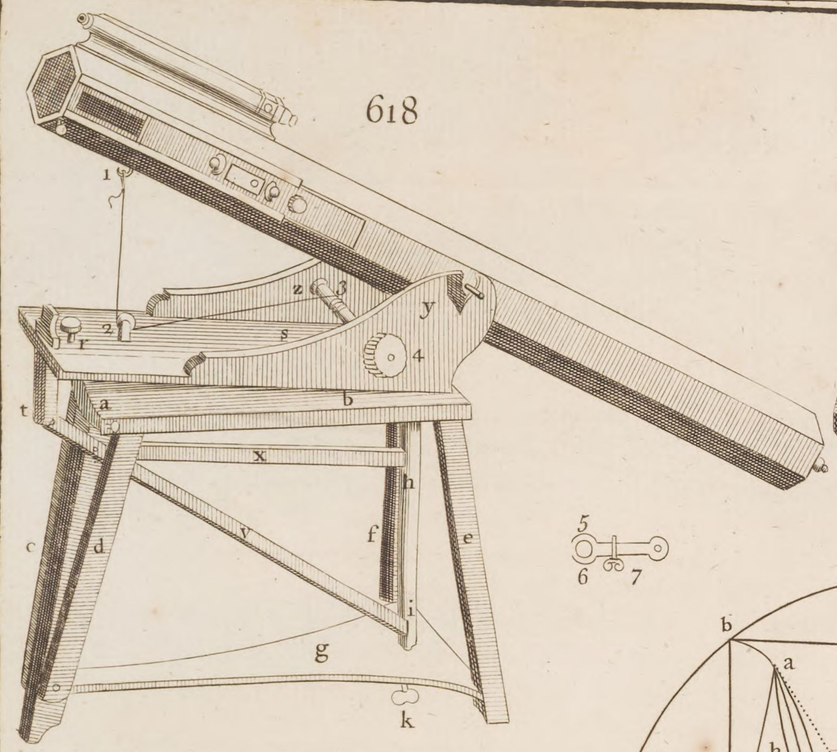
916. The obſerver is therefore to place himſelf at the ſide of the tube and to look in at *M* and he will then ſee the objects that lye at his left hand. This ſort of teleſcope in reſpect of a common glaſs one being to be uſed in the ſame manner as a German Flute is in reſpect of a common one. Some cautions are here to be given in reſpect of the little ſpeculum at *o*. In taking it out and putting it in again to its place in the box where it is laid by, when out of uſe, care muſt be taken not to ſhake or bend the arm; for the ſmalleſt accident of that kind will ſenſibly diſorder its ſituation. Three ſkrews may be ſeen at the back thereof, the middle one of theſe fixes it to the arm *mn*, the other two only preſs upon the back and ſerve only to adjuſt its ſituation to an exact angle of 45° with the axis of the great ſpeculum. This has been already done, but if any ſhock or other accident ſhould put it out of its place, it may be reſtored by theſe ſkrews. For whenever this inſtrument is uſed, the due performance of it abſolutely requires a true ſituation and direction in this ſpeculum.

The eye-glaſſes and magnifying power.

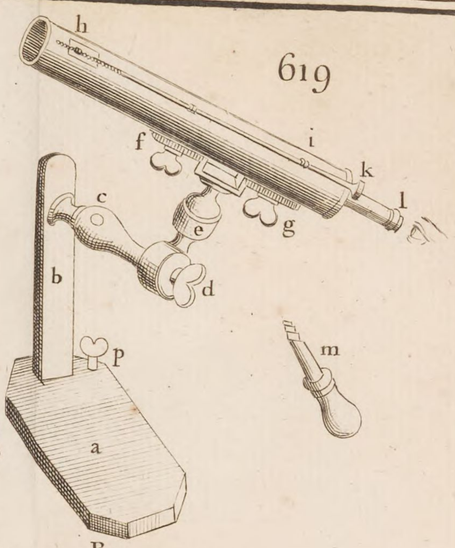
917. There are two eye-glaſſes whereof one that hath the largeſt aperture being made uſe of, the inſtrument will magnify as much as a common refracting teleſcope of about 20 or 22 feet long; and with the eye-

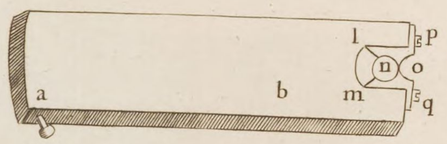


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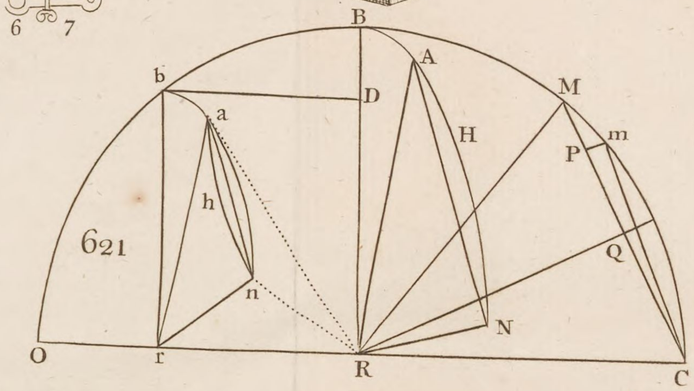


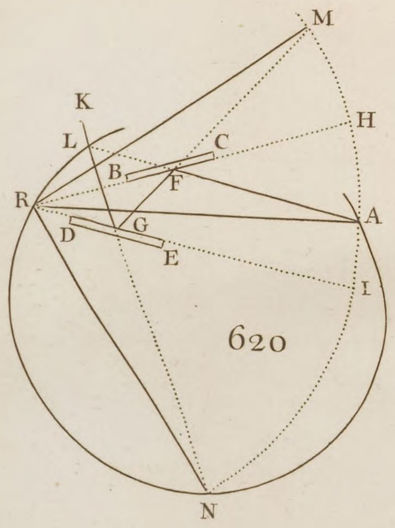
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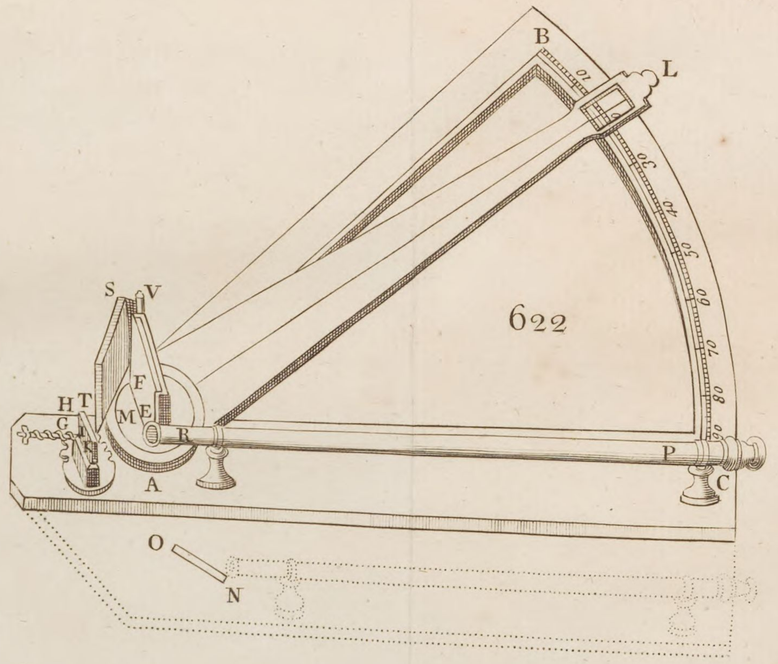


621





620



622

eye-glaſs that hath the ſmalleſt apertures it will magnify as much and as diſtinctly as one of 35 or 40 feet.

A deception in the magnifying power.

918. In comparing the effects of this inſtrument with thoſe of a teleſcope of that length, one very odd phenomenon is here to be mentioned; and that is, that ones imagination is always miſled in comparing their effects. For although by the minuteneſs of the parts viſible in the object obſerved, as alſo by the proportion between the focal diſtance of the eye-glaſs made uſe of and the focal diſtance of the great ſpeculum, it be demonſtrable that the diſtinctneſs and magnifying power thereof exceeds that of a refracting teleſcope of 35 feet; yet the ſpectator will always fancy that the refracter hath the advantage of this inſtrument. This may perhaps be attributed to the eye looking through ſo ſmall a hole as is neceſſary in this inſtrument; ſome other accounts may perhaps be given of this odd deception, but we need ſay no more here but that it will be certainly found upon a ſtrict examination to be but a deception, and probably an univerſal one to all perſons.

Diſtinct viſion how procured.

919. At *P* ſtands a round button of ivory, and at *Q* is repreſented a ſmall pin of ivory, which may be ſeen with a ſmall white thread fixt to it, at the end of the tube *H*. This thread at the other end is fixt in the inſide of the tube; and towards the middle of it, it is wound once round the inward end of the ivory button *P*; from this diſpoſition by turning round the ivory button *P*, the whole ſlider of black ebony wood *NO*, with the ſmall ſpeculum and the eye-glaſs applyed at *M*, may be made to approach to or recede from the large ſpeculum at the other end *IK*; and by this means its true diſtance and the diſtinct appearance of the object muſt be found; for various diſtances of the ſame object and for the various eyes of different obſervers; which variety in different perſons, from the great magnifying power of the eye-glaſs in this inſtrument, will be conſiderably more ſenſible than in a refracting teleſcope. But the true diſtance of the ſpecula will immediately be found in all caſes by turning this ivory pin *P* backwards and forwards very ſlowly and gently; and for celeſtial objects the true diſtance being once found for the obſerver's eye, a ſmall mark may be made acroſs the ſlider, and upon the edge of the tube to bring it ſpeedily and without any difficulty to its proper place at another time. By the little ivory pin at *Q* the ſtring may be tightened or relaxed to make the ſlider *NO* move moſt eaſily as occaſion ſhall require. Either of the eye-glaſſes, being applyed in the cylindrical hole *p* in the ſquare piece *mp*, may alſo be made to approach or recede from the focus, by turning round the ſmall tube *q* in which they are inſerted, the outſide whereof is wrought into a fine ſkrew for that purpoſe. And diſtinct viſion may alſo by that means be obtained for different eyes without moving the whole ſlider *NO*.

920.

The finder described.

920. *RS* repreſents a ſmall refracting teleſcope whoſe axis is parallel to the axis of the reflecter. In its focus there are placed two croſs hairs, and its only uſe is the more readily to find any object by the reflecter. The eye being applyed at *S*, turn the two handles at *E* and *G* till the point of the object to be viewed in the reflecter, falls exactly on the croſs hairs; then the eye applyed at *M* to the reflecter will ſee the ſame object: with this caution that as the whole inſtrument with the whole baſis can eaſily be moved, the moſt convenient ſituation for the obſerver will be to keep the tube *HI* nearly at right angles to the ſide *AC*, and to ſit with the ſide *AC* flat againſt his right ſide near *C*, as hath been already mentioned. And in finding the object at firſt with the ſmall refracting teleſcope, it is moſt convenient to ſtand at the corner of the table *C*. The handle *G* may be inſerted at either ſide of the pillar *F* as convenience ſhall require. At the ſmall pillars *TV* which ſupport and hold the ſmall teleſcope *RS*, there are ſome ſmall ſkrews near *T*, which being relaxed the direction of the tube *RS* may be altered horizontally by puſhing the tube with the hand ſideways either way as occaſion requires and then tightening the ſkrews again. And at *V* there are ſkrews and a ſpringing piece of braſs, which being relaxed or tightened will in like manner alter its elevation, ſo as to reſtore the paralleliſm of the tubes in caſe of any accident that may have diſturbed them. In obſerving, it is convenient not to touch the table in any part, and only to move the handles as the motions of the ſtar obſerved requires; for in an inſtrument that magnifies ſo much, the ſmalleſt motion or trembling is alſo magnified proportionably, and will be found inconvenient.

A pure air neceſſary for obſervations.

921. In conſidering the effects of this reflecting teleſcope, it muſt farther be noted, that for ſeeing clearly and well with it there is required a very clear equable and undiſturbed air. For if there be any vapours moving and undulating in the atmoſphere, which often happens though the night appears clear to the naked eye, theſe will entirely deſtroy the diſtinctneſs of the appearance: and it often happens that the air in this reſpect, at leaſt here with us at *Kew*, will ſo ſuddenly and ſo totally alter, that the object will appear very diſtinct and very confuſed afterwards in 3 or 4 ſeconds of time; and the air is ſometimes ſo very variable that objects will appear inſtantaneouſly to change, from being very clear to be confuſed, and then to be clear again. It will therefore be proper to accuſtome ones ſelf to the fluctuating appearances of ſome land-objects, ſeen in the day time through the reflecter; leſt the undulating appearances of the planets in the night may deceive one, and incline one to think this inſtrument doth not ſucceed ſo well as it is certain it will in a pure undiſturbed air.

Mr. *Hadley*'s machine for moving this teleſcope.

922. This other machine, for giving the horizontal and vertical motions to the teleſcope, is drawn according to Mr. *Hadley*'s deſign a little altered

altered by experience[a]. Here *ab* is an oblong board firmly ſupported by four legs *c*, *d*, *e*, *f*; but the two foremoſt *c*, *d*, being inclined inwards and united in one foot, the machine reſts upon 3 feet connected by a ſtrong triangular board *g* lying parallel to the board *ab*. Between the hinder ends of theſe boards there paſſes a ſtrong wooden axis *hi*, having a ſteel point fixt in the lower end of it, which reſts in a hollow conical point, punched in the end of a thick braſs pin *k*, ſkrewed through a nut in the lower board *g*. The upper end of the ſaid axis *hi*, paſſes through a large notch *plmq* cut in the end of the upper board *ab*, repreſented ſeparately below the figure: and the ſaid axis is here ſurrounded by a braſs hoop *n* well poliſhed and firmly fixt to it; and this hoop is touched in two places by two pieces of poliſhed ſteel, drove into the ſides of the notch *lm* and in a third place *o*, by the back of a ſpringing arch *poq*, preſſed againſt it by the heads of two ſkrews that go into the end of the board. To the top of this axis *hi* another board *rs*, lying parallel to *ab*, is firmly fixt; and under its fore-end *r*, which projects a little beyond the under board *ab*, a ſhort piece *t* is fixt, and is joined to the axis *hi* by the braces *v*, *x*, ſo as to form a ſort of a crane; which by turning about the axis *hi*, gives an horizontal motion to the tube of the teleſcope, while it reſts upon two oppoſite pins fixt in the ſides of it and lodged in two braſs notches, ſkrewed to the tops of the parallel wooden cheeks *y*, *z*, that are fixt upright upon the oppoſite ſides of the board *rs*. And this motion is gradually communicated by turning the head of a pin *r*, which goes down through a hole in the board *rs*, juſt clear of the end of the board *ab*. For this end being formed into an arch concentrick to the axis *hi*, there is a ſtring fixt to one end of the arch, which being applyed againſt the convexity of the arch and lapped once round the pin *r*, is alſo fixt to a pin at *a* at the other end of it, and being lapped about the pin, is tightened thereby.

[a] Phil. Tranſ. N°. 376. Fig. 618.

923. The hinder end of the tube of the teleſcope is juſt made to preponderate by the weight of the metal lodged in it. And then by means of a ſilk line 1, 2, 3 hung by a loop upon a hook 1, fixt under the fore-end of the tube, and thence paſſing under a pully 2 fixt to the moveable board *rs*, and from thence winding round a cylinder 3, 4, that turns round in two holes in the cheeks *y*, *z*, the tube is gradually elevated or depreſſed, by turning the knob 4 at the end of the cylinder; whoſe motion is adjuſted to a proper degree of ſtiffneſs by compreſſing two braſs arches 5, 6, ſhaped like a pair of tongs, againſt the far end of the cylinder, by a ſkrew, paſſing through their legs, as repreſented at the ſide of the figure. The tube of the teleſcope ſhould have been drawn octangular; and under the board *ab* a drawer is omitted, where the metals and eye-glaſſes are laid by when out of uſe.

924. A little machine for ſupporting and managing Mr. *James Gregory*'s reflecting teleſcope, brought into uſe by Mr. *Hadley*, and generally made

Mr. *James Gregory*'s reflecting teleſcope.

Fig. 619. made about 16 inches long, (being then equivalent to a common dioptrick teleſcope of 15 or 16 feet,) is contrived in this manner. The baſe of the pedeſtal *ab* is a thick board *a*, reſting upon 4 braſs feet underneath it; one of which being a pin, *p*, that ſkrews through the board, will make it ſteady upon any uneven plane: *b* is a ſmall upright pillar about a foot long, fixt in the board *a*; and *cd* is a braſs arm that ſkrews into it; *de* is a ſhort braſs piece that turns round upon the end of the arm *cd* and is tightened and ſtayed by the ſkrew *d*; *e* is a hollow ſocket having a round braſs ball in it, moveable any way, and tightened and ſtayed by a ſkrew or two not here expreſſed. The neck of this ball is fixt to the middle of a long braſs piece *fg*, which is fixt along the ſide of the tube *hi*, by the ſkrews *f*, *g*; and may be taken off at pleaſure. Thus the tube is ſupported and made capable of being gradually moved and ſtayed in any poſition. The larger concave metal, having a hole in the middle of it, is lodged at the bottom of the thicker tube *hik*; and the ſmaller concave metal is held in the axis of the tube, near the mouth of it, by a ſmall braſs arm coming through a ſlit in the tube at *h*. The long iron wire *hik*, on the outſide of the tube, ſkrews through a hole in the arm at *h*; and is confined from moving lengthways by two ſhoulders on each ſide of a ſmall hole in a fixt plate at *i*; and being turned round it ſelf by the knob at *k*, it draws the arm and little concave backward or forward, in order to procure diſtinct viſion of objects at various diſtances, and for eyes of different ſorts; while the obſerver is looking in at the end *l*, of a ſhort ſlender tube that is ſkrewed into the end of the larger, and carries the eye-glaſſes. When this teleſcope is uſed at home, the pedeſtal *ab* may be placed upon a table near a window or upon the window-board; but when it is uſed abroad the pedeſtal may be left at home. For having tapped a hole in the ſide of a tree or any piece of wood with the hand-auger *m*, the wood-ſkrew at the end *c* of the arm *cd* may be preſently ſkrewed into it; the auger *m* being put through the hole *c* to give power to the hand in turning the ſkrew. The theory of this teleſcope has been ſhewn by the by in art. 302, and ſhall be farther explained in the Remarks upon art. 125.

CHAPTER XII.

The deſcription of a new reflecting inſtrument for taking angles at Sea. By John Hadley *Eſquire, Vice-Preſident of the* Royal Society.

Phil. Tranſ. N°. 420. 925. THE inſtrument is deſigned to be of uſe, where the motion of the objects, or any circumſtance occaſioning an unſteadineſs in the common inſtruments, renders the obſervations difficult or uncertain.

926.

926. The contrivance of it is founded on this obvious principle in catoptricks: that if the rays of light diverging from, or converging to any point, be reflected by a plane poliſhed ſurface, they will, after the reflection, diverge from, or converge to another point on the oppoſite ſide of that ſurface, at the ſame diſtance from it as the firſt; and that a line perpendicular to the ſurface paſſing through one of thoſe points, will paſs through both. Hence it follows, that if the rays of light emitted from any point of an object be ſucceſſively reflected from two ſuch poliſhed ſurfaces; that then a third plane, perpendicular to them both, paſſing through the emitting point, will alſo paſs through each of its two ſucceſſive images made by the reflections; and that theſe three points will be at equal diſtances from the common interſection of the three planes: and if two lines be drawn through that common interſection, one from the original point in the object, the other from that image of it which is made by the ſecond reflection; they will comprehend an angle double to that of the inclination of the two poliſhed ſurfaces.

927. Let *RFH* and *RGI* repreſent the ſections of the plane of the figure by the poliſhed ſurfaces of the two ſpecula *BC* and *DE*, erected perpendicularly thereon, meeting in *R*; which will be the point where their common ſection, perpendicular likewiſe to the ſame plane, paſſes it; and *HRI* is the angle of their inclination. Let *AF* be a ray of light from any point of an object *A* falling on the point *F* of the firſt ſpeculum *BC*, and thence reflected into the line *FG*, and at the point *G*, of the ſecond ſpeculum *DE*, reflected again into the line *GK*; produce *GF* and *KG* backwards to *M* and *N*, the two ſucceſſive repreſentations of the point *A*; and draw *RA*, *RM*, and *RN*. Fig. 620.

928. Since the point *A* is in the plane of the ſcheme, the point *M* will be ſo alſo by the known laws of catoptricks. The line *FM* is equal to *FA*, and the angle *MFA* double the angle *HFA* or *MFH*; conſequently *RM* is equal to *RA*, and the angle *MRA* double the angle *HRA* or *MRH*. In the ſame manner the point *N* is alſo in the plane of the ſcheme, the line *RN* equal to *RM*, and the angle *MRN* double the angle *MRI* or *IRN*: ſubſtract the angle *MRA* from the angle *MRN*, and the angle *ARN* remains equal to double the difference of the angles *MRI* and *MRH*, or double the angle *HRI*, by which the ſurface of the ſpeculum *DE* is reclined from that of *BC*; and the lines *RA*, *RM* and *RN* are equal.

929. *Corol.* 1. The image *N* will continue in the ſame point; although the two ſpecula be turned together circularly on the axis *R*, ſo long as the point *A* remains elevated on the ſurface of *BC*: provided they retain the ſame inclination.

930. *Corol.* 2. If the eye be placed at *L*, (the point where the line *AF* continued cuts the line *GK*;) the points *A* and *N* will appear to it

at the angular distance ALN, which will be equal to ARN. For the angle ALN is the difference of the angles FGN and GFL; and FGN is double FGI, and GFL double GFR, and consequently their difference double FRG or HRI: Therefore L is in the circumference of a circle passing through A, N, and R.

931. *Corol.* 3. If the distance AR be infinite, those points A and N will appear at the same angular distance, in whatever points of the scheme the eye and specula are placed: provided the inclination of their surfaces remains unaltered, and their common section parallel to it self.

932. *Corol.* 4. All the parts of any objects will appear to an eye viewing them by the two successive reflections, as before described, in the same situation as if they had been turned together circularly round the axis at R, (keeping their respective distances from one another, and from the axis,) with the direction HI, *i. e.* the same way the second speculum DE reclines from the first BC.

933. *Corol.* 5. If the specula be supposed to be at the center of an infinite sphere; objects in the circumference of a great circle, to which their common section is perpendicular, will appear removed by the two reflections, through an arch of that circle, equal to twice the inclination of the specula, as is before said. But objects at a distance from that circle will appear removed through the similar arch of a parallel: therefore the change of their apparent place will be measured by an arch of a great circle, whose chord is to the chord of the arch equal to double the inclination of the specula, as the sines complements of their respective distances from that circle are to the radius: and if those distances are very small, the difference between the apparent translation of any one of these objects, and the translation of those which are in the circumference of the great circle aforesaid, will be to an arch equal to the versed sine of the distance of this object from that circle, nearly as double the sine of the angle of inclination of the specula, is to the sine complement of the same.

Fig. 621.

934. For let OBC in the annexed figure represent an infinite sphere, at whose center R are placed the two specula inclined to one another in any given angle, and let their common section coincide with the diameter ORC. Let BAN be the circumference of a great circle, to the plane of which the common section of the specula ORC is perpendicular, and BR its radius: let ban be the circumference of a circle parallel to BAN, and at the distance from it Bb: draw bD the sine, and br the sine complement of the arch Bb: BD is the versed sine of the same. Let A be a point of an object placed in the circumference of the great circle BAN, and N the point in which its image is formed by the two successive reflections,

flections, as before deſcribed; and let *a* be a point of another object placed any where in the circumference of the parallel *ban*, and *n* its image; and let *abn* be an arch of a great circle paſſing through the points *a* and *n*. The point *a* is at the ſame diſtance from the great circle *BAN*, as the point *b*, *i. e.* at the diſtance *Bb*. Draw *AR*, *AN*, *RN*, *ar*, *an*, *rn*, *aR* and *nR*.

935. By the fourth corollary the figures *ARN* and *arn* are ſimilar, and conſequently the line *AN* is to the line *an* as *AR* or *BR* is to *ar* or *br*, *i. e.* as the radius is to the ſine complement of the diſtance *Bb*. But *AN* is the chord of the arch *AHN* of the great circle *BAN* equal to the tranſlation of the point *A*, or double the inclination of the ſpecula, and *an* is the chord of the arch *ahn* of a great circle, meaſuring the angle *aRn*, by which the point *a* appears removed by the two reflections, to an eye placed in the center *R*. Therefore the tranſlation, or apparent change of place of the point *a* is meaſured by an arch of a great circle, whoſe chord is to the chord of the arch *AHN* (equal to double the inclination of the ſpecula) as the ſine complement of its diſtance from the great circle *BAN* is to the radius.

936. From any point *C* of the circumference *OBC* draw the chords *CN* and *Cm*, to the ſame ſide of the point *C*, and equal to the chords *AN* and *an* reſpectively, draw the radius *RM*, and from *R* and *m* draw *RQ* and *mP*, both perpendicular to *CM*, and cutting it in *Q* and *P*. *RQ* is the ſine complement, and *CM* double the ſine of half the angle *MRC*, or *ARN*, or of the angle of inclination of the ſpecula. The little arch *Mm* will repreſent the difference of the apparent tranſlations of the objects in *A* and *a*; and if it be very ſmall, may be looked on as a ſtraight line, and the little mixed triangle *MmP* as a rectilinear one, which will be ſimilar to *RMQ*, becauſe *RM* is perpendicular to *Mm* and *RQ* to *CM*, and the angles at *Q* and *P* right angles. The line *CP* may be taken as equal to *Cm*, and *MP* as the difference of the lines *CM* and *Cm*. Therefore the little arch *Mm* is to the line *MP* nearly as *RM* to *RQ*: but *CM* (i. e. *AN*) was to *Cm* (i. e. *an*) as *BR* to *br*, and the difference *MP* of *CM* and *Cm* to the difference *BD* of *BR* and *br*, as *CM* to *BR*. Therefore *Mm*, the difference of the apparent tranſlations, is to *BD*, the verſed ſine of the diſtance *Bb*, or to an arch equal to it, in the compound ratio of *RM* the radius to *RQ*, the ſine complement of the angle of inclination of the ſpecula, and *CM* double the ſine of the ſame to *BR* the radius, *i. e.* as *CM* to *RQ*.

937. The obſervation may be corrected by one eaſy operation in trigonometry, as will appear from the firſt part of this corollary, *viz.* by taking the half of the angle obſerved, and then finding another angle, whoſe ſine is to the ſine of that half, as the ſine complement of the diſtance *Bb* is to the radius: this angle doubled, will be the true diſtance

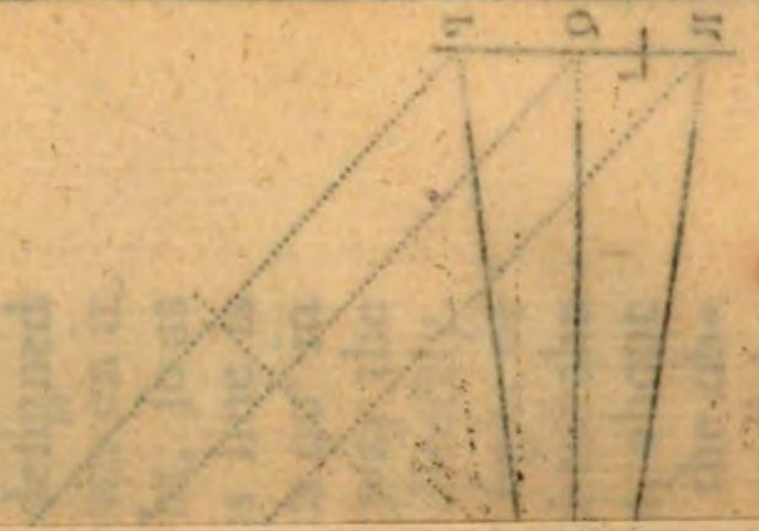
of the objects. But as this operation, though easy, will require the use of figures, I rather chose the method of approximation, because by that the observer, retaining in his memory the proportions of the sines of a few particular arches to the radius, may easily estimate the correction without figures, when the angle is not great, and by a line of artificial numbers and sines, may always determine it with greater exactness than will ever be necessary.

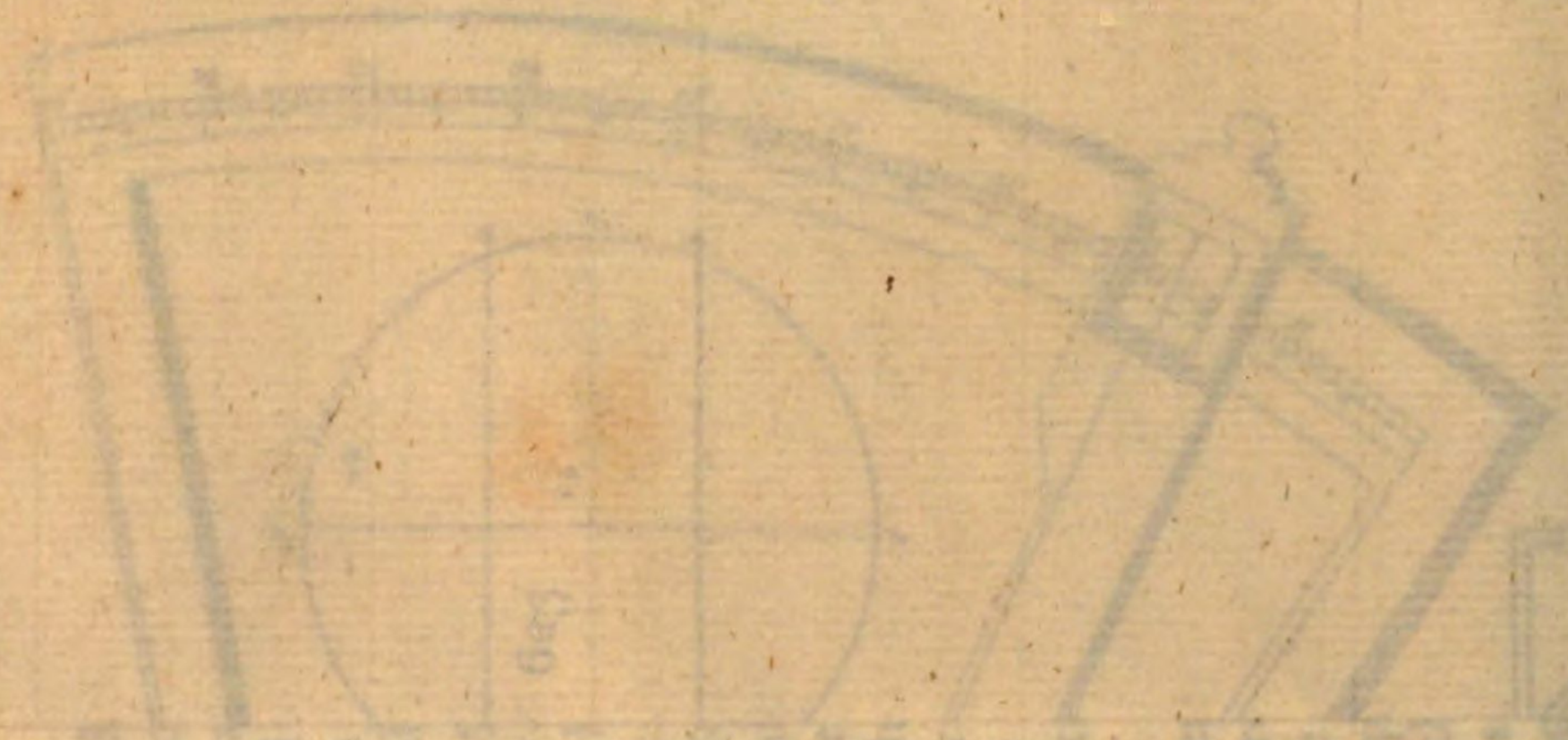
938. When the angle observed is very near 180 degrees, the correction may be omitted; for then it will be easy to keep the plane of the instrument so near that of the before-mentioned great circle, as not to want any, if the situation of that circle be known: if it be not, the observer, when he sees the two objects together, may turn the instrument on the axis of the telescope, till he finds that position of it by which he obtains the least angle; and this (if the specula are set truly perpendicular to the plane of the instrument) will always happen when the objects appear to coincide in the line *gh*, as expressed in the figure.

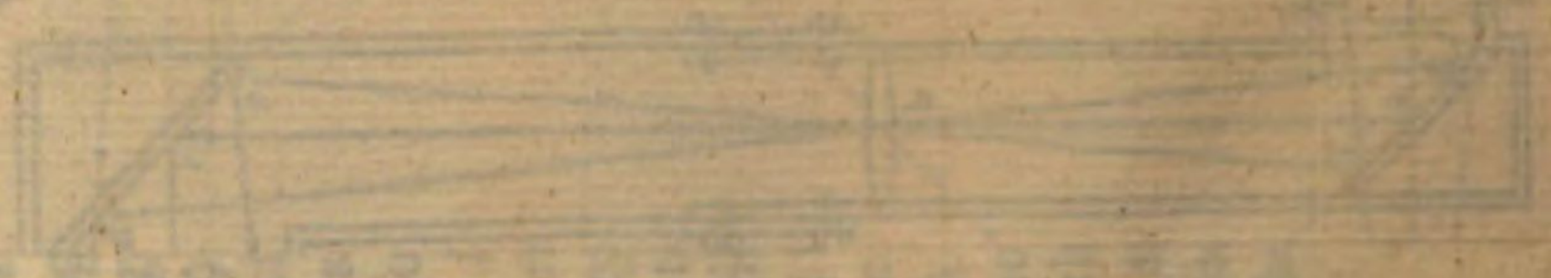
Fig. 622. 939. The instrument consists of an octant *ABC*, having on its limb *BC* an arch of 45 degrees, divided into 90 parts or half degrees; each of which answers to a whole degree in the observation. It has an index *ML* moveable round the center, to mark the divisions: and upon this, near the center, is fixed a plane speculum *EF* perpendicular to the plane of the instrument, and making such an angle with a line drawn along the middle of the index, as will be most convenient for the particular uses the instrument is designed for; (for an instrument made according to fig. 622 the angle *LFM* may be of about 65 degrees.) *IKGH* is another smaller plane speculum, fixed on such part of the octant as will likewise be determined by its particular use, and having its surface in such direction, that when the index is brought to mark the beginning of the divisions (*i. e.* 0°) it may be exactly parallel to that of the other; this speculum being turned towards the observer, and the other from him. *PR* is a telescope fixed on one side of the octant, having its axis parallel to that side, and passing near the middle of one of the edges *IK* or *IH* of the speculum *IKGH*; so that half its object-glass may receive the rays reflected from that speculum, and the other half remain clear to receive them from a distant object. The two specula must also be disposed in such manner, that a ray of light coming from a point near the middle of the first speculum, may fall on the middle of the second in an angle of 70 degrees or thereabouts, and be thence reflected into a line parallel to the axis of the telescope, and that a clear passage be left for the rays coming from the object to the speculum *EF* by the side *HG*. *ST* is a dark glass fixed in a frame, which turns on the pin *V*; by which means it may be placed before the speculum *EF*, when the light of one of the objects is too strong: of these there may be several.

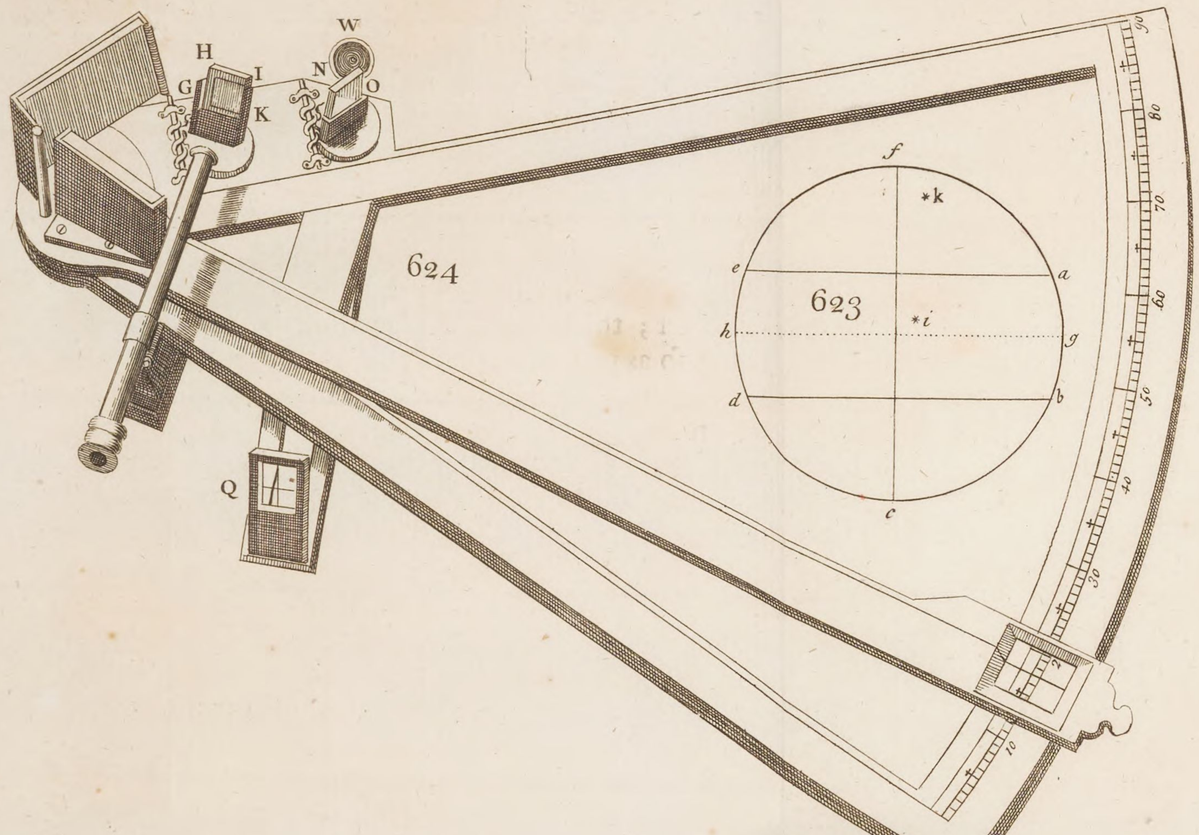
940.



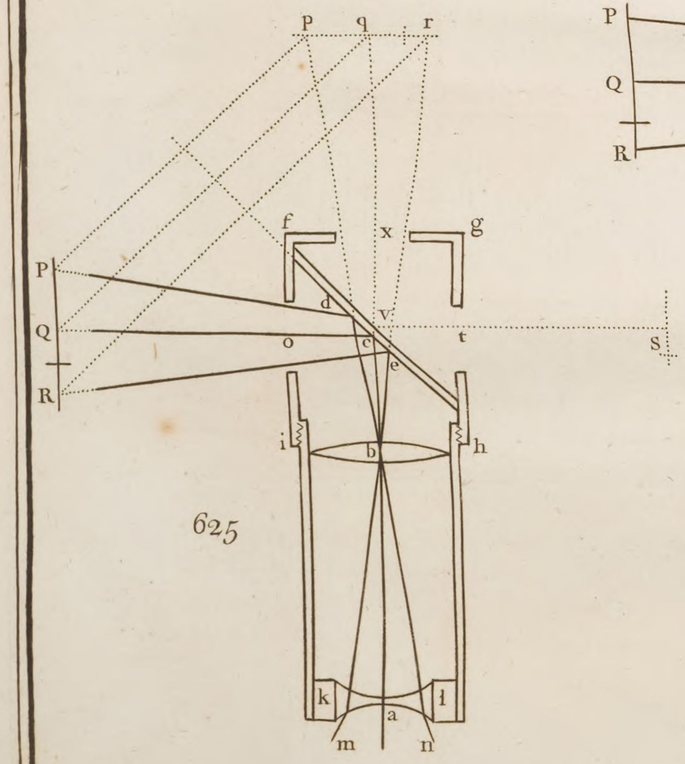

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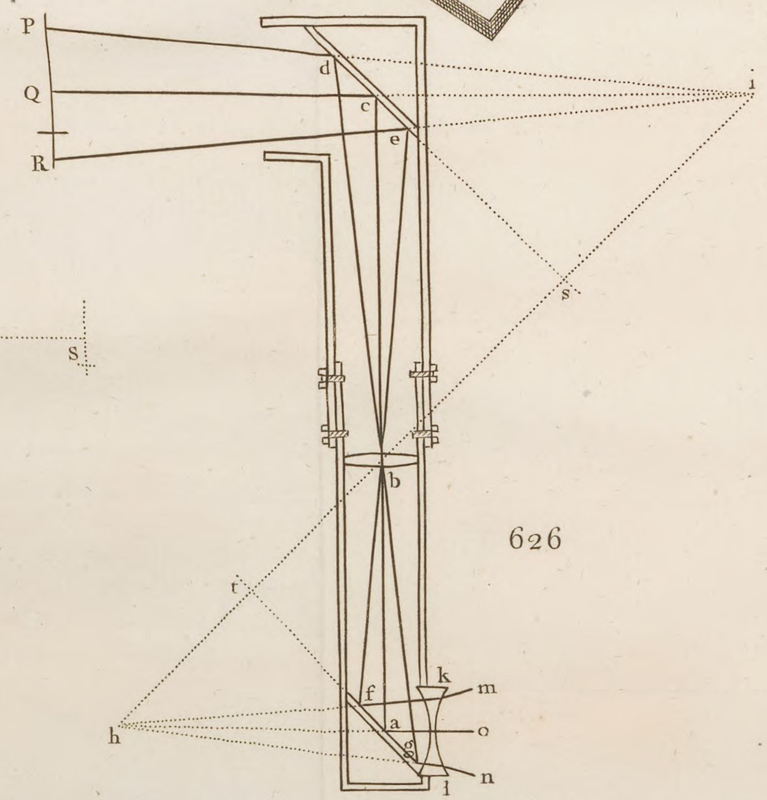



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940. In the distinct base of the telescope, represented by the circle *abcdef*, are placed three hairs, two of which, *ac* and *bd*, are at equal distances from, and parallel to the line *gh*, which passes through the axis, and is parallel to the plane of the octant: the third *fc* is perpendicular to *gh* through the axis. Fig. 623.

941. The instrument, as thus described, will serve to take any angle not greater than 90 degrees; but if it be designed for angles from 90 to 180 degrees, the polished surface of the speculum *EF* fig. 622 must be turned towards the observer; the second *IKGH* must be brought forward to the position *NO* so as to receive on its middle the rays of light from the middle of the first in an angle of about 25 degrees, their surfaces being perpendicular to one another when the index is brought to the end of the divided arch next *C*; and this second must stand five or six inches wide of the first, that the head of the observer may not intercept the rays in their passage towards it, when the angle to be observed is near 180°. The smaller speculum is fixed perpendicularly on a round brass plate, toothed on the edge; and may be adjusted by an endless skrew.

942. In order to make an observation, the axis of the telescope is to be directed towards one of the objects, the plane of the instrument passing as near as may be through the other, which must lie to that hand of the observer, as the particular form of the instrument may require; *viz.* the same way that the speculum *EF* does from *IKGH*, if it be composed according to this figure and description. The general rule is, that when the index is brought to the beginning of the scale (*i. e.* to 0° when the instrument is designed for angles under 90°, or to 90° when it is designed for angles from 90° to 180°) if then a line be imagined to be drawn on it parallel to the axis of the telescope, or line of direction of the sight, so as to point towards the object seen directly; which ever way this line is carried by the motion of the index along the arch from 0° towards 90° in the first case, or from 90° towards 180° in the second, the same way the object seen by reflection ought to lie from that which is seen directly. The observer's eye being applied to the telescope, so as to keep sight of the first object; the index must be moved backward and forward till the second object is likewise brought to appear through the telescope, about the same distance from the hair *cf* (fig. 623) as the first: if then the objects appear wide of one another, as at *i* and *k*, the instrument must be turned a little on the axis of the telescope, till they come even, or very nearly so, and the index must be removed till they unite in one, or appear close to one another in a line parallel to *cf*, both of them being kept as near the line *gh* as they can. If the instrument be then turned a little on any axis perpendicular to its plane, the two images will move along a line parallel to *gh*, but keep the same position in respect of one another; so

that

that in whatever part of that line they may be obſerved, the accuracy of the obſervation will be no otherwiſe affected than by the indiſtinctneſs of the objects. If the two objects be not in the plane of the inſtrument, but equally elevated on, or depreſſed below it, they will appear together at a diſtance from the line *g h*, when the index makes an angle ſomething greater than their neareſt diſtance in a great circle: and the error of the obſervation will increaſe nearly in proportion to the ſquare of their diſtance from that line; but may be corrected by help of the fifth corollary. Suppoſe the hairs *a e* and *b d*, each at a diſtance from the line *g h*, equal to $\frac{1}{4}\frac{0}{1}\frac{0}{6}$ of the focal length of the object-glaſs, ſo as to comprehend between them the image of an object, whoſe breadth to the naked eye is a little more than $2^{\circ}\frac{3}{4}$; and let the images of the objects appear united at either of thoſe hairs: then as the ſine complement of half the degrees and minutes marked by the index, is to the doubled ſine of the ſame; ſo is one minute to the error which is always to be ſubſtracted from the obſervation. Other hairs may alſo be placed in the area *a b c d e f*, parallel to *g h*, and at diſtances from it proportional to the ſquare roots of the numbers 1, 2, 3, 4, &c. and then the errors to be ſubſtracted from the ſame obſervation made at each of thoſe hairs reſpectively, will be in proportion to the numbers 1, 2, 3, 4, &c. This correction will always be exact enough if the obſerver takes care (eſpecially when the angle comes near 180°) to keep the plane of the inſtrument from varying too much from the great circle paſſing through the objects. When the angle is very near 180° the correction may be omitted, for then it will be eaſy to keep the plane of the inſtrument ſo near that of the beforementioned great circle, as not to want any, if the ſituation of that circle be known: if it be not, the obſerver when he ſees the two objects together, may turn the inſtrument on the axis of the teleſcope till he finds that poſition of it by which he obtains the leaſt angle; which (if the ſpecula are ſet truly perpendicular to the plane of the inſtrument) will always happen when the objects appear to coincide in the line *g h*, as expreſſed in fig. 623.

943. In regard to the workmanſhip, if an exactneſs be required in the obſervations, the arch ought to be divided with the greateſt care; becauſe all errors committed in the diviſion are doubled by the reflections. The index muſt have a ſteady motion on the center, ſo that the axis of it remain always perpendicular to the plane of the octant; for if that alter, it will be liable to vary the inclination of the ſpeculum it carries to the other: the motion muſt likewiſe be eaſy, leſt the index be ſubject to bend edge-ways: for the ſame reaſon it ſhould be as broad at that end next the center as conveniently can be. The ſpecula ſhould have their ſurfaces of a true flat; becauſe a curvature in either of them, beſide rendering the object indiſtinct, will vary its poſition, when ſeen by reflection from different parts of them: they muſt alſo be of a ſufficient length and breadth

fo-

for the teleſcope to take in a convenient angle without loſing the uſe of any part of the aperture of its object-glaſs, and that in all the different poſitions of the index. They may be either of metal or glaſs plates foiled, having their two ſurfaces as nearly parallel as they can; yet a ſmall deviation may be allowed; provided either their thickeſt or thinneſt edges (and conſequently the common ſection of their ſurfaces) be parallel to the plane of the octant: for in that caſe, though there are ſeveral repreſentations of the object, they will be always very near one another in a line parallel to *cf*; and any of them may be uſed, except when the angle to be obſerved is very ſmall. The chief inconvenience will be, that a ſmall ſtar will be more difficultly diſcerned, the light being divided among the ſeveral images. The teleſcope may be contrived to alter its ſituation, ſo as to receive the reflected rays on a greater or leſs part of its object-glaſs if the objects differ in brightneſs. The ſecond ſpeculum if it be made of a glaſs plate, may have a part unfoiled, that when one of them is ſufficiently luminous, the leſs bright may be ſeen through it by the whole aperture. If the ſun be one of the objects, or the moon be compared with a ſmaller fixt ſtar; their reflected images muſt be ſtill farther weakened by the interpoſition of one or more of the dark glaſſes *ST*. An exact poſition of the teleſcope is not neceſſary; and the inſtrument may be uſed without one, the diſpoſition of the ſpecula with regard to the ſector and index, being ſuch as may allow the eye to be brought as near the ſecond ſpeculum as may be, and make the inſtrument the moſt commodious for the obſerver.

944. It will be eaſy to judge, that ſcarce any greater degree of ſteadineſs is requiſite in the pedeſtal, or machine which carries this inſtrument, than what is ſufficient for the teleſcope uſed with it: for although the vibrating motion of the inſtrument may occaſion the images of the objects alſo to vibrate croſs one another; their apparent relative motion will be very nearly in lines parallel to *cf*; and it will not be difficult to diſtinguiſh whether they coincide in croſſing one another, or paſs at a diſtance: and if the ohjects are near one another, and the teleſcope magnifies but about four or five times, it may be held in the hand without any ſtanding ſupport. In this manner the altitude of the ſun, moon, or ſome of the brighter ſtars from the viſible horizon may be taken at ſea, when it is not too rough.

945. Fig. 624 ſhews an inſtrument deſigned for this purpoſe; differing from the foregoing deſcription chiefly in the placing the ſpecula and teleſcope, with regard to the ſector and index; in this the line drawn along the middle of the index falls on the fore ſurface of the larger ſpeculum, in an angle of about 4 or 5 degrees. The line of direction of the ſight or the axis of the teleſcope (if that be uſed) falls on the ſurface of the ſpeculum *IKGH* in an angle of about 70° or 71°. It has alſo a third ſpeculum *NO* Fig. 624.

diſpoſed

diſpoſed according to the directions when the angle is greater than 90 degrees, whoſe uſe is to obſerve the ſun's altitude by means of the oppoſite part of the horizon. On this the line of direction of the ſight falls in an angle of about 32 or 33 degrees. In placing theſe two ſmaller ſpecula, it will be farther neceſſary to take care that the ſpeculum *IKGH* does not ſtand ſo as to intercept any of the rays coming from the greater one, fixed on the index, to the third *NO*; nor either of them hinder the index from coming home to the divided arch. *W Q* is a director for the ſight; which is neceſſary when the teleſcope is not made uſe of. This conſiſts of a long narrow piece, which ſlides on another fixed on the back of the octant, and carries at each end a ſight erected perpendicularly on it: it may be removed at pleaſure, and exchanged for the teleſcope, which ſlides on in the ſame manner, both ſerving indifferently with either of the two ſmaller ſpecula. The eye is to be placed cloſe behind the ſight at *W*; and the thread ſtretched acroſs the opening of the other ſight at *Q* perpendicular to the inſtrument, is to aſſiſt the obſerver in holding it in a vertical poſture; who is to keep this thread as near as he can parallel to the horizon, and the object near the upright one. How far an inſtrument of this kind may be of uſe at ſea to take the diſtance of the moon's limb from the ſun or a ſtar, in order to find the ſhip's longitude, when the theory of that planet is perfected, I leave to trials to determine.

a Phil. Tranſ. N°. 425.

946. This inſtrument Mr. *Hadley* tells us[a] was made of wood, and was intended chiefly for taking altitudes of the ſun, moon or ſtars above the viſible horizon, either forwards or backwards: and that he has ſince procured another to be made of braſs, by Mr. *Siſſon*, for taking the diſtance of any kind of objects. It is ſupported by a ſingle ſtem ſkrewed to it on the under ſide; the lower end of which may reſt on the ground, to eaſe any obſerver of the weight of the inſtrument. This ſtem is alſo made to lengthen or ſhorten, by which means the inſtrument is brought to the proper height for any obſerver's eye, either ſtanding or ſitting. Inſtead of a ball and ſocket it has two circular arches fixed on its back, by which it is readily ſet to any poſition, which the ſituation of the objects may require.

The reſult of a trial of the inſtruments.

947. This moſt ingenious gentleman has alſo given us a particular account of a trial of theſe inſtruments, made at ſea by order of the Right Honourable the Lords Commiſſioners of the Admiraly, in the preſence of many ſkilful perſons. The reſult of the trial was this, that after all due allowances were made, three obſervations of the interval between two ſtars, taken with the braſs inſtrument, differed from Mr. *Flamſteed*'s obſervations at land, but about a minute at a medium; and that a dozen obſervations of the ſun's altitudes, taken with the wooden inſtrument while the ſhip lay at anchor, were ſo conſiſtent with one another as to differ from the truth but about half a minute at a medium; and laſtly that another dozen

taken

taken under ſail while the wind blew hard, differed but a minute from the truth, and at another time they agreed together more exactly. Notwithſtanding ſo cloſe a conſiſtence among theſe obſervations, it might in all probability have been much exacter, had not ſeveral accidents concurred at that time to diſturb it; as that the horizon was not always clear of land, and conſequently not ſo readily diſtinguiſhable; that all the obſervers were perſons quite unaccuſtomed to the motion of a ſhip; which was here very great and quick; this veſſel being very light and ſmall, and conſequently more ſubject to be toſſed up and down by the waves. Now if the difference of the obſervers heights, thus occaſioned, was about 4 or 5 feet, as it was judged to be; it muſt neceſſarily raiſe and ſink the viſible horizon by turns near one minute, which is nearly the whole error in the altitudes abovementioned: by which it appears at laſt that a more accurate and commodious inſtrument for the buſineſs of navigation can hardly be wiſhed for.

CHAPTER XIII.

Of reflecting Perſpectives.

948. THERE is an inſtrument ſold in the ſhops which ſome call an Opera-glaſs, others a diagonal perſpective; it is properly a reflecting perſpective, ſo contrived for viewing a perſon in a publick place that no one can diſtinguiſh who it is you look at, even though they know the deſign of the inſtrument. The 625th figure repreſents a ſection of it made by a plane paſſing through its axis *abc* and cutting the plane of a little ſpeculum perpendicularly in the line *dce*. This reflecting plane is fixt obliquely in a round ſhort tube *fghi*, ſkrewed upon the end of the perſpective's tube *hikl*, ſo that its axis *abc* ſhall make about half a right angle *bcd* with the ſpeculum, whoſe poliſhed ſide faces the object-glaſs *hbi*. Therefore ſuppoſing rays as *abc* to go from the eye at *a*, and after reflection at *c* to paſs through a round hole at *o* in the ſide of the ſhort tube, and to fall upon a diſtant object at *Q*; this object will be ſeen by rays returning back in the ſame lines *Qcba*. The way then to find the object intended to be viewed in the perſpective, is to direct its axis at a right angle to the rays that come from the object. For the angle *acd* being made equal to half a right one, and being equal to *ecQ* *, the angle *Qca* muſt be a right angle to make up two right ones with the reſt. And if the object perceived through the perſpective, be higher or lower than the object ſought for, this will be brought into view by turning the perſpective to and fro about its axis.

The opera-glaſs. Fig. 625.

* Art. 9.

949. If the object be too near you to be ſeen diſtinctly with the perſpective, turn the other end of it towards your eye; and by looking

through the hole *x* you will ſee the object *s* by the rays *stvx* coming through another hole at *t*, oppoſite to the former hole *o*, and reflected at *v* from another plane ſpeculum parallel to the former and facing the eye at *x*. If the ſpectator be ſhort ſighted, a concave glaſs muſt be placed in the hole *x*, otherwiſe a plane one, to make the inſtrument more like a common perſpective. But though it be diſcovered to be a reflecting perſpective by the holes in its ſides; yet theſe holes being oppoſite, make it impoſſible to be known on which hand the object is placed that the ſpectator is viewing.

950. In both caſes the reflecting plane neither magnifies nor diminiſhes the appearance of the object. For in the perſpective, if its axis *ac* be produced till *cq* equals *c*Q, the reflected rays will diverge as from an image at *q* equal to the object at Q; and therefore will fall upon the object-glaſs *b* in the ſame manner as if they had come from the object it ſelf removed to *q*; with this difference only that the right ſide of the object will appear on the left hand, and the left ſide on the right. Becauſe the lines *Pp*, *Qq*, *Rr*, that join the correſponding points of the object and its image, are all perpendicular to the reflecting plane *de* produced[a], and conſequently parallel to one another. But the object appears upright, either by the ſingle reflection or through the perſpective provided its eye-glaſs be concave; becauſe higher points of the image *q* anſwer to higher points of the object Q, for the reaſon juſt mentioned. The whole inſtrument is generally about 4 inches long, and is very well made by Mr. *Scarlet* near St. *Anne*'s Church, *London*.

[a] Art. 23.

The Polimoſcope.

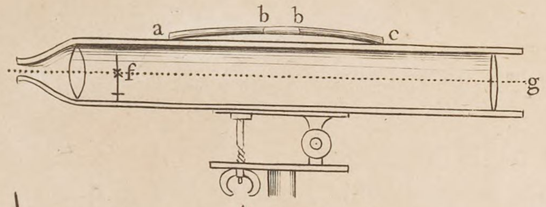
951. *Hevelius* in the preface to his *Selenography* deſcribes an inſtrument of this kind of his own invention, and commends it as ufeful in war eſpecially in ſieges, for diſcovering what the enemy is doing, while the ſpectator lies hid behind any obſtacle; and therefore he calls it a Polimoſcope. For this purpoſe he inlarges the interval *bc* between the object-glaſs and the ſpeculum by a tube of a length ſufficient to project the ſpeculum beyond the obſtacle that covers the ſpectator. And for a farther convenience of looking forward, as it were, he propoſes to place another plane ſpeculum *fg* at the other end of the tube, to reflect the rays through a hole *kl* in its ſide, in a direction *ao* parallel to, and according to the tendency of, the incident rays Q*c*; and to place the concave eye-glaſs in this hole. By this means the object will ſtill appear upright, and magnified juſt as much as if the two ſpeculums were removed, and the ſame eye-glaſs was placed in the axis of the tube. For in the rays Q*c*, *oa* produced through the ſpeculums *de*, *fg*, take *ci* equal to *cb* and *ah* equal to *ab*; and ſuppoſing rays to flow both ways through *b* the center of the object-glaſs, after reflection from the ſpeculums they will diverge from the points *h*, *i**. Let two of them fall upon the object in *P* and *R*, and ſince the angle *PiR* or *die* is equal to *dbe* or *fbg* or *fhg*; if the eye-glaſs was removed the object

Fig. 626.

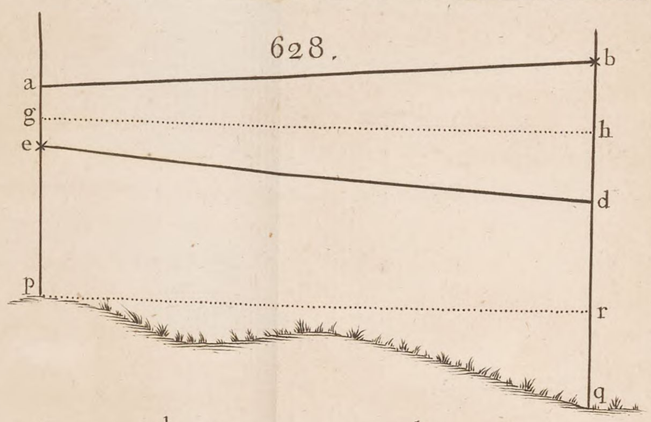
* Art. 23.



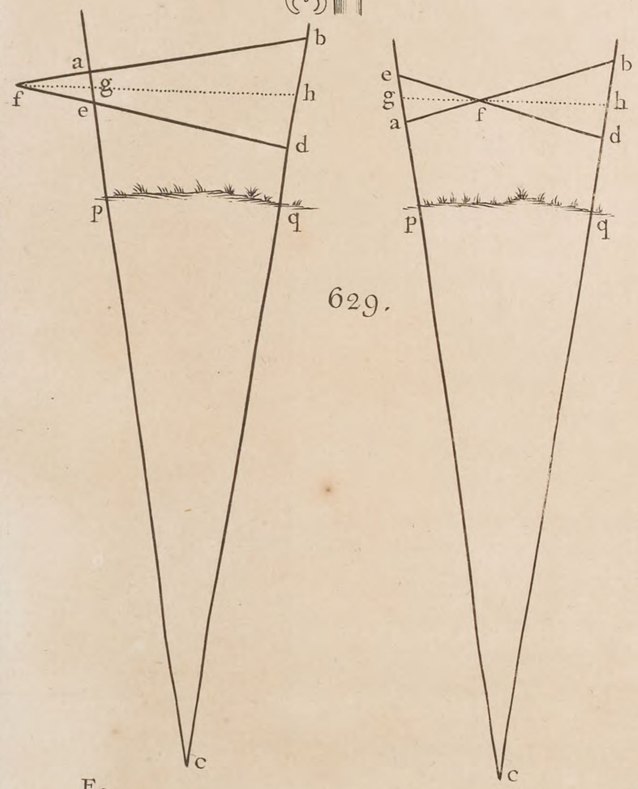
627.



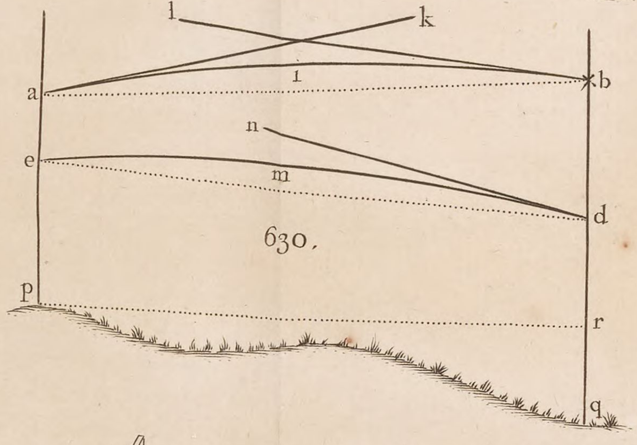
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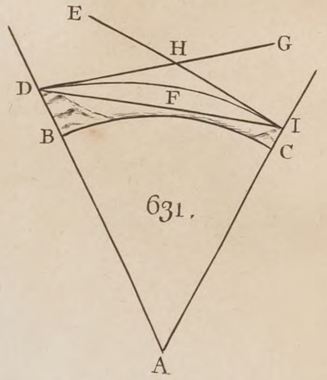
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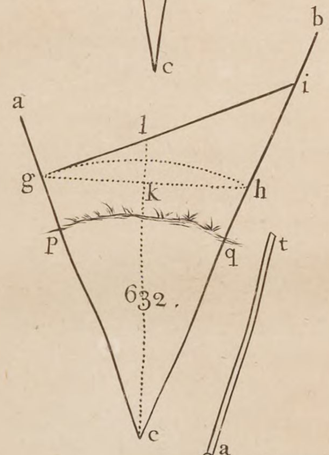
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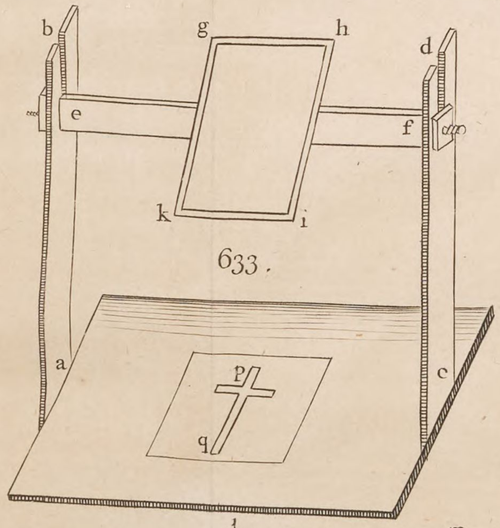
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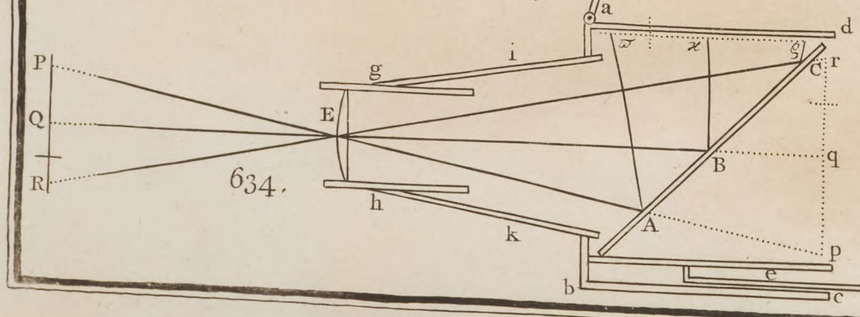
632.

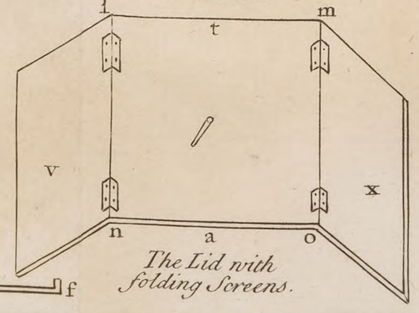


633.



634.





The Lid with folding Screens.

object would appear under the ſame angle *fhg* or *khl* as it would appear under to the naked eye placed at *i*; but the reflected rays *fk*, *gl*, after refractions into *km*, *ln* through the eye-glaſs *kl*, are inclined in the ſame angle to each other as they would be, if the ſpeculum *fg* being removed they had been refracted through the ſame eye-glaſs placed in the axis of the tube, at the ſame diſtance from *b* as it is now from *h*. And by tracing an oblique ray *Rebfkm*, it is manifeſt that the object appears upright; it alſo appears in the ſame poſture with reſpect to right and left as to the naked eye.

952. The length of the perſpective *ab* muſt not be very great; for then it will take in ſo little at one view as to make it difficult to find an object. And this was the Author's reaſon I ſuppoſe for ordering a longer tube to be joined to it. He ſays the length of his ſhorter tube was 8 inches and of the longer 22: by the ſhorter tube I ſuppoſe he means the perſpective. Thoſe that would be farther informed in the mechaniſm of the inſtrument, may conſult this Author in his Selenography p. 24.

CHAPTER XIV.

A teleſcopick Level and its uſes.

953. A TELESCOPE is of ſingular uſe in the art of levelling; that is in finding out ſuch tracts of ground as have a continued and a ſufficient declivity, for conducting water from place to place; either for making rivers navigable, or for draining low grounds, and the like uſeful purpoſes. Along the ſide of a teleſcope 2 or 3 feet long, having croſs hairs in its focus, a tube of a ſpirit-level is fixt immutably in a poſition parallel to the teleſcope; and when the teleſcope is placed nearly horizontal upon a three legged ſupporter, there is uſually a contrivance by a ſkrew for altering its elevation very gradually and ſteadily, in order to bring the extremities of the air bubble to reſt exactly at two marks, made upon the glaſs tube with the point of a diamond. The line *fg* repreſents the line of ſight drawn from *f*, the interſection of the croſs hairs, through *g* the center of the object-glaſs; and the points *b*, *b* are the marks upon the glaſs tube *abc*. Now while theſe parts of the inſtrument are immutably fixt with reſpect to each other, it is manifeſt that as often as the air-bubble is exactly reduced to the marks *b*, *b*, the line of ſight will always be reduced to the ſame poſition with reſpect to the horizon or to a plumb-line. It is not at all neceſſary in the buſineſs of levelling that the line of ſight and plumb-line ſhould be exactly at right angles; but only that the angles they make ſhall be always the ſame.

Deſcription of a level.

Fig. 627.

954. The whole art of levelling conſiſts in but little more than a repetition of the following problem: *p* and *q* being given points in two remote places,

The method of levelling. Fig. 628.

ces, it is required to find which is the lower and how much. Let *p a* and *q b* repreſent two ſtraight ſtaffs or poles fixt upright, by means of a plumb-line. Having placed the teleſcope by the ſide of the pole *p a*, and directed the line of ſight to the pole *q b*, alter its elevation by the ſkrew above-mentioned, till the air-bubble reſts exactly at the marks upon the tube. Then let an aſſiſtant mark the point *b*, which appears to be covered by the croſs hairs; and alſo the point *a* exactly upon a level with the croſs hairs; which is eaſily done by a common ſquare applyed to the ſide of the pole *p a*. Then remove the teleſcope to the pole *b q*, and here let the ſame things be repeated; that is let *d* be the place upon a level with the croſs hairs, and *e* the point upon the other pole *p a*, that appears to be covered by them while the air-bubble reſts at the ſame mark as before. Biſect the interval *a e* in *g*, and the interval *b d* in *h*, and the points *g*, *h* will be upon a level: that is, if we ſuppoſe *g p q h* to repreſent a long canal full of ſtagnating water, the points *g*, *h* will both be in its ſurface; and conſequently taking the leſſer depth *p g* from the greater *q h*, their difference *q r* ſhews how much the point *q* is below the point *p* or *r*.

955. If the places *p*, *q* cannot be ſeen from each other, or if the difference of their heights be greater than the length of any common poles; then one or more intermediate ſtations muſt be choſen; and by repeating the ſame practice between every two ſucceſſive ſtations, we ſhall find the level of the extreams.

956. When the points *g*, *h* are once found upon two poles not far aſunder, it will be convenient by moving the croſs hairs, to rectify the line of ſight, ſo as to be nearly coincident with the line *g h* or with a line parallel to it; for then in future levellings, at greater diſtances, the marks *b*, *e* will be leſs ſubject to fall above or below the poles.

The reaſon of it.

957. The reaſon of this practice will appear from the following principles. 1ſt That a plumb-line is perpendicular to the ſurface of ſtagnating water directly under it, or to a plane conceived to touch the water at the point where the plumb-line produced would cut it. 2dly That the ſurface of a canal or lake of ſtagnating water, extended over the country to be levelled, may be taken for a portion of a ſpherical ſurface, notwithſtanding that the figure of the earth is not exactly ſpherical. 3dly That any two points are upon a level which lie in the ſurface of the lake; and conſequently are equidiſtant from the center of it, or from the concourſe of two plumb-lines produced.

Fig. 629.

958. Now let *a c b* be the angle made by the plumb-lines *a e*, *b d* produced downwards, till they meet at *c*. At any point *a* of the line *a e* make an angle *c a b* of any given magnitude; and at any point *d* of the other line *b c*, make the angle *c d e* equal to *c a b*; and let *a b* and *d e* (produced) meet in *f*; and let the line *f g h*, which biſects the angle *b f d*, cut the plumb-line *a e* in *g*, and *b d* in *h*. I ſay the points *g*, *h* will be upon a level

level, or that *cg* and *ch* will be equal. For the angles *cab*, *cde* being made equal, and the angle at *c* being common to the triangles *cab*, *cde*; the remaining angles *cba*, *ced* are alſo equal [a]. Hence the exterior angle *chf* = (*hbf* + *hfb* = *ced* + *hfb* = *gef* + *gfe* = the exterior angle *agf* =) *cgh*; and by conſequence the legs *cg*, *ch* of the triangle *cgh* are equal, which was to be proved.

[a] Euc. I. 32.

959. Now in the practice of levelling the intervals *ae*, *bd* are very ſmall in compariſon to the diſtance *gh*, and conſequently the angle *afe* or *bfd* is very ſmall; and therefore its legs *af*, *fe* and *bf*, *fd* are very nearly in a ratio of equality; and conſequently the line *gh*, which biſects the angle at *f*, will alſo biſect the intervals *ae*, *bd* very nearly; becauſe the ſegment *ga* : *ge* :: *af* : *fe* [*], and likewiſe the ſegment *hb* : *hd* :: *bf* : *fd*, that is in a ratio of equality without ſenſible error in the practice of levelling.

[*] Euc. VI. 3.

960. This reciprocal way of levelling ſeems to be the moſt exact of any I have met with, eſpecially if it be performed by two inſtruments made to agree together beforehand; which may be done by placing them together, and by altering the croſs hairs in either of them, till the ſame mark upon a remote object, is covered by both the croſſes, while both the bubbles reſt at their marks upon the tubes. Then may two obſervers find the marks upon the oppoſite poles at the ſame time; and conſequently the refractions of the rays in the air, whatever be their quantities, will be equal as near as poſſible; and then the reſult of the practice will be as accurate as if there had been no refractions at all. For let the curve *bia* repreſent the courſe of the viſual ray from *b* to *a*; and let the lines *ak*, *bl* touch it at *a* and *b*. Then becauſe the points *a*, *b* are very nearly upon a level, the denſity and conſtitution of the air and vapours at the ſame inſtant, will be nearly the ſame in each place; and by conſequence the curve *aib* and its tangents at *a* and *b*, will be equally inclined to the chord *ab*. For the ſame reaſons the curve *emd* will be ſimilar and equal to the curve *aib*, being ſituated ſo very near to it. Therefore the angle *edn*, under the chord *ed* and tangent *dn*, will be equal to the angle *abl* or *bak*; and conſequently ſince the angles *qdn*, *pak* are made equal in the two obſervations, by taking away the equal angles *edn*, *bak*, cauſed by the equal refractions, the remaining angles *qde*, *pab* will be equal to each other, as if there were no refractions at all.

The refraction of the rays by the air, conſidered.

Fig. 630.

961. If the reciprocal obſervations be made about the middle of the ſame day when the air is the pureſt, there will ſcarce be any occaſion for two inſtruments; but if they be made near the morning or evening, even on the ſame day, an equality of refractions cannot be depended upon unleſs they are made at the ſame inſtant. The members of the Royal Academy of Sciences at *Paris* tell us, in their account of the Meaſure of the Earth,

Earth*, they often found, "that an object which at break of the day appeared in the level and ſometimes a little above it, did afterwards, when the ſun was up, appear below it. And on the contrary, after the ſetting of the ſun, objects far diſtant appeared to be raiſed ſo ſenſibly, that in leſs than half an hour their apparent height was augmented more than three minutes. As to the cauſe of theſe appearances they add, that the coolneſs of the night condenſes the vapours, which deſcend to a lower place, leaving the air in the higher ſtations more pure than in the day time. And on the contrary when the heat of the ſun has made a part of the vapours to mount to more elevated ſtations, there muſt be leſs difference of the mediums, and conſequently a leſs refraction. They alſo teach us the following method whereby the quantity of any particular refraction may be determined by obſervations.

Fig. 631. 962. "Let *A* be the center of the earth, *BC* its ordinary ſurface, and *D*, *I* the tops of two mountains. We are to conſider that the earth being inveloped with an atmoſphere or vaporous air compoſed of different regions which are gradually more ſubtil the farther they are removed from the earth; it comes to paſs that the viſual ray, which comes from a higher place to a lower, as from *D* to *I*, and paſſes obliquely from a more ſubtil to a more groſs air, is continually bent in its way in proportion as it changes the medium; which gives it the poſition of a curve line much like that of *DFI*. But the eye placed at *I* receives the curve ray as if it were the tangent *IE*, in which direction it ſees the object *D*. For the ſame reaſon if we ſuppoſe another eye at *D*, it ſees the object *I* in the ſtraight line *DG*, tangent to the ſame bended ray *DFI*. And ſuppoſing the two tangents *IE* and *DG*, which are inſtead of the viſual rays, to cut each other in *H*; one may imagine, that there happens the ſame thing as if the two objects *D* and *I* were reſpectively ſeen with only one refraction, which ſhould be made in *H*, and which ſhould be equivalent to all thoſe of the true ray *DFI*.

963. For diſcovering theſe refractions, and alſo for knowing the total value of them, which we ſuppoſe reduced to the angle *DHE* or *IHG*, the two angles *AIE* and *ADG* muſt be obſerved; and moreover the angle at *A* muſt be determined by meaſuring the diſtance *BC* or *ID*, and by changing it into minutes and ſeconds of a great circle of the earth. Becauſe the exceſs of theſe three angles *AIE*, *ADG* and *DAI* above 180 degrees, is the total refraction equal to the angle *IHG* or *DHE*.

964. They alſo add the following experiment to make it appear (contrary to the opinion of ſome authors) that even at noon day there remains ſomewhat of refraction; when the diſtance is ſo great that the viſual ray cannot paſs from one place to another without approaching the

* Tranſlated by Mr. *Waller* and joined to the Memoirs for a Natural Hiſtory of Animals. *London* 1688.

earth

earth. "The laſt ſummer (ſay they) being on the top of the towers of *Noſtre Dame* at *Paris*, we pointed the quadrant towards the tower of *Montlehery*, and we found that the foot of this tower was preciſely in the apparent level *. This was about noon in a very ſerene time. Some days after at the ſame hour, the top of the tower of *Noſtre Dame* obſerved from the foot of *Montlehery* appeared below the apparent level 11′. 30″. whereas conformable to the diſtance of 12796 toyſes, which there are between theſe two places, this angle ought to have been 13′. 30″. Whence it appears that the ray had two minutes of refraction in the whole; that is, the angle *DHE* contained two minutes. In this calculation they took the diameter of the earth to be 6538594 toyſes.

Other methods of levelling.

965. Setting aſide the curvity of a ray, which Mr. *Picard* tells us is ſcarce ſenſible about noon, when the diſtance of the object does not exceed 1000 toyſes; the line of ſight through the teleſcope may be ſet perpendicular to a plumb-line or parallel to the horizon in this manner. Having found two points *g*, *h* upon a level as before; let *gi* be perpendicular to *gc* and cut *ch* in *i*, and having computed the line *hi* (as follows) and made a mark at *i*, place the level at *g* and alter the place of the croſs hairs in the focus, till they appear to cover the point *i* when the air-bubble is at its marks, and the buſineſs is done. Now the line *hi* is equal to the ſquare of *gh* applyed to 2*gc*; and conſequently may be found by meaſuring the diſtance *gh* and dividing its ſquare by the diameter of the earth; which may be ſuppoſed equal to 2*gc*, though it is not exactly ſo, the earth being not exactly ſpherical. For biſecting *gh* in *k*, draw *ck* cutting *gi* in *l*; and ſince the triangles *kgl*, *kcg* are ſimilar, we have *kl* : *kg* :: *kg* : *kc*, and by doubling them all we have *ki* : *gh* : : *gh* : 2*kc*. Mr. *Picard* computes that when the diſtance *gh* is 300 toyſes or 1800 *Paris* feet, the line *hi* is one inch; and hence any other *hi* may be found for any other known diſtance; it being as the ſquare of the diſtance *gh*.

Fig. 632.

966. Hence when the inſtrument is thus rectified, the point *h* upon the level with *g*, may be found by one obſervation; that is by marking the point *i* covered by the croſs hairs, and by computing *ih* by the rule above. As the intervals between the ſtations muſt be but ſmall in this method, becauſe of refractions as was ſaid above, the readieſt way is to make them all equal; which may be known exact enough for this purpoſe, by obſerving whether the pole be removed to ſuch a diſtance, that its image (or the image of any given part of it) in the focus of the teleſcope ſhall be always of the ſame length, being meaſured by the diſtance between two parallel hairs in the focus: and then the ſame allowance muſt always be made for the depth of the point *h* below *i*.

* A line parallel to the horizon of the ſtation they call the apparent level.

967.

967. Laſtly by means of theſe parallel hairs, it is eaſy to find when the teleſcope is placed in the middle between two ſtations; and then the points upon a level at each pole are preſently found, by directing the teleſcope firſt to one pole and then to the other, and by marking the points covered by the croſs hairs. And theſe points will be upon a level notwithſtanding any refractions of the viſual rays, becauſe the refraction of each ray will be equal. Thoſe that are deſirous of fuller inſtructions in the practice of levelling may conſult Mr. *Picard*'s Traite du Nivellement 8°. who deſcribes ſeveral forms of inſtruments for levelling, and gives ſeveral hiſtories of levels actually taken.

CHAPTER XV.

Optical machines for making pictures of objects; and their uſes in drawing.

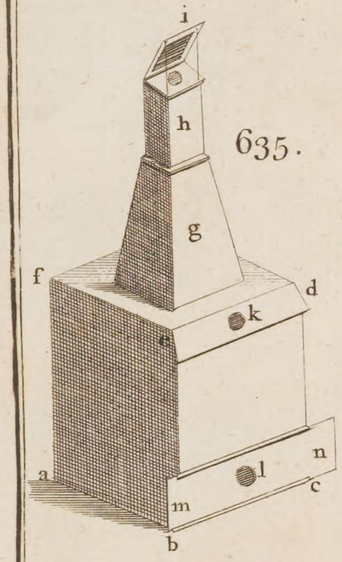
Of pictures in a dark room.

968. IN the 67th article I explained the reaſon of that famous and admirable experiment of forming pictures of external objects upon a white paper, by a lens placed in the window-ſhutter of a dark room; and ſhewed how the inverted pictures might be viewed upright by reflecting them downwards upon a table. For the convenience of directing the axis of the lens towards any object, it is uſually placed in a large cylindrical hole, bored through the middle of a wooden ball, commonly called a ſky-optick-ball; which is eaſily moveable about its center within a hollow zone made of wood and faſtened to the window-ſhutter: this zone conſiſts of two half zones ſkrewed together in the middle after the ball was let in; and the concavity of the zone hinders the light from paſſing between it and the ball. The pictures of objects will be ſo much the larger as the focal diſtance of the lens is longer, and ſo much the brighter as its aperture is larger. The focal diſtance of the lens being 8 or 10 feet, it is convenient to receive the pictures upon a large ſkreen, covered with linnen or white paper, and to have it moveable upon ſmall wheels for the purpoſe of placing it readily at the due diſtance from the lens.

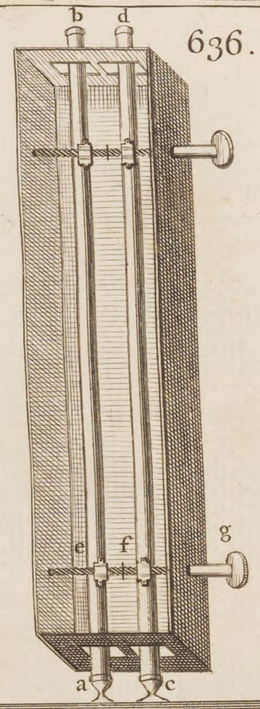
A remarkable appearance in theſe pictures.

969. The broadeſt lens of this kind that ever I ſaw, is at Mr. *Scarlet*'s ſhop by St. *Anne*'s Church *London*: where I remember I was not only diverted with ſeeing the experiment in ſo great perfection, but was ſurpriſed alſo with an unexpected appearance in it: *viz.* that the pictures of perſons walking along the ſtreet, beſides their progreſſive motion, had alſo a perpetual dancing or undulating motion up and down, like the appearance of a perſon carried in a chair, but quicker and more ſenſible. Poſſibly this may ariſe from nothing elſe but a real undulation in the progreſſive motion of a perſon in walking; for I queſtion whether he could walk freely between two parallel boards, if the upper board be

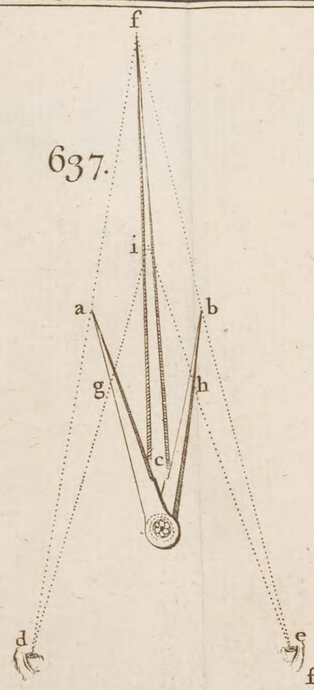
635.



636.



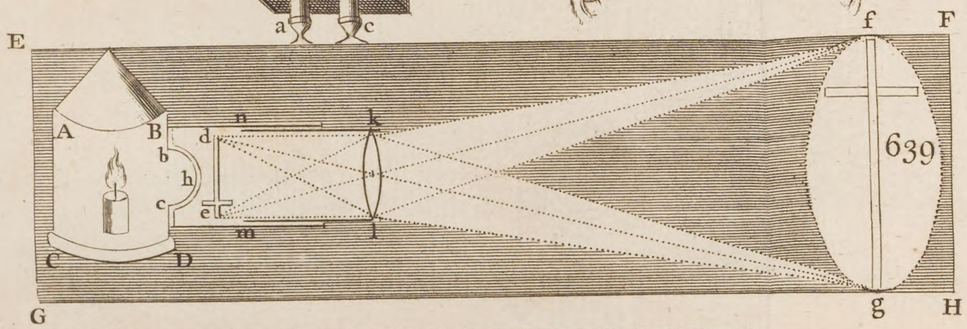
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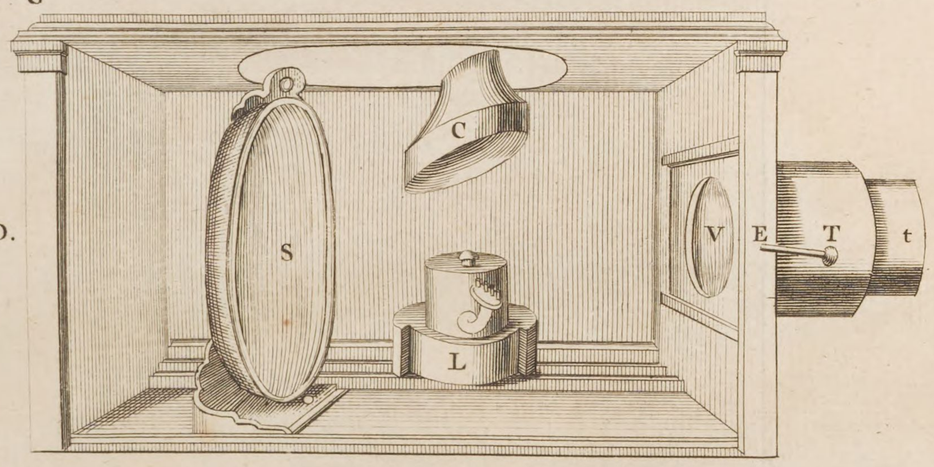
638.



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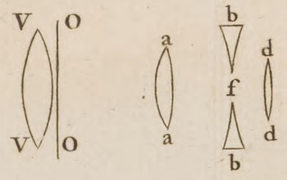


640.





642.



be but little higher than his head when he ſtood ſtill; and if not, whether the undulation of his head compared with the fixt board would not be ſenſible to a by-ſtander. And if ſo, it will be more ſenſible in the picture upon the ſkreen, in proportion as the picture is nearer to the eye than to the lens. For if the eye be placed by the ſide of the lens, looking firſt at the picture and then at the man himſelf, their apparent motions would be equal, becauſe the rays croſs in the center of the lens. Now upon the ſkreen the picture of a board over the man's head is needleſs; ſince the fixt parts of the ſkreen are ſeen at the ſame view with the picture upon them, and are ſufficient for diſtinguiſhing its motion. If theſe pictures upon the ſkreen be viewed by reflection from a looking-glaſs held in ones hand almoſt horizontal, ſo that the rays coming from the picture may be reflected upwards to the eye, the people in the ſtreet will appear upright and without any undulating motion of their heads; which confirms the cauſe I have been given of it. For now they do not appear like flat paintings upon the ſkreen, but to walk upright upon level ground; the ſkreen being ſcarce taken notice of now, as it is when we view it directly or without a looking-glaſs.

970. For drawing copies of prints or paintings, or the perſpective of ſolids, by tracing the out-lines of their pictures formed by the lens; let the original be placed without doors at a proper diſtance, and let its picture be received upon a ſheet of paper or upon a large plane glaſs, left unpoliſhed on one ſide. This glaſs being fixt upright with its rough ſide turned from the window, it is eaſy with a black lead pencil to trace upon it the out-lines of the pictures. Then having ſtrained a ſheet of fine paper over the glaſs, the ſtrokes of the pencil will appear through it, when held againſt the ſky-light; and thus the picture may be drawn upon the paper. The eaſieſt way of making the image diſtinct upon the glaſs when fixt, is to place the lens in a tube that ſhall ſlide within another ſhort tube fixt in the window-ſhutter. Uſes in drawing.

971. But to ſave the trouble of making two draughts, we may proceen in this manner. Having ſtrained the drawing paper upon a ſmooth board, let it be laid upon a ſmall firm table, to be placed under the lens in the window-ſhutter; and let an inclined looking-glaſs, to reflect the picture upon the paper, be fixt over the table in ſome ſuch manner as this: *ab* and *cd* are two boards fixt upright on the table on each hand of the drawing paper; and *ef* is a third board equal in length to the interval between the upright boards, and having a round pin at each end of it. When this board is laid over the backſide of a looking-glaſs and ſkrewed to the frame of it, the pins muſt be lodged in two ſlits at the tops of the upright boards; and by two nuts that ſkrew upon the pins the glaſs may be ſtayed at a proper inclination, for throwing the picture directly down Fig. 633.

upon the paper below; and then it may be made distinct by moving the tube inwards or outwards that holds the lens.

The portable *camera obscura*. Fig. 634.

972. The portable *camera obscura*, or dark chamber, commonly sold in the shops, will require but a short description. The theory is this; the rays that come from the object PQR after passing the lens E are tending to form an image pqr; but being reflected upwards by the looking-glass ABC, they form an horizontal image $\varpi\varkappa\varrho$ upon a glass plane, whose unpolished side lyes uppermost; upon which a copy of the picture may be sketched out with a black lead pencil; and to the spectator facing the object, the picture appears upright. This figure represents a section of the machine through the axis of the tube that holds the lens, and through the middle of the square box and the looking-glass within it. The section of the side opposite to the tube is not here represented, it being a door that opens sideways; the edges of the rough glass at the top are placed in two grooves upon the sides of the box; and being taken off, it is placed in a drawer ef at the bottom of the box; the looking-glass ABC may also be drawn out of the grooves in the sides of the box and lodged in the same drawer. The square wooden tube consists of 3 parts; the innermost that carries the lens, draws outwards or inwards to make the pictures distinct. The parts gh and ik, being fixt together and to the box with small bolts, may be taken asunder and put into the box; then the lid at at the top, and the door at the end, being both shut and fixt, the machine becomes more commodious for carriage. The inside of the lid whose section is at, has two wings, that open to right angles on each side of it, and rest upon the sides of the box, to shade the image upon the rough glass.

Fig. 635.

973. Another portable *camera* for the business of drawing is made by Mr. *Scarlet* in this manner. Into the middle of the top of a square box there is inserted an upright tube, or rather a piece of a square pyramid; in the top of which there slides a short square tube, having a broad object-glass fixt in a hole at the top of it; the focal distance of the glass being somewhat less than its utmost height above the bottom of the box, where the pictures of objects are to be formed by it. Upon one edge of the top of this tube there turns a lid, having a plane looking-glass fixt flat upon the inside of it; and thereby capable of being stayed at any inclination proper for reflecting the rays, that come from an object, directly downwards through the object-glass, to the drawing paper fixt upon the bottom of the box; where the picture of the object will be distinctly painted when the object-glass is adjusted to a proper height. The box being quite closed and dark within, this picture is viewed through a small hole in the upper edge of the box sloped off, from the side opposite to the object, where the draughts-man stands and puts his hand and pencil through a hole made in this side, or rather in a long piece that slides in it horizontally, accord-

according as he has occaſion to move his hand. The ſquare box is *abcdef*; the fixt tube, *g*; the ſliding tube, *h*; the object-glaſs, *o*; the inclined looking-glaſs, *i*; the hole for the eye, *k*; the hole for the hand, *l*, in the ſlider *mn*; the picture *p* of the object *P*.

CHAPTER XVI.

A Binocular Teleſcope deſcribed.

974. IT is a common obſervation that objects ſeen with both eyes appear more vivid and ſtrong than to a ſingle eye; eſpecially when both are equally good. A perſon not ſhort-ſighted may ſoon be convinced of this fact by looking attentively at objects pretty romote, firſt with one eye and then with both. *P. Cherubin* in his *Viſion Parfaite*, ſays the experiment ſucceeds alſo in looking at near objects; to me it does not anſwer ſo well. But a ſhort-ſighted perſon may be perfectly ſatisfied of it by applying a concave glaſs firſt to one eye, and then another concave to his other eye. This obſervation gave riſe to the contrivance of a binocular teleſcope, in which the ſaid appearance is ſtill more manifeſt.

975. The binocle conſiſts of two diſtinct teleſcopes ſeverally directed from each eye to the ſame object, and combined together as follows. In the 636th figure, *ab* and *cd* are two equal teleſcopes laid in a long box nearly parallel to each other; the interval between the eye-ſtops *a*, *c* being equal to the interval of the pupils, and that of the centers of the object-glaſſes ſomewhat leſs than the other. Both ends of the teleſcopes paſs through oblong ſlits in both ends of the box; and the interval between them may be widened or contracted at either end by a long ſkrew-pin, laid over each end of both the teleſcopes; the threads of each half of the ſkrews being wrought contrary ways and called a right and left handed ſkrew. For theſe halves being put through two nuts *e*, *f* fixt to the upper ſides of the teleſcopes, it comes to paſs that by turning the ſkrew-pin one way, the two teleſcopes will accede to, and the other way, they will recede from, each other; till by one of theſe ſkrew-pins the interval between the eye-ſtops *a*, *c* becomes equal to the interval of the pupils of the obſerver, and by the other, the axes of the teleſcopes become directed to the ſame object: which will be known exactly if there be croſs hairs in the focus of each teleſcope; or even without them. For before this poſition is obtained the objects will appear double, and afterwards ſingle; and then alſo beſides a much ſtronger and brighter appearance of the object through both teleſcopes, than through either alone, there is another phenomenon very remarkable. Fig. 636.

976. In the focuſes of the two teleſcopes there are two equal rings as uſual, which terminate the pictures of the objects there formed; and of con- A remarkable appearance in the binocle.

consequence the visible area of the objects themselves. These equal rings by reason of the equal eye-glasses, appear equal and equally remote when seen separately by each eye, the other being shut; but when seen united by both eyes, they appear much larger and remoter too; and the objects seen through them do also appear larger, though circumscribed by the united rings in the same places as when seen separately.

The like appearance to the naked eye.

977. Endeavouring to solve this odd appearance, I found out another still more surprising. Having opened the points of a pair of compasses somewhat wider than the interval of your eyes; with your arm extended hold the head or joint in the ball of your hand, with the points outwards and equidistant from your eyes, and somewhat higher than the joint. Then fixing your eyes upon any remote object lying in the line that bisects the interval of the points, you will first perceive two pair of compasses (each leg being doubled [a]) with their inner legs crossing each other, not unlike the old shape of the letter w. But by compressing the legs with your hand, the two inner points will come nearer to each other; and when they unite, (having stopt the compression,) the two inner legs will also entirely coincide and bisect the angle under the outward ones; and will appear more vivid, thicker and longer than they do, so as to reach from your hand to the remotest object in view, even in the horizon it self, if the points be exactly coincident. This appearance will continue the same wheresoever you direct your eyes to any other collateral objects; nor will it vanish by variously inclining the plane of the legs to the horizon; or by any other means than by looking directly at them. The like appearance will happen when two equal round slices of a cork, or any two equal surfaces, are stuck upon the points of the compasses; and likewise when two straws, or two tobacco-pipes, or any two things of equal thickness are used in the shape and instead of the compasses; with this difference only, that the ends of the pipes, or the corks, will not shoot out so far as the naked points of the compasses. If the pipes be held parallel to each other and perpendicular to the visual rays, their apparent union will also seem parallel to them, and somewhat remoter; and the same will happen when two tapering legs of two pair of compasses, are also held parallel to each other; but if they converge as towards a joint, the points will shoot out farther than the other parts of the legs, as in the first experiment, when the plane of them was not perpendicular to the visual rays. In the 637th figure, the points of the compasses are *a* and *b*; the head or joint *c*; the eyes of the observer *d* and *e*; the imaginary leg *cf*, bisecting the angle *acb* and extended to the object *f*.

[a] Art. 137.

Fig. 637.

The reason of it.

978. I apprehend the reason of the phenomenon to be this: by shutting the eyes *d*, *e* alternately, it will be evident that the points *d*, *a*, *f*, are in one straight line, and the points *e*, *b*, *f* in another; and so the reason of the apparent union of the points *a*, *b* at the object *f*, is because their pictures

pictures are upon the ſame points of the retina's as thoſe of the object it ſelf. For the ſame reaſon, while the legs of the campaſſes are gradually compreſſed, if your eye be kept fixt upon the imaginary point of the united legs, this point will appear to approach gradually: which ſhews why any two points of the legs, as *g*, *h*, equidiſtant from the extreams, appear nearer and nearer to the eyes as they lye nearer and nearer to the head of the compaſſes. The appearance of the round corks upon the points of the compaſſes, is ſo exactly analogous to that of the two holes in the focuſes of the two teleſcopes, that what has been ſaid of the corks is ſufficient to explain the phenomenon of the holes.

979. For want of a two handed ſkrew, the ends of the teleſcope may be made to accede to, or recede from, each other, by ſeveral other contrivances, to be ſeen in *P. Cherubin*'s *Viſion parfaite*; who has wrote immenſely upon this inſtrument. To exclude all uſeleſs and hurtful light from the eyes, the eye-ſtops are made hollow and very broad, to cover ſome part of the temples; and their inner parts are cut away to admit the upper part of the noſe between them. Mr. *Scarlet*, who makes theſe binocles at *London*, ſays that 3 plano-convex eye-glaſſes for each teleſcope, with their ſides ſituated as in the 638th figure, are better than 3 double convex ones. But a binocle made with two of Mr. *James Gregory*'s reflecting teleſcopes hereafter deſcribed, would be leſs cumberſome and magnify more than theſe. The inſtrument may be ſupported upon a ſtand, whoſe top has an horizontal circular motion, and alſo a vertical one upon an horizontal axis; which may be governed by a ſtring, tied to the lighter end of the teleſcope and wound about a peg, that turns eaſily in a hole made in the pillar of the ſtand. Fig. 638.

CHAPTER XVII.

An explication of the Magick Lantern, ſometimes called Lanterna Megalographica.

980. THE contrivance is briefly this; *ABCD* is a tin lantern, from whoſe ſide there proceeds a ſquare or round arm or tube *bnkclm*, conſiſting of two parts; the outermoſt whereof *nklm* ſlides over the other, ſo as that the whole tube may be lengthened or ſhortened thereby. In the end of the arm *nklm* is fixt a convex glaſs *kl*: about *de* there is a contrivance for admitting and placing an object *de* painted in dilute and tranſparent colours on a plane thin glaſs; which object is there to be placed inverted. This is uſually ſome ludicrous or frightful repreſentation, the more to divert the ſpectators: *bhc* is a deep convex glaſs, ſo placed in the other end of the prominent tube, that it may ſtrongly caſt the

Deſcription of it by Mr. *Molyneux*. Fig. 639.

the light of the flame *a* on the picture *de* painted on the plane thin glaſs. And here it is to be noted, that the glaſs *bhc* is only deſigned for the ſtrong illumination of the picture *de*, and has nothing to do in the repreſentation; and therefore in ſome of theſe lanterns, inſtead of the glaſs *bhc*, we ſhall find a concave ſpeculum ſo placed, that it may ſtrongly caſt the light of the flame *a* on the picture at *de*; and ſometimes both are uſed.

981. Wherefore let us now conſider the picture *de* as a very lightſome object of diſtinct colour and parts. And let us conceive *de* more remote from the glaſs *kl* then its focus. It is then manifeſt that the diſtinct image of the object *de*, ſhall be projected by the glaſs *kl* on the oppoſite white wall *FH* at *fg*; and here it ſhall be repreſented erect. For now the whole chamber *EFGH* is dark, the lantern *ABCD* incloſing all the light; ſo that in effect this appearance of the magick lantern is no more than what is already declared concerning the repreſentation of outward objects in a dark room by a convex glaſs; and here we may obſerve, that if the tube be contracted, and thereby the glaſs *kl* brought nigher the object *de*, the repreſentation *fg* ſhall be projected ſo much the larger; and ſo much the more diſtant from the glaſs *kl*; according to the rules be-
a Art. 48. fore laid down [a]. So that the ſmalleſt picture at *de* may be projected at *fg* in any greater proportion required, within due limits. From whence the name of Lanterna Megalographica. And conſequently, protracting the tube and drawing the glaſs *kl* more diſtant from the object *de*, will diminiſh the repreſentation *fg*, and project it nigher the glaſs *kl*. As to the mechanick contrivance of this lantern, the moſt convenient proportion of the glaſſes, &c. this is ſo ordinary amongſt the common glaſs-grinders, that it is needleſs to inſiſt farther thereon in this place. It is ſufficient to me that I have explained the theory thereof.

Agother magick lantern deſcribed by Dr. 's *Graveſande*.

982. "There are ſeveral machines made by the combination of mirrors and lenſes, which afford uſeful and pleaſant appearances; whoſe explanation may be eaſily deduced from what has been ſaid. Among many other, I ſhall only chuſe to explain one; by which, figures that are painted upon ſmall pieces of glaſs, are repreſented monſtrouſly large upon a white plane. This is a phenomenon wonderful enough to deſerve a particular explanation. The inſtrument that performs this, is called a magick lantern, which optick writers have not altogether paſſed by, but yet have not ſufficiently improved.

Fig. 640.

983. Let there be a wooden box about a foot and an half long, 14 inches high, and as wide; there muſt be a concave mirror *S*, of 8 inches diameter, and a portion of a ſphere of 18 inches: this mirror is fixt to a foot which moves upon rulers, along the length of the box. There is alſo in this box a lamp *L*, ſuſtained by a wooden foot which is moveable lengthways between two rulers, in the ſide of the box. The pipe of the lamp ſtands forward in ſuch manner, that the center of the flame is over againſt

againſt the center of the ſurface of the mirror; this flame is made up of four little flames, which by touching one another, make one ſquare flame, two inches wide. In the top, or upper plane of the box, there is an oblong hole, which has a cover that ſlides in two grooves, or between two rulers or ledges: through this cover paſſes the chimney *C*, which (as you ſee in the 641ſt figure) ſtands up about one half foot above the box. Fig. 641. The chimney is moveable with the cover, whilſt the opening remains ſhut; that the chimney may be always over the lamp. In one of the little ſides of the box, which is over againſt the mirror, there muſt be a round hole about 5 inches wide; which muſt have in it a convex lens of glaſs of the ſame bigneſs *V*, convex on both ſides, which are portions of a ſphere of one foot diameter: the axis of this glaſs, being continued to the ſurface of the mirror, will be perpendicular to it, and fall upon its center, as likewiſe to the plane of the flame, through whoſe middle point it alſo paſſes. This hole is ſhut and opened by a plane moveable in a groove, which is moved by a cylinder that ſtands out of the box at *E*. To this hole without the box anſwers the tube *T*, whoſe length and diameter is of about 6 inches, at the end of which there is a ring, in which the ſecond tube *t* moves, of about 4 inches diameter, and 5 or 6 inches long. In the leſſer tube there are two lenſes; the firſt in that end which is thruſt into the tube *T*, and it is of the ſame convexity as the glaſs *V*, and three inches and a half diameter. The ſecond lens is three inches from the firſt, and flatter, being terminated by portions of a ſphere of four feet diameter. Between theſe lenſes, at the diſtance of about one inch from the ſecond, there is placed a wooden circular ſtop, or aperture, which ſhuts up all the tube, except a hole of an inch and a quarter diameter in the middle of the wood. The objects that are to be repreſented, are to be painted upon a flat thin piece of glaſs, which muſt be moved without the box over againſt the glaſs *V*, between it and the tube *T*, the picture being in an inverted poſition. If theſe pictures are round, they may be of 5 inches diameter: that they may be moved eaſily, they are put into flat boards, three in one board. The picture alſo may be painted upon long glaſſes, which may be ſucceſſively made to ſlide before the glaſs *V*.

984. This whole box ſtands upon a frame or foot, made ſo that it may be fixed at different heights. There are flat pieces of wood fixed to the box, at bottom, which ſlide in grooves in the frame; each of them has a ſlit in it; ſo that the box may be made faſt at any deſired height, by the help of ſkrews joined to the frame, and moveable in the ſlits. The whole machine is placed at the diſtance of 15, 20, or 30 feet from a white plane, which diſtance muſt be different according to the bigneſs of the plane; for this diſtance may be equal to the length of the plane: the box muſt be juſt at ſuch an height that the tubes, in the ſide of the box, may be exactly oppoſite to the middle of the plane. The lamp being lighted, the box

muſt

muſt be ſhut, and the figures which are painted upon the glaſs will be repreſented upon a white plane. By moving backwards and forwards the tube that has the two lenſes in it, you will find the proper poſition of the glaſſes required to give a diſtinct repreſentation. As for the diſpoſition of the ſeveral parts of the machine, which immediately ſerve for exhibiting this appearance, we ſhall here more particularly explain it.

Fig. 642. 985. In this figure the parts are ſhewn ſeparately; *SS* is the mirror, *ll* the flame which conſiſts of four flames in the line *ll*; *VV* is the glaſs *V* of the 640th figure; *OO* is a picture painted upon a flat thin piece of glaſs; *aa* the bigger lens; *dd* the flatteſt lens; *bb* the wooden ſtop between the lenſes; *f* the aperture or hole in the middle of the wooden circle. Theſe things being diſpoſed as has been already explained, and as may be ſeen in the figure, the rays which proceed from a point of the picture *OO*, by going through the lens *aa*, become leſs diverging, and fall upon the lens *dd*, as if they came from a point more remote; from this lens they go out converging, and are collected upon the ſurface of the white plane, where they exhibit the point of the picture that is painted on the glaſs. This picture is illuminated both by the rays that proceed from the flame *ll*, and by the rays reflected by the mirror *SS*. For the perfection of this machine it is required, 1ſt, that the picture *OO* be enlightened as much as poſſible; 2dly, that it be equally enlightened in all its points; 3dly, that all the light, by which every point of the picture is enlightened, go through the lenſes *aa* and *dd* to the white plane, and ſerve to make the repreſentation; 4thly, that no other light but that go out of the box, leſt the repreſentation ſhould be leſs lively, by reaſon of extraneous light. The firſt requiſite depends upon the bigneſs of the flame and of the mirror, and of its concavity; the more concave it is, the nearer it is to be brought to the flame, and then the more rays will be intercepted and reflected; yet care muſt be taken that the mirror (which may be made of very good glaſs) be not too much heated. When the flame and mirror are ſo contrived, that the picture is the moſt enlightened that it can poſſibly be, and every where equally enlightened, the flame and mirror muſt be ſo placed, that the inverted image of the flame ſhall fall juſt upon the picture. Now as the repreſentation of the flame can be increaſed and diminiſhed, the mirror and flame muſt be ſo diſpoſed, that the repreſentation of the flame ſhall cover the whole picture upon the glaſs, but ſo as not to exceed it: for then the picture is as much enlightened by the reflected light as it can be, and all its points are equally illuminated; the direct light alſo does pretty near equally fall upon all the points of the picture; this light would indeed be increaſed by bringing the flame nearer; but the reflected light would be diminiſhed, and the diminution of this laſt would be greater than the increaſe of the other. The glaſs *VV* ſerves to inflect the light, by which the picture *OO* is illuminated, before





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fore it comes to the picture; by which inflection all the light comes to the lens *a a*, and ſerves for the repreſentation on the white plane. All the light that is of uſe for this repreſentation, goes through the hole *f*; and the rays coming from different points, interſect one another there. Wherefore the picture upon the glaſs, which is placed inverted, is repreſented erect upon the white plane. By the ring *b b* all the rays, which do not ſerve to form the repreſentation, are intercepted, leſt they ſhould enter the room, and make the picture leſs diſtinct. This ring or aperture alſo intercepts thoſe rays by which one point is more enlightened than another, whereby the light, which (from what has been ſaid) is equally enough diffuſed, is yet made more equal. But unleſs the ſtop or aperture *b b* be juſt where the rays interſect, it does a great deal of miſchief.

CHAPTER XVIII.

The mechaniſm of various Microſcopes, with ſome microſcopical obſervations.

986. HAVING conſidered the theory of ſingle and double microſcopes in the 118, 129, 127, 690, 705, 715, 720, &c. articles, and in the remarks upon them, I proceed here to a deſcription of ſeveral ingenious contrivances for placing the objects at a due diſtance from the glaſſes, and for caſting a proper light upon them. But firſt I will ſhew how the little glaſs globules are made for ſingle microſcopes; for the manner of making little lenſes, being much the ſame as of making object-glaſſes for teleſcopes, need not here be repeated.

To make glaſs globules for microſcopes.

987. Mr. *Butterfield* tells us[a] he had tryed ſeveral ways of making glaſs globules of the bigneſs of great pins heads and leſs, as in the flame of a candle made of tallow or of wax; but that the beſt ſort of flame for making them clear and without ſpecks was that of a lamp, made with rectified ſpirit of wine. Where inſtead of a cotton wick, he made uſe of fine ſilver wire doubled up and down like a ſkein of thread. Then having prepared ſome fine glaſs, beaten to powder and waſhed very clean, he took a little of it upon the ſharp point of a ſilver needle wetted with ſpittle, and held it in the flame, turning it about till it melted and became quite round, but no longer for fear of burning it. The art lyes in giving the globule an exact roundneſs, which can only be learned by experience. When a great many globules are thus formed, he rubs them clean with a ſoft leather. Then having ſeveral ſmall pieces of thin braſs plates, twice as long as they are broad, he doubles them up into the form of a ſquare, and punches a fine hole through the middle of them: and having rubbed off the bur about the holes with a whetſtone, and blacked the inſides of the plates with the ſmoke of a candle, he places a globule between

[a] Phil. Tranſ. N°. 141.

between the two holes, and tacks the plates together with two or three rivets. Then he tries how they magnify ſmall objects, and keeps the beſt of them for uſe.

988. Dr. *Hook* uſed to take a very clear piece of glaſs and to draw it out into long threads in a lamp; then he held theſe threads in the flame, till they ran into round globules hanging to the end of the threads. Then having fixt the globules with ſealing wax to the end of a ſtick, ſo that the threads ſtood upwards, he ground off the ends of the threads upon a whetſtone and poliſhed them upon a ſmooth metal plate with a little putty.

a Phil. Tranſ. N°. 221, 223.

989. Mr. *Stephen Gray* tells us[a] that for want of a ſpirit lamp, he laid a ſmall particle of glaſs, about the bigneſs of the intended globule, upon the end of a charcoal; and by the help of a blaſt-pipe, with the flame of a candle he ſoon melted it into a globule. By this means he made them indifferently clear, and the ſmalleſt very round; but the larger by reſting upon the coal were a little flatted, and received a roughneſs on that ſide. Therefore he was was wont to grind and poliſh them upon a braſs plate, till he reduced them to hemiſpheres. But he found that the ſmall round globules, beſides that they magnified more, ſhewed objects more diſtinct than the hemiſpheres.

Particles within a tranſparent globule how magnified.

990. Theſe experiments led him to another which is very curious. Having obſerved, ſays he, ſome irregular particles within the glaſs globules, and finding they appeared diſtinct and prodigiouſly magnified, when held cloſe to my eye; I concluded that if I conveyed a ſmall globule of water cloſe to my eye, in which there were any opacous or leſs tranſparent particles than the water, I might ſee them diſtinctly. I therefore took on a pin a ſmall portion of water, which I knew to have in it ſome minute animals, and laid it on the end of a ſmall piece of braſs wire, that then lay by me, about $\frac{1}{10}$ of an inch in diameter, till there was formed ſomewhat more than an hemiſphere of water. Then keeping the wire erect, I applyed it to my eye, and ſtanding at a proper diſtance from the light, I ſaw them and ſome irregular particles as I had predicted; but moſt enormouſly magnified. For whereas they were ſcarce diſcernable by my glaſs microſcope, they appeared within the globule not much different in form nor leſs in magnitude than ordinary peaſe. They cannot well be ſeen by day-light, unleſs the room be darkened, after the manner of the famous dioptrical experiment, but moſt diſtinctly by candle-light. They may alſo be ſeen very well by the full moon-light.

Fig. 643.

991. Mr. *Gray* explains the reaſon of this appearance as follows. Let the circle $DBBD$ repreſent a ſphere of water, A an object placed in its focus, ſending forth a cone of rays, two of which are AB, AB; which coming into the water at B and B, will be refracted from their direct courſe and become BD, BD; and at D, D they will, at their paſſing into

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to air, again be refracted into DE, DE, and ſo run parallel to one another and to the axis of the ſphere $AFCG$. Now it is a known principle in opticks, that the angle of reflection is equal to the angle of incidence: wherefore let the rays BD, BD, be imagined to have come from ſome point F of an object placed within the ſphere of water, by being reflected from the interior ſurface of the ſphere at B, B. But CBD is the angle of reflection, to which making CBF equal, the point F will be the place from whence if an object ſends forth a cone of rays, two of them as FB, FB, will be reflected into the lines BD, BD; and then coming to the other ſide of the globule at D, D, they will be refracted into DE, DE, as before: and conſequently will be fit for diſtinct viſion, whether the object be placed in F within, or in A without, the ſphere, if its interior ſurface be conſidered as a concave ſpeculum.

And in what proportion.

992. This explanation of Mr. *Gray*'s may be carried a little farther, to ſhew the reaſon why the animals appeared ſo enormouſly great, as he deſcribes them. The concluſion is this; the animals within the water globule are magnified $3\frac{1}{3}$ times more in diameter, than they would be if placed without the globule at its principal focus: and this increaſe of the magnifying power of the ſmalleſt globules is very conſiderable. To demonſtrate this concluſion, let the perpendiculars Aa, Ff to the diameter HFI be cut by any other diameter hfi in a and f; and then the unequal objects Aa, Ff, whereof the former is ſeen by refraction, and the latter by reflection, will appear under the ſame angle HCh, and conſequently of the ſame magnitude. Therefore one and the ſame object placed at F will appear bigger than at A in the ratio of CA to CF*. Imagine the rays ED, GH to go backwards or from the eye, and after the firſt refraction let them meet at K, and after the ſecond refraction at A; which will biſect the line IK*. Therefore biſecting CI in T, we have $CA:CT$ :: ($2CA$ or $HK:HC$, that is as) the ſine of incidence (m) to the difference of the ſines ($m-n$)*; and diſjointly we have $TA:TC :: n : m-n$. Hence $CA = \frac{m}{m-n}\,CT$ and $TA = \frac{n}{m-n}\,CT$. Now becauſe K is the focus of incident rays as DB upon the ſurface BI, their focus F after reflection will be had by taking $TF:TI$ or $TC :: TC : TK$*, and conjointly $CF : CT$ :: ($CK:TK$ that is :: $2TA : 2TA-TC :: \frac{2n}{m-n}\,TC : \frac{2n}{m-n}\,TC-TC$ ::) $2n : 3n-m$. Therefore $CF = \frac{2n}{3n-m}\,CT$, and $CA:CF :: \frac{m}{m-n} : \frac{2n}{3n-m}$. Hence putting $m : n :: 4 : 3$ as in water, we have $CA:CF :: 10 : 3 :: 3\frac{1}{3} : 1$, which was to be proved.

Fig. 644.

* Art. 222.

* Art. 227.

* Art. 224.

* Art. 207.

993. In a glaſs globule putting $m : n :: 3 : 2$, we have $CA:CF :: 2\frac{1}{4} : 1$, which ſhews how much any ſmall particles within the glaſs, are more magnified than if they were placed beyond it at its principal focus.

994. Hence alſo the animals within the water globule are more magnified than if they were ſeen through a glaſs globule of the ſame bigneſs, in the ratio of $2\frac{1}{2}$ to 1. For in glaſs $CA = 3\,CT$, and in water $CF = \frac{6}{5}CT$. Therefore $CA : CF :: 2\frac{1}{2} : 1$.

995. Hence alſo the animals within the water globule, appear of the ſame magnitude as they would do through a glaſs globule, whoſe diameter is $\frac{2}{5}$ths of the diameter of the water globule. For ſuppoſing CA or $3\,CT$ in glaſs, equal to CF or $\frac{6}{5}\,CT$ in a larger water globule; we have CT in the glaſs globule, to CT in the water globule, as $\frac{6}{5}$ to 3, or as $\frac{2}{5}$ to 1.

996. But Mr. *Gray* tells us theſe little animals will appear more diſtinctly, if drops of water be conveyed by a pin's point into a round hole made in a braſs plate, whoſe thickneſs is about one tenth of an inch, and the diameter of the cylindrical hole a little leſs than half a tenth, obſerving to fill it till near an hemiſphere of water be extant on each ſide of it. Now ſuppoſing the axis of this cylinder of water to be terminated by equal ſpherical ſurfaces, and to be exactly equal to three diameters of the ſphere of thoſe ſurfaces, I find the little animals, ſeen by reflection from the farther ſurface, will appear juſt twice as big in diameter, as if they were placed in the focus of one of thoſe ſpheres of water, and were ſeen through it as in common microſcopes.

Deſcription of little animals in water &c.

997. The author's account of the little animals thus obſerved, being very curious, deſerves to be tranſcribed. " They are of a globular form and but little leſs tranſparent than the water they ſwim in. They have ſometimes two dark ſpots diametrically oppoſite; but theſe are rarely ſeen. There are ſometimes two of theſe globular inſects ſticking together; where they are joined it is opacous: poſſibly they may be in the act of generation. They have a twofold motion; a ſwift progreſſive regular one, and at the ſame time a rotation about their axes, at right angles to the diameter that joins the dark ſpots: but this is ſeen only when they move ſlowly. They are almoſt of an incredible minuteneſs. Mr. *Leeuwenhoek* is moderate enough in his computation, when he tells us (Phil. Tranſ. N°. 213. p. 198.) he ſaw inſects in water ſo ſmall that 30000 could hardly equal a courſe ſand. But I believe it will ſeem a paradox to him, when he is told he may ſee them by only applying his bare eye to a portion of water wherein they are contained. I have examined many tranſparent fluids, as water, wine, brandy, vinegar, beer, ſpittle, urine, &c. and do not remember to have found any of theſe liquors without more or leſs of theſe inſects. But I have not ſeen many in motion, except in common water that has ſtood, for ſometimes a longer at others a ſhorter time, as has been obſerved by Mr. *Leeuwenhoek*; though I do not remember he has obſerved that they are exiſtent in the water before they revive. In the river, after the water has been thickened by rain, there are ſuch infinite numbers of them, that the water ſeems in great part to owe its opacity and whiteneſs

neſs to theſe globules. Rain water ſo ſoon as it falls has many, and ſnow water has more of them. The dew that ſtands on glaſs windows has many of them: and foraſmuch as rains and dews are continually aſcending or deſcending, I believe we may ſay the air is full of them. They ſeem to be of the ſame ſpecifick gravity with the water they ſwim in, the dead remaining in all parts of the water. Of many thouſands that I have ſeen I could diſcern no ſenſible difference in their diameters; they appear of equal bigneſſes in water that has been boiled; they retain their ſhapes and will ſometimes revive.

998. There is another ſort of inſects I have ſeen this way, but theſe are not ſo frequently to be found, at leaſt this winter ſeaſon. They are much longer than the former, and can transform themſelves into many ſhapes. They are elliptical for the moſt part; but ſometimes they contract themſelves ſo as to be almoſt globular; and ſometimes they extend themſelves ſo as to be two or three times longer than broad. Theſe ſometimes turn themſelves round, on their axes and diameters, as they go, and conſiſt of tranſparent and opacous parts.

Another water microſcope by Mr. *Gray*. Fig. 645.

999. The ſame ingenious author deſcribes another water-microſcope of his own invention, as follows. "*AB* I call the frame of the microſcope; it may be about $\frac{1}{16}$ inch in thickneſs. At *A* there is a ſmall hole, near $\frac{1}{30}$ inch in diameter, in the middle of a ſpherical cavity about $\frac{1}{8}$ inch in diameter, and in depth ſomewhat more than half the thickneſs of the braſs. Oppoſite to this at the other ſide of the braſs, there is another ſpherical cavity, half as broad as the former; and ſo deep as to reduce the circumference of the ſmall hole abovementioned, to almoſt a ſharp edge. In theſe cavities the water is to be placed, being taken upon a pin or a large needle, and conveyed into them till there be formed a double convex lens of water; which, by the concavities being of different diameters, will be equivalent to a double convex lens of unequal convexities. By this means I find the object is rendered more diſtinct, than by a plano-convex of water or by a double convex, formed on the plane ſurfaces of a piece of metal. *CDE* is the ſupporter whereon to place the object, if it be water in the hole *C*, if a ſolid on the point *F*. This is fixt to the frame of the microſcope by the ſkrew *E*, where it is bent upwards, that its upper part *CF* may ſtand at a diſtance from the frame *AB*. It is moveable about the ſkrew *E* as a center to the end that either the hole *C* or the point *F* may be expoſed before, the microſcope *A*, and that the object may be brought to, and fixt in, its focus. There is another ſkrew about half an inch in length, which goes through a round plate in the frame of the microſcope *AE*, the ſkrew and plate taking hold of the ſupporter about *D*; where there is a ſlit ſomewhat larger than the diameter of the ſkrew; which is requiſite for the admiſſion of the hole *C* or point *F* (according to the nature of the object) into the focus of the watery lens

at

at *A*. For by turning the ſkrew *G*, the ſupporter is carried to or from the ſame; which may be ſooner done, if while one turns the ſkrew with one hand, the other holds the microſcope by the end *B*; and one be looking through the water till the object be ſeen moſt diſtinctly. The ſupporter muſt be made of a thin piece of braſs well hammered, that by its ſpring it may better follow the motion of the ſkrew. I chuſe rather to fix the ſupporter by the ſkrew *E* than by a rivet; becauſe it may now, by the help of a knife be unſkrewed, and by the other ſkrew *G* be brought cloſe to the frame of the microſcope, without weakening its ſpring, and ſo become more conveniently portable. If the hole *C* in the ſupporter be filled with water, but not ſo as to be ſpherical, all objects that will bear it are ſeen therein more diſtinctly.

1000. The hole at *B* is made for ſeeing animals in water by reflection from its farther ſurface, as above deſcribed. So far Mr. *Gray*.

A deſcription of Mr. *Wilſon*'s pocket microſcope, taken from Phil. Tranſ. N°. 281. Fig. 646, 647.

1001. This ſet of microſcopes has nine different magnifying glaſſes; eight of which may be uſed with two different inſtruments, for the better applying them to various objects. One of theſe inſtrumens is repreſented at *AABB* and is made of ivory or braſs; it has 3 thin braſs plates at *E* and a ſpiral ſpring of ſteel-wire *H* within it; to one of the thin plates of braſs is fixed a piece of leather *F*, with a ſmall furrow *G* both in the leather and braſs to which it is fixed: in one end of this inſtrument there is a long ſkrew *D* with a convex glaſs *C*, placed in the end of it: in the other end of the inſtrument there is a hollow ſkrew *oo*; wherein any of the magnifying glaſſes, *M*, are ſkrewed when they are to be made uſe of. The 9 different magnifying glaſſes are all ſet in ivory, 8 of which are ſet in the manner expreſſed at *M*. The greateſt magnifier is marked upon the ivory, wherein it is ſet, with N°. 1, the next N°. 2, and ſo on to N°. 8; the ninth glaſs is not marked, but is ſet in the manner of a little barrel box of ivory, as at *b*. At *ee* is a flat piece of ivory, whereof there are 8 belonging to this ſet of microſcopes; (though any one who has a mind to keep a regiſter of objects may have as many of them as he pleaſes;) in each of them there are 3 holes *fff*, wherein 3 or more objects are placed between two thin glaſſes, or talks, when they are to be uſed with the greater magnifiers.

1002. The uſe of this inſtrument *AABB* is this. Having taken the handle *W* from the inſtrument in fig. 647 and ſkrewed it upon the button *S* in fig. 646, take one of your flat pieces of ivory *ee*, or ſliders, (if you pleaſe to call them ſo,) and ſlide it betwixt the two thin plates of braſs at *E*, through the body of the microſcope, ſo that the object you intend to look upon be juſt in the middle; remarking that you put that ſide of the plate *ee*, where the braſs rings are, fartheſt from the end *AA*: then you are to ſkrew into *oo* (the hollow ſkrew in the end of the body of your microſcope) the 3d, 4th, 5th, 6th or 7th magnifying glaſs *M*; which being

done put the end *AA* close to your eye, and while you are looking through your magnifying glass upon the object, you are to skrew in or out the long skrew *D*, which moving round upon the leather *F*, held tight to it by the spiral wire *H*, will bring your object to the true distance; which you will know by seeing it clearly and distinctly: but seeing that in the greater magnifiers you can see but a small part of the object, *viz.* the legs or claws of a flea; while you are looking upon any part of the object, if you take hold of the end of the plate or slider, *ee*, whereon the object lyes, and move it gently, you may see the whole object successively, or any part of the object you please; and if that part of the object you design to look upon be out of the true distance, remember your end skrew *D*, can always bring it in, by skrewing it one way or the other.

1003. After this manner may be seen all transparent objects, dusts, liquids, chrystals of salts, small insects, such as fleas, mites, &c. If they be insects that will creep away, or such objects as one intends to keep, they may be placed between the two register glasses *ff*. For by taking out (with the point of a pen-knife or small plyers) the ring that keeps in the glasses *ff* where the object lies, they will fall out of themselves; so you may lay the object between the two hollow sides of them, and put the ring in as it was before: but if the objects be dusts or liquids, a small drop of the liquid, or a little of the dust laid on the outside of the glass *ff* and applyed as before, will be seen very easily.

1004. As to the first second and third magnifying glasses, being marked with a + upon the ivory wherein they are set, they are only to be used with those plates or sliders that are also marked with a +, wherein the objects are placed between two thin talks; because the thickness of the glasses in the other plates or sliders, hinder the object from approaching to the true distance from these greater magnifiers. But the manner of using them is the same with the former. Only remember to be careful when you put in or pull out the plate or slider *ee*, whereon the object lyes, or move it from one object to another, not to let it rub your magnifying glass; which is done by unskrewing a little the end skrew *D*, when you put in or pull out your plate, or move it from one object to another.

1005. For seeing the circulation of the blood at the extremities of the arteries and veins, in the transparent parts of fishes tails, &c. there are two glass tubes, the one bigger and the other lesser, as expressed at *gg*, wherein the fish is to be put; when these tubes are to be used, you are to unskrew the end skrew *D* in the body of the microscope, until the tube *gg*, can be received easily into that little cavity *G* of the brass plate fastened to the leather *F*, under the other two thin plates of brass at *E*: when the tail of your fish lies flat to the glass tube, set it opposite to your magnifying-glass, and by skrewing in or out your end skrew *D*, as is said before,

fore, you may eaſily bring it to the true diſtance, and ſee the blood circulate with great pleaſure.

1006. If you would ſee the blood circulate in a frog's foot, chooſe ſuch a frog as will juſt go into your tube; then with a little ſtick expand the hinder foot of the frog, and apply it cloſe to the ſide of the tube, obſerving that no part of the frog hinders the light from coming on its foot; and when you have it at the juſt diſtance, by means of the ſkrew *D*, as aboveſaid, you will ſee the rapid motion of the blood in its veſſels, which are very numerous, in the tranſparent thin membrane that is between the frog's toes. For this object the 4th and 5th magnifiers will do very well; but you may ſee the circulation in the tails of water-newts with the 6th and 7th glaſſes, by reaſon the globules of the blood of thoſe newts are as big again as the globules of the blood of frogs or ſmall fiſh, as has been taken notice of in N°. 280. of the Philoſophical Tranſactions, pag. 1184.

1007. The circulation cannot ſo well be ſeen by the firſt ſecond and third magnifyers, becauſe the thickneſs of the glaſs tube wherein the fiſh lyes, hinders the approach of the object to the focus of the magnifying glaſs.

1008. The other inſtrument fig. 647 is made of braſs or prince's metal, with joints *P P P* to turn eaſily any way, and with a ſmall pair of tongs *G G*, which open at the points *K*, by preſſing together the two heads of the pins *I I*, for taking up of objects. At the other end of theſe tongs *G G*, is ſkrewed on a round piece of black wood *H*, with a piece of ivory let into it, for placing opake objects on, according to their difference of colour. Upon the end L there is a ſkrew, upon which the glaſs *b* ſet in the barrel-box may be ſkrewed. When the other glaſſes are to be uſed, there is a ring *R* of braſs to be ſkrewed on the end *L*, into which ring all the other glaſſes *M*, may be ſkrewed. So when any object is taken up in the points of the tongs *K*, or laid upon the other end *H*, it may very eaſily (as one who ſees the inſtrument will perceive) be applyed to the true focal diſtance of any of the glaſſes *M*, by the help of the joints *P P P*, and by means of the ſkrew *C*, with the wheel *D*, which being regulated by a ſpring *N*, will bring the object to the exact diſtance for diſtinct viſion.

1009. The glaſs placed in the manner of a barrel-box at *b*, is only to be uſed with the braſs inſtrument (or in your hand) being the leaſt magnifier, for greater objects, ſuch as flies and common inſects, &c. remembring to put the hole *b* next to your eye.

1010. In the viewing objects, one ought to be careful not to hinder the light from falling upon them, by the hat, peruke, or any other thing, eſpecially in looking at opake objects: for nothing can be ſeen with the beſt of glaſſes, unleſs the object be at a due diſtance, with a ſufficient light. The beſt lights for the plates or ſliders, where the object lyes

between

between the two glaſſes, is a clear ſky-light, or where the ſun ſhines on any white thing, or the reflection of the light from a looking-glaſs. The light of a candle is likewiſe good for the viewing of very ſmall objects, though it be a little uneaſy to thoſe who are not practiſed in microſcopes to find it out. The only uſe of the convex lens at *C* is to collect the light into a narrower compaſs where it falls upon the object, after it has paſſed through a moderate hole in the leather *F*.

1011. For the convenience of thoſe who would draw, or make any ſketches or deſigns of microſcopical objects, they may alſo have a pedeſtal to fix the two inſtruments above deſcribed, and make them ſtationary to any convenient light. This pedeſtal may be placed on a table, and after the object and light are fixed, as many perſons as pleaſe may view the object, without any trouble or difficulty in finding the light.

A deſcription of Mr. *Marſhal*'s double microſcope by Dr. *Harris* in his Lexicon Technicum. Fig. 648.

1012. The eye-glaſs is at *W*, the object-glaſs at *C*, the middle glaſs at *A* 1; *B* is the cover or lid, to keep out the duſt from the eye-glaſs *W*; *X* is the place of the eye, *W* a ſkrew where the eye-glaſs lyes; *A* 1 a ſkrew where the middle glaſs lyes; *A*2 the draw, where the outermoſt tube *A* 1 *A*2 is disjoined from the inner one, of the ſame length; *Z* the frame or baſis on which the microſcope ſtands firm; *T* a ſmall drawer in the frame or baſis, with a ledge or till in it, having ſix partitions to hold ſo many ſeveral object-glaſſes, one magnifying more than another, and fixed in braſs cells ready to ſkrew on at *C*, and marked 1, 2, 3, 4, 5, 6; theſe partitions are alſo marked, 1, 2, 3, 4, 5, 6; the other part of the drawer ſerves to hold the object-plate *a*; a pair of ſmall nippers *b*, to take up, or handle any object conveniently; another object-plate *d*, having one ſide white and the other black, to fix your objects upon, as black upon white, and white objects on black. *L*, *M* is a braſs ball and ſocket, on which the whole body of the microſcope is moveable, ſo as to lye in any poſition for the light. *K*, *O* a ſquare braſs pillar on which the microſcope is moveable up and down, by means of the collar *E*, into which the arm *D*, (holding the microſcope) is continued. *G* another braſs collar ſliding up and down on the pillar *KO*, having a ſmall ſkrew *H* by which it is, as occaſion ſerves, fixed faſt to the ſaid pillar, at any height. *I* a large braſs nut, in whoſe center is a female ſkrew, fitted to the male ſkrew *F*, which is fixed in the collar *E*: by the turning of which nut *I*, (the collar *G* being firſt fixed to the pillar by the ſkrew *H*) the microſcope is raiſed up or down on the pillar, and made to come nearer or go farther from the object *P c*: and, which is alſo a very great advantage, the axis of the microſcope is always kept perpendicular to that point of the object, over which it was firſt placed; ſo that here is not the inconvenience which occurs in other glaſſes, of often loſing the ſight of the object, by ſkrewing the glaſs *C* higher or lower. *PQ* is a glaſs object-plate fixed in a braſs frame, whoſe arm *NN*, is fixed to the pillar by means of the nut *O*. The arm *NN* hath in

it a ſlit, by which it is eaſily put on, or taken off the pillar, and by which it may be fixed upon it at any diſtance. *P* a ſmall fiſh lying on the glaſs-plate, that the circulation of the blood may be ſeen in the end of the tail-fin, at *c*. *R* a convex-glaſs, by whoſe help a bright ſpot of light is brought from a candle at *S*, ſtanding on the ground, while the microſcope ſtands on the edge of a table or ſtool, which ſpot of light *c* ſerves to render the circulation more conſpicuous. *V* a lead coffin to be put on the fiſh, to hinder it from ſpringing away, and moving his tail out of the light. 1, 2, 3, 4, 5, 6, are marks on the pillar *KO*, to ſhew the reſpective diſtances of the object-glaſs from the object you look upon, according as the object-glaſſes you make uſe of magnify more or leſs. Thus, for inſtance, if you uſe the object-glaſs 5 or 6 (either of which will ſhew the circulation of the blood) you muſt fix the upper edge of the collar *E*, at the mark 5 or 6 on the pillar. And then the microſcope will be very near its exact diſtance from the object; ſo that by a ſmall turn or two of the nut *I* one way or the other, to be found by tryal, you may ſoon fit it exactly to your own eye.

1013. By this microſcope liquors alſo may be very commodiouſly examined; for if you place a ſmall drop of any liquor on the glaſs-plate juſt in the middle of the ſpot of light *c*, the parts of it will become very viſible, and its animalcula, if it have any, will be diſcovered. And thus may the eels in vinegar, the ſmall creatures in black-pepper water, or in waters where wheat, barly, &c. has been infuſed, the eels and other ſmall living creatures in puddle water, be as plainly ſeen as by almoſt any other microſcope.

1014. There is one thing I ought not to omit to mention on this occaſion; which is, that I have often with this glaſs, ſeen the circulation of the blood in the fins of the tails of tadpoles; and indeed more conſpicuouſly here than in any other creature: for the fins growing all round the tail, and coming but a little way out beyond the body of it, both the ejaculation of the blood out by the arteries, and its return again by the veins, is much quicker than in the tails of fiſhes; and abundance more ſtreams, turns, and windings of the moving blood are here viſible, than I could ever ſee in any other animal. To which I may add, that the creature will live a good while out of the water, and will lie very ſtill. So far Dr. *Harris*.

1015. Dr. *Hook* tells us in the preface to his Micrography that in moſt of his obſervations he uſed a double microſcope; retaining the broad middle glaſs, when he wanted to ſee very much of an object at one view, and taking it out when he would examine the ſmall parts of the objects more accurately. For always the fewer the refractions, the more bright and clear the object appears. Accordingly he tells us that objects appear more diſtinct and more magnified through a ſingle glaſs globule, prepared

as

as above [a], than through any double microſcope. Nevertheleſs he ſeldom made uſe of a ſingle microſcope upon account of the difficulties in placing the objects ſo near the globule as they ought to be.

[a] Art. 987.

Illumination of microſcopical objects.

1016. Since the aperture of the object-glaſs of a double microſcope muſt be very ſmall to ſhew the object diſtinct, and conſequently can receive but little of its light, the way to remedy this inconvenience is to illuminate the object as much as poſſible. Dr. *Hook* made choice of a room having but one window towards the ſouth; where at the diſtance of three or four feet from the window, he placed his double microſcope upon a table, and by a glaſs globe full of water and a thick plano-convex lens he collected the light upon the object. Or when the ſun ſhone he placed a piece of oiled paper very near the object, and with a very large burning-glaſs he threw the ſun's rays upon the paper, ſo that a great quantity of them might be tranſmitted through it to the object. But the paper being ſubject to take fire, if the focus of the rays fell upon it, inſtead thereof he ſometimes ſubſtituted a piece of plane glaſs, not poliſhed but only rough ground with fine ſand; which when warmed gradually would endure much more heat than oiled paper, and conſequently would ſuffer more light to paſs through it to the object. By this means he ſays the light of the ſun was more equally diffuſed upon the parts of the object than the ſun's direct light; which coming but in one direction, would be ſo ſtrongly reflected from ſome parts of the object, as to drown the appearance of all the reſt that lay in the ſhade. In the night time he illuminated his object with the light of a lamp, firſt refracted through his globe full of water, (or clear brine which refracts more than water) and then collected into a ſmaller ſpot upon the object by a plano-convex lens. On the ſide of the lamp oppoſite to the globe, he alſo placed a poliſhed concave metal, to reflect part of the rays upon the globe. And thus he ſays he could illuminate an object with a ſmall lamp as much as it would well bear; and that he drew moſt of his repreſentations of objects by the light of his lamp. Why the Dr. always preſcribes the uſe of a plano-convex lens, I do not underſtand. For it is evident that a double convex lens will contract a given pencil of rays into a much ſmaller ſpace than a plano-convex one of the ſame ſphere, becauſe its principal focus is as near again to the lens. But after all though greater degrees of illumination are great helps to greater degrees of magnifying with ſmaller object-glaſſes, yet after a certain degree of magnifying, he obſerves, they leave us again in the dark.

Mr. *Folkes*'s account of *Leeuwenhoek*'s microſcopes. Phil. Tranſ. N°. 380.

1017. Mr. *Leeuwenhoek* on the contrary in all his obſervations which he continued to communicate to our Royal Society for above 50 years together, made little or no uſe of any other microſcopes but ſingle ones, as we are informed by our worthy Vice-Preſident *Martin Folkes* Eſquire, in the account he has given us of Mr. *Leeuwenhoek*'s legacy to the Society of

of 26 of theſe microſcopes. Thoſe that are converſant in microſcopical obſervations may be uſefully entertained with this account, part of which I will here tranſcribe. " Theſe microſcopes are all ſingle conſiſting each of a ſmall double convex-glaſs, let into a ſocket, between two ſilver plates riveted together, and pierced with a ſmall hole. The object is placed on a ſilver point or needle, which by means of ſkrews of the ſame metal, provided for that purpoſe, may be turned about, raiſed or depreſſed, and brought nearer or put farther from the glaſs, as the eye of the obſerver, the nature of the object, and the convenient examination of its ſeveral parts may require. Mr. *Leeuwenhoek* fixed his objects, if they were ſolid, to this ſilver point with glew; and when they were fluid or of ſuch a nature as not to be commodiouſly viewed unleſs ſpread upon glaſs, he firſt fitted them on a little plate of talk, or exceſſively thin-blown glaſs; which he afterwards glewed to the needle in the ſame manner as his other objects. The obſervation indeed of the circulation of the blood and ſome others require a ſomewhat different apparatus; and ſuch a one he had, to which he occaſionally fixed theſe ſame microſcopes. But as it makes no part of this legacy, I ſhall omit giving any farther account of it; only taking notice it may be ſeen in a letter to the Royal Society, on the 12th of *January* 1689, and printed in his *Arcana Naturæ detecta* N°. 96. But I was willing to mention juſt ſo much to ſhew the univerſal uſe of theſe ſingle microſcopes; and as it induces me, among other things to believe, theſe were the kind of microſcopes generally, if not ſolely, uſed by this curious gentleman in all his obſervations, and to which we are obliged for his moſt ſurpriſing diſcoveries.

1018. The glaſſes are all exceedingly clear and ſhew the object very bright and diſtinct; which muſt be owing to the great care this gentleman took, in the choice of his glaſs, his exactneſs in giving it the true figure, and afterwards among many, reſerving ſuch only for his uſe, as upon tryal he found to be moſt excellent. Their power of magnifying are different as different ſorts of objects may require; and as on the one hand, being all ground glaſſes none of them are ſo ſmall and conſequently magnify to ſo great a degree as ſome of thoſe drops frequently uſed in other microſcopes, yet on the other hand, the diſtinctneſs of theſe very much exceeds what I have met with in glaſſes of that ſort; and this was what Mr. *Leeuwenhoek* ever principally propoſed to himſelf, rejecting all thoſe degrees of magnifying in which he could not ſo well obtain that end. For he informs us in one of his letters, where he is ſpeaking of the exceſſive praiſe ſome give to their glaſſes on this account, that though he had above 40 years had glaſſes by him of an extraordinary ſmalneſs, he had made but very little uſe of them; as having found in a long courſe of experience, that the moſt conſiderable diſcoveries were to be made with ſuch

glaſſes,

glaſſes, as magnifying but moderately, exhibited the object with the moſt perfect brightneſs and diſtinction.

1019. Nor ought we to forget a piece of ſkill in which he very particularly excelled, which was that of preparing his objects in the beſt manner to be viewed by the microſcope: and of this I am perſwaded any one will be ſatisfied who ſhall apply himſelf to the examination of ſome of the ſame objects as do yet remain before theſe glaſſes. At leaſt I have my ſelf found ſo much difficulty in this particular, as to obſerve a very ſenſible difference between the appearances of the ſame object, when applyed by my ſelf and when prepared by Mr. *Leeuwenhoek*, though viewed with glaſſes of the very ſame goodneſs. I have the rather inſiſted upon this, as it may be a caution to us, that we do not raſhly condemn any of this gentleman's obſervations, though even with his own glaſſes we ſhould not be able to verify them ourſelves. We are under great diſadvantages for want of the experience he had, and he has himſelf put us in mind, that thoſe who are the beſt ſkilled in the uſe of magnifying glaſſes, may be miſled, if they give too ſudden a judgement upon what they ſee, or till they have been aſſured by repeated experiments. But we have ſeen ſo many and thoſe of his moſt ſurpriſing diſcoveries, ſo perfectly confirmed by great numbers of the moſt curious and judicious obſervers; that there can ſurely be no reaſon to diſtruſt his accuracy in thoſe others which have not yet been ſo frequently or ſo carefully examined. So far Mr. *Folkes*.

Micrometers applicable to double microſcopes.

2020. Thoſe that are curious in making exact draughts of the appearances of objects ſeen in double microſcopes, may be very much aſſiſted by a lattice made with fine ſilver wires, or with the ſtrokes of a diamond upon a plane glaſs, put into the place of the image formed by the object-glaſs; and by transferring the parts of the object, ſeen in the ſquares of the lattice, into correſponding ſquares of a ſimilar lattice drawn upon paper. It may alſo be of ſingular uſe in philoſophical enquiries to know the exact meaſures of the ſeveral veſſels and other parts of animal and vegetable ſubſtances; and this, as Mr. *Balthaſaris* has obſerved in his little treatiſe upon Micrometers, may be done very exactly by a micrometer of the ſame form as is uſed in a teleſcope. For by opening the hairs of the micrometer till they exactly comprehend an object of a known length, ſuppoſe $\frac{1}{10}$ inch, and by obſerving the number of revolutions in this opening; the diameter of any other object, anſwering to a known number of revolutions, may be found by the golden rule.

Dr. *Jurin's* way of meaſuring microſcopical objects.

1021. But the moſt ingenious Dr. *James Jurin*, in his excellent Diſſertations upon Phyſico-mathematical ſubjects pag. 45, has taught us a very accurate and ready way of doing the ſame thing even without a micrometer. Firſt he twiſts a very fine ſilver wire a great many times about a pin or a ſlender cylinder, ſo cloſely as to leave no interval between the

wreaths

wreaths, which he carefully examines by a magnifying glaſs; then he takes the interval of the outermoſt wreaths between the points of a pair of fine compaſſes, and applies this extent to a diagonal ſcale of inches; and by dividing this meaſure of that extent, by the number of wreaths therein contained, he obtains the thickneſs of the wire it ſelf. Then he cus it into very ſmall bits, and ſcattering them upon the object-plate, he places the object upon them, if it be tranſparent, or the wires upon the object, if it be opake; and by the eye he compares the parts of the object with the thickneſs of thoſe wires that happen to lye contiguous to them.

Applyed to blood-globules.

1022. Thus he obſerved that 4 globules of human blood would generally cover the breadth of a wire which he had found to be $\frac{1}{485}$ part of an inch, and by conſequence that the diameter of a ſingle globule was $\frac{1}{1940}$ part of an inch. Which was alſo confirmed by Mr. *Leeuwenhoek*'s obſervations upon human blood, made with a piece of the ſame wire tranſmitted to him from Dr. *Jurin*. Phil. Tranſ. N°. 377.

Other obſervations on blood-globules by Dr. *Jurin*.

1023. We are alſo obliged to this learned and judicious gentleman, my highly eſteemed friend, for two other microſcopical obſervations, to rectify two vulgar errors. The firſt is, that the globules of blood were thought lighter than the ſerum; which gave riſe to a ſecond miſtake, that they contained either air or ſome elaſtical fluid within them. I have often placed, ſays the Dr, a drop of ſerum upon a clean glaſs before a microſcope, in which I had diſſolved a very ſmall quantity of blood; and obſerved, that when the glaſs was held in a perpendicular poſture, the blood-globules ſubſided to the bottom of the drop: and inverting the glaſs, the globules again deſcended through the ſerum to the bottom. I had alſo the ſame ſucceſs with a ſmall quantity of ſerum and blood in a capillary tube. And the ſame has alſo been obſerved by the famous Mr. *Leeuwenhoek*. Hence it is by no means probable that the blood globules are veſicles filled with air, or any other fluid lighter than the ſerum; and that they are not filled with any ſort of elaſtick fluid, will appear by the following experiment. In a ſmall quantity of ſerum I diſſolved ſo much human blood as that the globules might not lye too thick together, to hinder their being ſeen diſtinctly. Then having lodged a ſmall drop of this liquor on the ſide of a thin glaſs tube, I fitted the tube to the air-pump and placed a microſcope by it, ſo that I could ſee the blood-globules through the tube. This being done I cauſed the tube to be exhauſted, keeping my eye upon the globules all the time, in order to obſerve whether they dilated themſelves, as the air was withdrawn; but could not perceive the leaſt alteration; they appearing exactly of the ſame bigneſs in the Vacuum as they had done before. Whereas if they had been filled with an elaſtick fluid, they would either have burſt, or have been dilated to at leaſt 70 or 80 times their former magnitude. The ſtop-cock being afterwards turned and the air ſuffered



649.

650

652

Sun's disk seen inverted

651.

May. 18. Aug. 20. Nov. 19. Feb. 15.

Eclip- -tick

E F G H I K L M

Jun. 10. Sep. 12. Dec. 10. Mar. 9.

Eclip- -tick

fered to re-enter the tube, the blood-globules ſtill retained the ſame bigneſs as in vacuo. So far Dr. *Jurin.* Phil. Tranſ. N°. 361.

A double microſcope, in which the objects are illuminated by reflection. Fig. 649.

1024. In this microſcope, made by Mr. *Culpeper* and Mr. *Scarlet*, the inner tube *a b*, which ſlides in the outer *c d*, holds all the glaſſes. The eye-glaſs is at *a a*, the broad middle glaſs at *b b*, and the object-glaſs, being ſet in a button at *e*, is ſkrewed upon the end of a narrower tube *f g*; which being fixed in the baſe of the inner tube, paſſes freely through a hole in the baſe of the outer. The buttons that contain the ſeveral object-glaſſes are marked 1, 2, 3, &c. and the convexity of the inner tube is alſo marked with dotted circles, numbered 1, 2, 3, &c. in order to bring that circle to coincide with the mouth *c c* of the outer tube, whoſe number is the ſame as that of the object-glaſs then made uſe of. But if the object does not yet appear quite diſtinct, the button *e* upon the ſnout *f g* muſt be gradually unſkrewed, to bring the glaſs nearer to the object placed below it. Of theſe glaſſes the greater magnifiers are known by their having ſmaller apertures.

1025. The baſe *d d* of the outer tube is ſupported by three braſs pillars, fixt into a wooden pedeſtal *h*; and a little below the object-glaſs *f*, a circular plate *i k l* is lodged like a ſtage between the pillars, three ſemicircular notches being cut in its circumference to receive the pillars, and to reſt upon three rings that ſurround them. Three ſmall braſs circles *m n* with holes through the middle of them, are to be placed over the hole *k* in the middle of the ſtage; and then the ivory plate *o* may be put between the two uppermoſt of theſe circles, which are preſſed together by a ſpiral ſpringing wire lodged between the two undermoſt; the two outermoſt being held together by three ſmall pillars paſſing through three holes in the circumference of the middle circle. For viewing the circulation of the blood, the button *p*, on the under ſide of the frame of a broad plane glaſs *q r*, being put through a ſlit *i*, made in the ſtage, a ſmall braſs plate *s* under the ſtage, muſt be ſhoved inwards, till a ſmaller ſlit in it embraces the neck of the ſaid button; and then the fiſh being laid upon this glaſs and covered with the coffin *t*, its tail may be brought exactly under the object-glaſs by turning the glaſs *p q* about the button, or by ſhoving it inwards or outwards along the ſlit *i* in the ſtage. The circular object-plate *v x* has a like button in its center, to be put into the ſame ſlit as before; and then the different objects, placed between two talks in the holes made round the circumference of the plate, may be viewed ſucceſſively by turning the plate about its center.

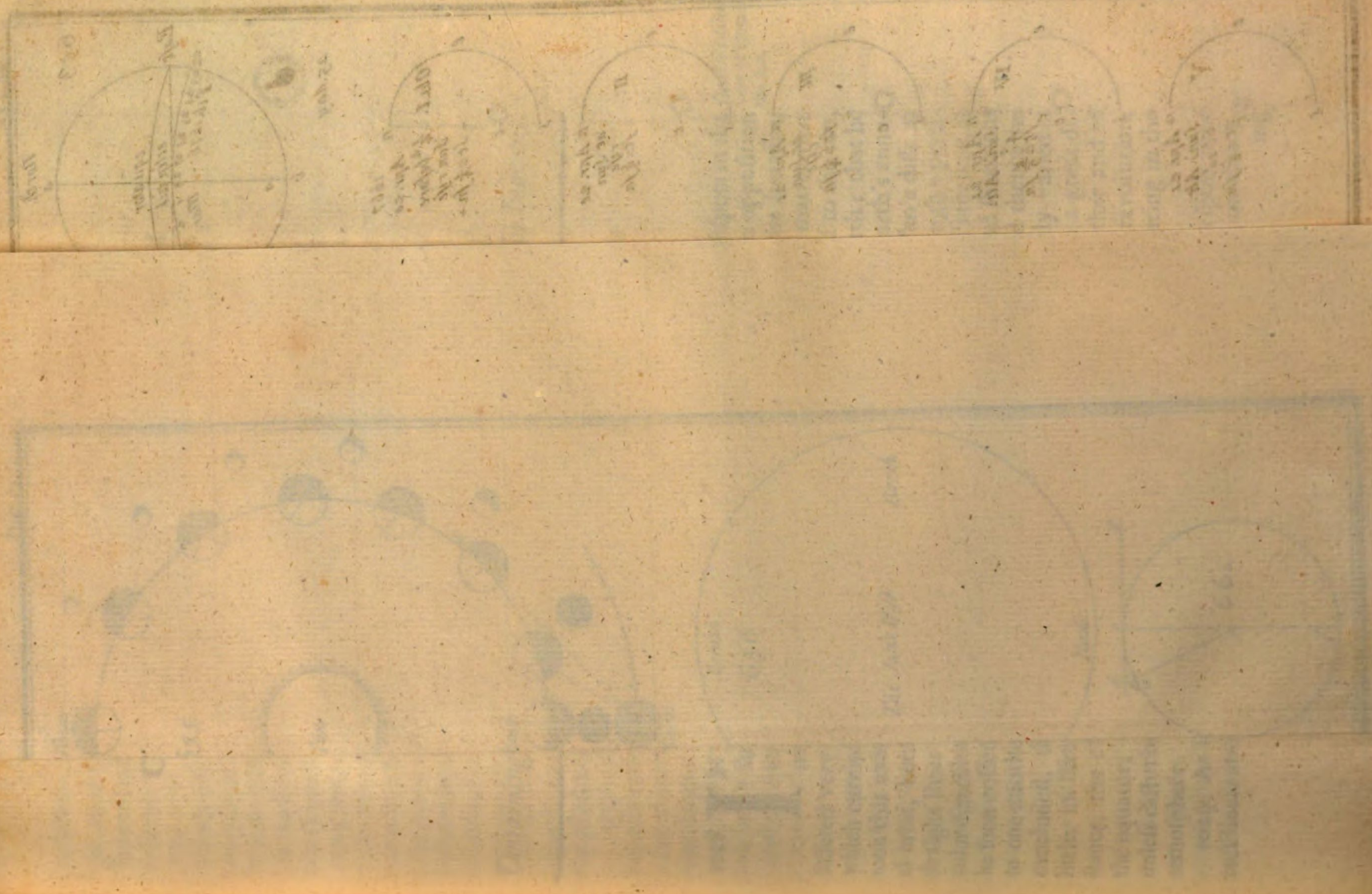
1026. All theſe tranſparent objects are illuminated mighty well in this microſcope, either by candle-light or ſky-light reflected upwards from a concave looking-glaſs *y z* placed in a frame upon the center *h* of the pedeſtal. While you are viewing the object through the microſcope, turn this concave upon its horizontal poles *y*, *z*, and you will ſoon find out that

that poſition of it wherein it reflects the moſt light through the hole *k* upon the object; and this happens when it reflects the rays very obliquely. Opake objects, when laid upon a black ebony or a white ivory plate, put into the hole *k* upon the ſtage, may be illuminated by candle-light tranſmitted through a double convex lens $\alpha\beta$; the foot γ, of the frame $\alpha\beta\gamma$ in which it turns, being put into the hole *l* in the ſtage. The candle muſt be placed in a line drawn from the object through the middle of this lens, at ſuch a diſtance from it as ſhall cauſe the ſpot of light upon the object-plate to be the narroweſt. By day-light this lens gives little or no advantage to the direct ſky-light.

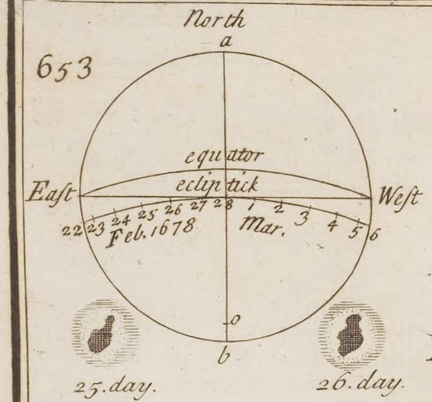
An extempore apparatus for a reflecting microſcope. Fig. 649, 650.

1027. For want of a proper apparatus for a reflecting microſcope, I made the tryals I mentioned in art. 738 with this before us. Having removed the concave looking-glaſs and its frame *xyz*, and having cemented the back of a concave metal *AB* to the top of a ſmall wooden cylinder *C* about an inch long, I placed this cylinder within a hollow braſs tube *D*; whoſe ſides were ſlit downwards to make it ſpring towards the cylinder *C*, and thereby ſupport it at any given height. Then I ſkrewed the bottom of this tube to the center of the pedeſtal *b* by a ſkrew whoſe head lay under it. I placed my object over the hole *k* upon the glaſs-plane *qr*, for laying fiſh upon; and enlightened it, in the night time, by candle-light refracted through the lens $\alpha\beta$: then having took out the object-glaſs and the broad middle glaſs *bb*, I adjuſted the diſtance of the eye-glaſs *aa* from the concave, by raiſing or depreſſing the inner tube that contained it; or by inſerting a longer tube. I believe the experiments would have ſucceeded better, had I encompaſſed the reflecting metal with a wide looſe tube, blacked within and extended from the pedeſtal up to the hole *k* under the object-plate; to exclude all other light from falling upon the metal, but that which deſcended from the object it ſelf.

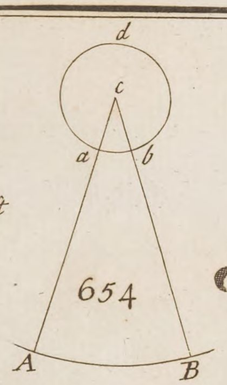
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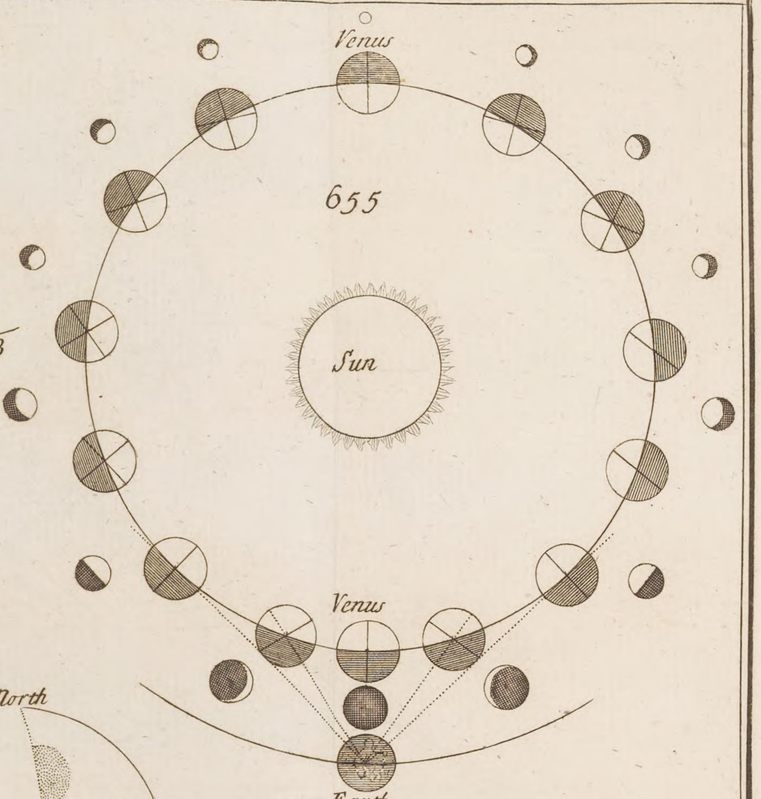
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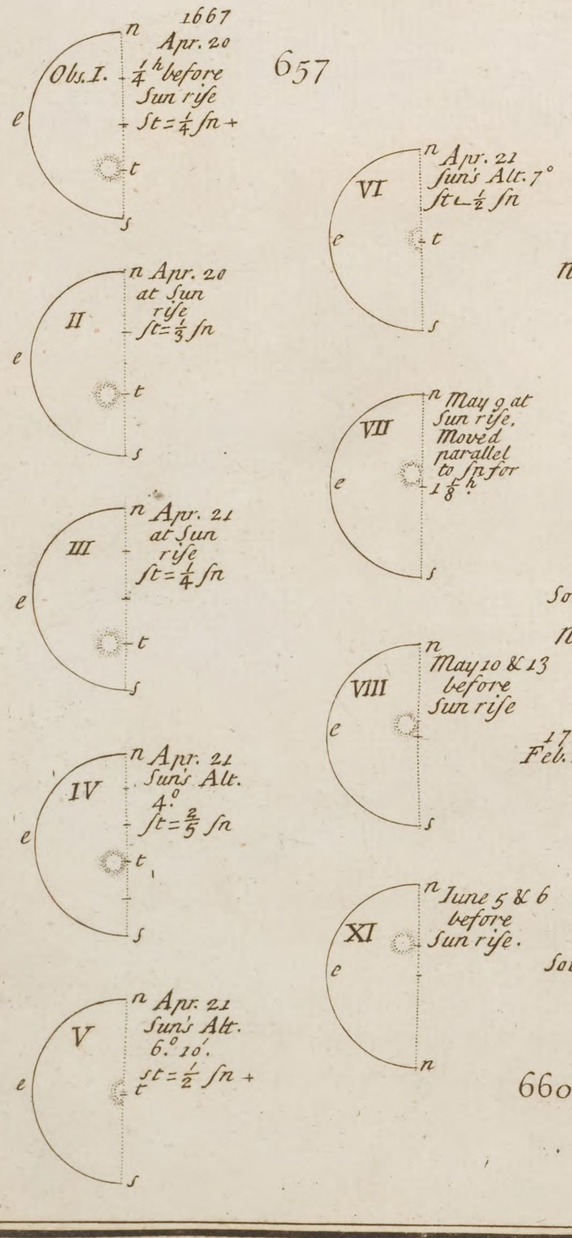
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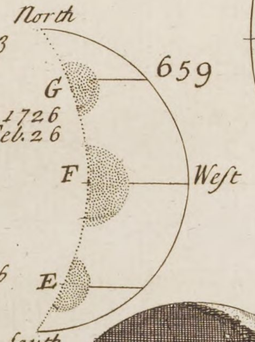
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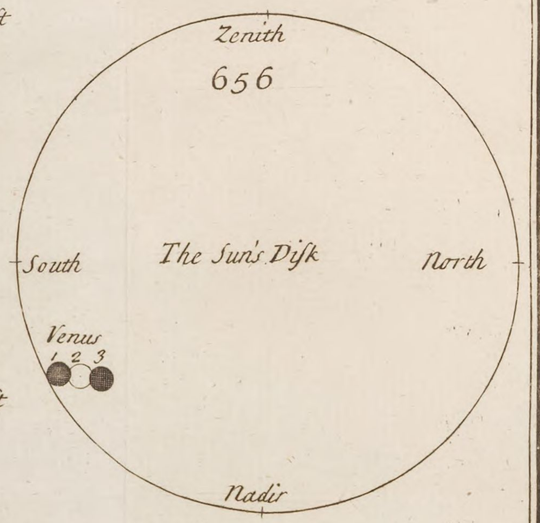
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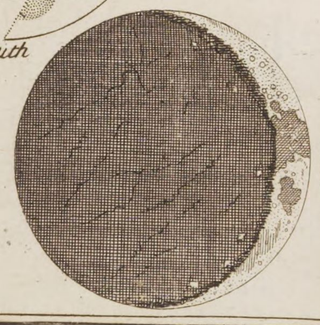
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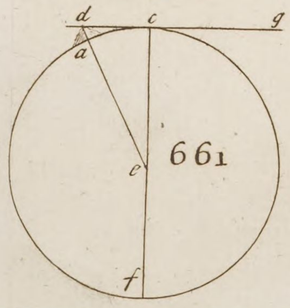
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A
COMPLEAT SYSTEM
OF
OPTICKS.

BOOK IV.

A

Philoſophical Treatiſe,

Containing the hiſtory of teleſcopical diſcoveries in the heavens.

CHAPTER I.

Teleſcopical diſcoveries in the ſun.

1028. IN the year 1610 *Galileo* made the firſt diſcovery of ſpots in the ſun, ſoon after he had finiſhed his teleſcope; and in oppoſition to the received opinion of the unalterable ſtate of the heavens, maintained, that theſe ſpots conſiſted of a kind of matter that ſuffered very great and ſudden alterations; that they adhered to the ſun, which carried them round an axis of his globe in about a month; that he took this axis at firſt to be perpendicular to the plane of the earth's annual orbit, becauſe the courſes of the ſpots appeared on the ſun's diſk as ſtraight lines parallel to that plane; which then happened through a particular direction of the floating motions of the ſpots upon the ſun's ſurface, as he ſoon collected from the viſible alterations in their ſhapes and ſituations to one another when ſeveral of them appeared together; and thence he concluded, that the general courſe of the ſpots was frequently varied a little, in like manner as, to a perſon viewing the earth from a great diſtance, the clouds would appear to move parallel to one another and to the equator, with the uniform velocity of the earth's diurnal revolution, unleſs diſturbed a little from that regular courſe by their floating in the atmoſphere. *Galileo*'s diſcovery of ſpots in the ſun.

1029. At firſt, I ſay, he took the apparent motion of the ſun's ſpots to be rectilinear and parallel to the ecliptick; but a year or two after, upon obſerv-

ing an entire tranſit of a large ſpot, and by marking its place at noon, day after day, upon a circular paper anſwering to the ſun's diſk, he found its courſe was a little incurvated; and thence he concluded that the ſun's axis was a little inclined to the plane of the ecliptick.

1030. The fame of this diſcovery ſoon excited others to make the like obſervations, and occaſioned ſome diſputes upon the nature of theſe ſpots. Thoſe who contended for *Ariſtotle*'s doctrine of the unalterable ſtate of the heavens, maintained, they were either illuſions of the teleſcope, or, if real bodies, were a large ſyſtem of ſatellites, which like Mercury and Venus moved round the ſun, in ſuch narrow circles as even in their greateſt digreſſions from the ſun to be totally obſcured by the ſplendor of his rays; and therefore never viſible but as ſpots on his body at the times of their inferior conjunctions with it.

1031. In oppoſition to that hypotheſis *Galileo* alledged two phænomena; firſt, that many ſpots are obſerved to break out near the middle of the ſun's diſk, others to decay and vaniſh there or at ſome diſtance from the limb; which plainly ſhews them to be generated and diſſolved. For if without generation and diſſolution, they were brought into view by progreſſive motion only, their ingreſs and egreſs at the limb of the ſun would always be viſible.

1032. Secondly, from the changes of their apparent velocities and ſhapes, conſtantly regular, and exactly agreeable to the rules of opticks, he inferred their contiguity to the ſun's ſurface; ſeeing that immediately after their ingreſs, and towards their egreſs, their apparent motions upon the diſk were exceeding ſlow, and quickeſt at the middle of it; and that their ſhapes were oblong and ſlender near the limb of the diſk, but grew broader and rounder towards the middle of it; and that theſe variations of velocity and ſhape, when duly obſerved and computed, agreed exactly to the motions and poſitions of ſpots adhering to the ſun, but differed enormouſly from their motions in concentrick circles, even at very ſmall diſtances from the ſun's ſurface. Beſides, ſays he, were they ſpherical bodies like other planets and ſatellites, they would appear always round, even at the limb of the diſk, as well as the middle; it being peculiar to a ſpherical body, to appear always ſpherical. Therefore the apparent contraction of the ſpots towards the margin of the diſk, and dilatation towards the middle, plainly ſhews them to be a ſort of ſubſtance whoſe thickneſs is but ſmall in compariſon to its length and breadth, and conſequently ſomething that adheres to, or floats upon, the ſurface of the ſun.

1033. From what has been ſaid it appears that *Galileo* made a compleat diſcovery of all that we know at preſent of the nature and properties of theſe ſpots; what others have ſince added, is only an exacter determination of the poſition of the ſun's axis, and of the periodical time of his motion

tion round it, which without ſome viſible ſpots could never have been known to us.

1034. Conceiving a perpendicular *ab*, to the plane of the earth's annual orbit *efghiklm*, to paſs through the ſun's center *c*, it is collected from the apparent curvilinear courſes of the ſun's ſpots, that his axis *pco* is inclined to the perpendicular *acb* in an angle of $7\frac{1}{2}$ degrees; and that a plane paſſing through the axis *po* and perpendicular *ab*, cuts the ecliptick in the 8th degree of Piſces, on the ſide next the ſun's north pole *p*, and conſequently in the 8th of Virgo on the ſide next his ſouth pole *o*. *Scheiner* determined the angle *acp* to be 7 degrees, and *Caſſini* 8, but chuſes to take a medium of $7\frac{1}{2}$; in the reſt they both agree. The inclination of the ſun's axis and equator to the ecliptick. Fig. 651.

1035. Conſequently the plane of the ſun's equator *æq* is inclined to the plane of the earth's annual orbit in an angle of $7\frac{1}{2}$ degrees, and being produced will cut the orbit in two oppoſite points *e*, *i*, at which when the earth arrives, the ſun's equator will appear to an eye in the plane of it, as a ſtraight line upon his diſk. Conſequently if the earth ſtood ſtill at *e* or *i* for about a fortnight, a ſpot upon the ſun's equator would appear to deſcribe a diameter of his diſk, inclined to the ecliptick in an angle of $7\frac{1}{2}$ degrees.

1036. But while the earth is moving in her orbit from *e* towards *g*, or from *i* towards *l*, the perpendicular diſtance of the ſpectator's eye from the plane of the ſun's equator produced, will increaſe continually; and conſequently the viſible projection of the ſun's equator upon his diſk (or any plane parallel to the diſk) will become elliptical and grow wider continually; and when the eye arrives at *g* or *l*, will be an ellipſis whoſe long axis is to its ſhort one, as the radius to the ſine of $7\frac{1}{2}$ degrees, or as 100 to 13; becauſe two lines drawn from *g* or *l* to *æ* and *q* will be nearly perpendicular to *ab*.

1037 The 8 figures below at *E*, *F*, *G*, *H*, *I*, *K*, *L*, *M*, repreſent the apparent ſhapes and poſitions of the ſun's equator ſeen upon his diſk *aæbq* from the places *e*, *f*, *g*, *h*, *i*, *k*, *l*, *m* of the earth's annual orbit, on the days noted at the ſides of theſe figures, and deduced from the ſun's apparent places in the 8th degree of ♊, ♍, ♐, ♓ abovementioned, and likewiſe on the intermediate days of the ſolſtices and equinoxes for the preſent age. Fig. 651.

1038. On theſe 8 days the meridian plane of any obſerver's place cuts the ſun's diſk at noon in the line *mn*, whoſe poſition and inclination to the ecliptick, and its axis *ab*, will be found on thoſe days, as here repreſented, by ſeeking the ſun's place in the ecliptick of a globe, by bringing it to the brazen meridian, and by obſerving the angle and poſition in which the meridian croſſes that part of the ecliptick. Apparent courſe of the ſun's ſpots at any given time.

1039. Therefore by turning towards the ſouth and holding the diameter *mn*, belonging to any one of thoſe eight days, in an upright poſture, that your eye may view it in your meridian, the line *æq* ſhews you the appa-

apparent ſhape and poſition of the ſun's equator to his vertical diameter *m n* at the noon of that day; and by holding the diameter *m n* in the plane of any other hour circle, paſſing through the ſun's center, you have the poſition of his equator and parallels at any hour of thoſe 8 days: which is ſufficient to give an idea of the courſe of the ſun's ſpots, not only upon thoſe particular days but any intermediate day in the year.

How to obſerve the courſe of a ſpot. Fig 652.

1040. The agreement of this theory with the courſe of any ſpot may be practically examined, by obſerving it day after day with a teleſcope furniſhed with four fine wires in its focus, croſſing one another in equal angles of 45 degrees, and fixt in ſuch a poſition to the courſe of the ſun's diurnal motion, that his upper or under limb may touch any one of the wires as *o p* and move along it *. At the inſtant of time when the ſun's anteeedent limb arrives at the wire *o q*, perpendicular to *o p*, and touches it in *r*, imagine his motion to be ſtopt till you draw his diameter *r s*, and from the ſpot *t* the line *t u x*, perpendicular to *o q*, cutting it in *u* and either of the oblique wires in *x*. Then by obſerving the number of ſeconds in which the ſpot *t* will be carried to *u* and to *x*, and the point *s* to *r* by the ſun's uniform motion, you have three numbers proportional to the lines *r s*, *t u*, *u x* or *u o*, whereby the poſition of the ſpot is determined with reſpect to the diameter *r s*, to which the diameter *m n* in this and the foregoing figures is perpendicular, as being a portion of an hour circle. And by repeating the like obſervation day after day, you may find as many points of the ſpot's courſe as you pleaſe; but be the obſervations ever ſo exact, thoſe points can never form an exact portion of an ellipſis or a ſtraight line; becauſe while the earth is moving in her orbit, the poſition of the obſerver's eye, to the circle deſcribed by the ſpot in turning about the ſun's axis, is continually changing. But if a correſponding number of points of the ſpot's courſe be determined upon the principles of the theory, they will agree very well with the poſitions of the obſerved points; of which many inſtances may be ſeen in the Memoirs of the Royal Academy of Sciences: I will tranſcribe only one.

* Art. 874.

An obſervation to confirm the foregoing theory. Recueil de Mem. de l' Acad. Tom. X. p. 601. Fig. 653.

1041. In the year 1678. Feb. 25. N. S. 8^h. a. m. *Caſſini* ſaw a ſpot in the ſun ſurrounded by the uſual nebuloſity, and advanced ſo far upon the diſk, that he judged it might have been ſeen three days before had the ſky been clear. It lay a little below the ecliptick upon the diſk, and at $8^h. 40^m$. its diſtance from the ſun's eaſtern limb was $\frac{1}{6}$ of the diameter of the diſk. This very day the ſun's ſouthern pole arrived at its greateſt elevation *b o* of $7\frac{1}{2}$ degrees above the ſouthern limb of the diſk, which always happens when the ſun is in the 8th degree of Piſces. Conſequently the ſun's equator was alſo elevated $7\frac{1}{2}$ degrees above the ecliptick of the diſk, and the ſpot was then about as much below the equator of the diſk; by which he concluded it would paſs very near the center of the diſk in the morning of Feb. 28, as repreſented in the figure; which he drew beforehand.

hand. According to this prediction, on Feb. $28^d. 9^h. 30^m.$ a. m. he ſaw the north ſide of the ſpot upon the center of the diſk, and at the ſame hour of the ſubſequent mornings he ſaw the ſpot in the very places he had marked in his figure; which entirely confirmed the elements of his theory; namely, that the ſun's axis was then inclined $7\frac{1}{2}$ degrees to the plane of his diſk, that the ſpot adhered to the ſun's ſurface, and made an apparent revolution to the center of the diſk in $27\frac{1}{2}$ days.

Periodical time of a ſpot's apparent revolution.

1042. In the year 1639 *Scheiner* publiſhed his *Roſa Urſina* containing, he ſays, near 2000 obſervations of ſolar ſpots for 20 years together, in which time he frequently ſaw above 50 at once, but for 20 years after, betwixt 1650 and 1670, ſcarce any appeared *. He found that many of his ſpots ſcarce exceeded 25 days in making an apparent revolution from a given place on the diſk to the ſame again, that others took up 27 days, and ſome others near 28. An apparent revolution reſults from a combination of three different motions; of the ſun about his axis, which no doubt is uniform; of the earth about the ſun, whoſe variations are regular and known; and of the ſpot upon the ſun's ſurface, whoſe irregularities are unknown. Since the ſun's motion about his axis, and the earth's about the ſun, have the ſame direction, it follows that, about midſummer, when the earth moves ſloweſt, the periodical time of a ſpot's revolution to a line connecting the centers of the ſun and earth, is the leaſt of all, *cæteris paribus*, and the greateſt about midwinter when the earth moves quickeſt. But the difference in time thus ariſing, cannot exceed four hours, and therefore the greater part of the difference in the periods abovementioned muſt ariſe from the floating motion of the ſpots upon the ſun's ſurface. *Scheiner* and *Caſſini* the ſon * have alſo obſerved that the periodical times of the nearer ſpots to the equator are ſhorter than thoſe of the remoter; and *Caſſini* the father obſerved the like property in Jupiter's ſpots *.

* Phil. Tranſ. N°. 75.

* Mem. de l'. Acad. 1703 p. 142. 8vo.

* Mem. de l' Acad. 1692

1043. Theſe irregularities ſeem to create great difficulties in determining the time of the ſun's exact revolution about his axis; nevertheleſs *Caſſini* the father, obſerving that ſeveral large ſpots had often appeared and diſappeared in the very ſame parallel to the ſun's equator, imagined that ſome particular place or places in it might be diſpoſed, at different times, to ſupply the matter of theſe ſpots; and if ſo, that they ſeldom went far from the place of their origin: juſt as the fumes of mount *Ætna*, if ſeen from the ſun, at the times of their greateſt eruption, would appear as a ſpot on the earth's diſk, returning conſtantly to the ſame ſituation after every 24 hours very nearly; ſometimes a little ſooner, at other times a little later, according as the wind might happen to drive the fumes eaſtward or weſtward from the mountain. He then conſidered that if this compariſon were applicable to the ſun, the apparent periodical time

of

of a ſpot, ſo nearly fixt to a certain place, would be nearly the ſame as that of the ſun it ſelf, and be a common meaſure of ſeveral large intervals of time betwixt the ſpot's appearances in the ſame apparent place of its parallel. Accordingly by comparing many diſtant obſervations together, he found a common meaſure of many ſuch intervals of time to be $27^d. 12^h. 20^m.$ very nearly *.

* Rec. de l' Acad. Tom. X. p. 727,

1044. For example, betwixt an obſervation of his own on May 14. N. S. 1684., and another on Apr. 29. 1686, there paſſed 715 days, that is 26 times $27\frac{1}{2}$ days; again by dividing the days betwixt the former of thoſe obſervations and a later on May 13. 1688, by 54 revolutions, the quotient gave him $27^d. 12^h. 40^m.$ for a mean revolution; and a mean betwixt this and the former is $27^d. 12^h. 20^m$; although he had obſerved that a ſingle revolution of ſome of thoſe ſpots was performed in $27\frac{2}{3}$ days, of ſome others in $27\frac{1}{3}$, and of others in $27\frac{1}{2}$.

1045. Laſtly by turning over the earlieſt obſervations made by *Scheiner* and *Hevelius*, for ſpots ſeen alſo in the month of May and in the ſame parallel with thoſe he had conſidered, he found one in 1625 and another in 1644, which he compared with the three abovementioned, and ſettled the limits of ſix grand intervals of time, whoſe common meaſure was about $27^d. 12^h. 20^m$, within 4 or 5 minutes at moſt. This period may therefore be attributed to the ſun it ſelf rather than any other, as being of a middling length among many ſingle periods of various ſpots, and deduced from obſervations all made in the ſame month of May or near it, and conſequently leſs affected by the inequalities of the earth's annual motion.

The time of ſun's revolution about his axis.

1046. Hence the periodical time of the ſun's revolution to a fixt ſtar is $25^d. 15^h. 16^m.$ For in $27^d. 12^h. 20^m.$ of the month of May, the earth deſcribes an angle AcB or acb, about the ſun's center c, of $26^o. 22'$, and as $360^o + 26^o. 22'$, to 360°, ſo is $27^d. 12^h. 20^m$, the time of the ſun's apparent revolution to the moveable line cbB, to the time of his abſolute revolution to the fixt line caA, which therefore is $25^d. 15^h. 16^m$.

Fig. 654.

Fig. 653.

1047. All ſpots conſiſt of a black part in the middle, of ſome irregular figure, encompaſſed with a nebulous border of a colour leſs dark; and it often happens, after a gradual decay and diſappearing of the black part, that its place ſeems brighter than the reſt of the ſun, and continues ſo for two or three days. Theſe brighter ſpots are called *faculæ*.

Abſolute magnitude of ſpots.

1048. The magnitude of the ſurface of a ſpot may be eſtimated by the time of its tranſit over a hair in a fixt teleſcope. *Galileo* reckons ſome ſpots to be larger than all *Aſia* and *Africa* put together. But had he known the ſun's parallax and diſtance as exactly as we do, he would have found them much larger than the whole ſurface of the earth. For in 1612 he obſerved a ſpot ſo large as to be plainly viſible to the naked eye; it therefore ſubtended an angle at the eye of about a minute *. The diameter of the earth, if removed to the ſun, would ſubtend an angle of but 20 ſeconds.

* Art. 97.

conds. Therefore the diameter of the ſpot, was to the diameter of the earth as 60 to 20 or 3 to 1, and conſequently the ſurface of the ſpot, if circular, to a great circle of the earth, as 9 to 1, and to the whole ſurface of the earth as 9 to 4 or $2\frac{1}{4}$ to 1. *Gaſſendus* obſerved a ſpot whoſe diameter was $\frac{1}{2\frac{1}{2}}$ of the ſun's, and therefore ſubtended an angle at the eye of above a minute and a half. Its ſurface was therefore above five times larger than the whole ſurface of the earth. He tells us he ſaw above 40 ſpots at once, but did not perceive that the light of the ſun was ſenſibly diminiſhed; nevertheleſs the paleneſs of the ſun mentioned by hiſtorians, as after the death of *Julius Cæſar*, might poſſibly be cauſed in this manner, provided we admit the fact, and do not regard it as an invention of flattery.

1049. From the perpetual emiſſion of the ſun's light, how minute and delicate ſoever, it is reaſonable to conclude that his body muſt waſte, and therefore could not have exiſted from eternity, as a luminous body, without a ſupply of new matter. Some ſupplies may come now and then, as Sir *Iſaac Newton* imagines, from the falling of comets into the ſun. He computes that the comet in 1680 approached towards the ſun's ſurface within leſs than a ſixth part of the ſun's diameter; and, by moving with an immenſe velocity in that nearneſs, he concludes that it muſt have been retarded by the reſiſtence of the ſun's atmoſphere, and conſequently muſt approach nearer and nearer after every revolution, till at laſt it falls into the ſun. It muſt be confeſſed that theſe ſupplies can happen but very rarely, and that the bodies of comets are very ſmall in compariſon to the vaſtneſs of the ſun; nevertheleſs, conſidering the inconceiveable minuteneſs of the particles of light, though continually flying from the ſun, a few ſuch ſupplies may abundantly ſuffice for keeping up his bulk for innumerable ages, without ſenſible diminution to the inhabitants of the ſyſtem. The moſt ancient obſervations of the ſun's diameter compared with the preſent, are quite inſufficient for determining whether it be diminiſhed or not. Sun's decreaſe yet imperceptible.

CHAPTER II.

Teleſcopical diſcoveries in Mercury and Venus.

1050. WHEN *Copernicus* revived the ancient Pythagorick Syſtem, aſſerting that the earth and planets moved round the ſun in the center of their orbits, the Ptolemaicks objected, if this were true, that the phaſes of venus ſhould reſemble thoſe of the moon. *Copernicus* replied, that ſome time or other that reſemblance would be found out. This prediction was fulfilled by *Galileo*, who ſent this firſt diſcovery of the phaſes of venus in a letter, written from *Florence* in 1611, to *William* Phaſes of Venus.

liam de' Medici, the Duke of *Tuſcany*'s Ambaſſador then reſiding at *Prague*, deſiring him to communicate it to *Kepler*, firſt mathematician to the Emperor *Rudolphus* II. The letter is extant in the preface to *Kepler*'s Dioptricks, and in engliſh is this. "Being undoubtedly certain of the truth of the diſcovery concealed in the cypher ſent you ſome weeks ago, it is now time to explain it to your Excellency, and by you to Sig^r. *Kepler*. Know then, that about three weeks ago, when venus became viſible in the evenings, I began to obſerve her very diligently with my teleſcope, hoping to ſee with my eyes, what my underſtanding did not at all doubt
ig. 655. of. At firſt then I ſaw venus perfectly round, neat and diſtinctly terminated, but very ſmall; which figure ſhe retained till ſhe approached nearer to her greateſt digreſſion from the ſun, increaſing continually in apparent bulk. From that time her figure began to fail of its roundneſs on its eaſtern ſide which lay from the ſun, and in a few days was reduced to a perfect ſemicircle; and continued ſo without the leaſt alteration, till ſhe left the tangent to her orbit, and began to return towards the ſun. At preſent the ſemicircle becomes more and more hollow every day, its angles being changed into horns, which will grow ſharper and ſharper, till they become ſo thin as to vaniſh at her occultation in the beams of the ſun. Hence paſſing by to her morning appearance, we ſhall ſee her fine horns again, pointing from the ſun, and growing fuller and fuller up to the angles of a ſemicircle, at the other greateſt digreſſion from the ſun, which ſhape will continue many days without ſenſible change. Hence forward the ſemicircle will ſwell out gradually till it be almoſt compleatly round, and will keep ſo for ſome months. At preſent the apparent diameter of venus is about five times greater than it was at her firſt evening appearance. By theſe obſervations of this admirable appearance, we have the moſt certain, ſenſible deciſion and demonſtration of two grand queſtions, which to this very day have been doubtful and diſputed among the greateſt maſters of reaſon in the world. One is, that the planets in their own nature are opake bodies, attributing to mercury what we have ſeen in venus. The other is, that venus neceſſarily moves round the ſun, as alſo mercury and the other planets; a thing well believed indeed by *Pythagoras*, *Copernicus*, *Kepler* and my ſelf, but never yet proved, as now, by ocular inſpection upon venus. *Kepler* and the Copernicans may therefore glory with good reaſon, in having philoſophiſed ſo well as not to ſeem vainly credulous, though hitherto looked upon as fools and bigots by all thoſe that ſeek philoſophy in books only. As to the cypher, ſent you in letters tranſpoſed into theſe words, *Hæc immatura a me fruſtra leguntur*, *o*, *y*, the ſame put in order run thus, *Cynthiæ figuras æmulatur mater amorum*, that is, venus imitates the phaſes of the moon. I am &c.

Galileo Galilei.

Florence 1. *Jan.* 1611.

Phaſes of the other planets.

1051. *Galileo*'s teleſcope ſcarce magnified enough to diſcover the like phaſes of Mercury, whoſe exceſſive brightneſs was another impediment. But from this and his ſhorter digreſſions from the ſun, than thoſe of Venus, he juſtly concluded, that the orbit of Mercury lay within that of Venus. Mars he plainly perceived was ſometimes gibbous, or deficient from perfect roundneſs. Jupiter appeared to him always round, and Saturn too, which he took to conſiſt of three contiguous globes, as explained hereafter. From theſe diſcoveries of the phaſes of the planets and their degrees of brightneſs in proportion to their nearneſs to the ſun, he ſoon compoſed the very firſt, and ſtill the beſt, as being the ſhorteſt and plaineſt demonſtration of the true ſyſtem of the univerſe; which till then had been always dubious and diſputed; followed indeed by the beſt judges, but for no other reaſon than that it afforded more ſimple and eaſy ſolutions of phænomena, than could be deduced from *Ptolemy*'s hypotheſis; which tho' ſufficient to determine their choice, was ſtill too far from a ſatisfactory proof of the real order of the heavenly bodies.

Venus and mercury viſible at noon day.

1052. M. *de la Hire* was the firſt perſon that diſcovered and obſerved the tranſits of fixt ſtars and planets over the meridian at noon day *. This he did through the moveable teleſcope belonging to the mural quadrant at the Royal Obſervatory at *Paris*, and ſays, that he never failed of ſeeing the tranſit of venus, though within two degrees of the ſun, to the right or left, upwards or downwards; that in 1700 Aug. $27^d.\ 0^h.\ 24^m$, he obſerved her tranſit with a 16 foot tube, which magnified her diameter (of one minute) about 90 times, and conſequently ſhewed it three times larger than that of the moon viewed by the naked eye; that ſhe then appeared in the form of a fine ſlender creſcent, with her horns in an horizontal line and her back upwards; that in the interior arch of the creſcent he ſaw ſome inequalities more conſiderable than thoſe of the new moon; that he had obſerved the like at other times, by which he judged ſhe might have ſpots upon her, viſible at other times, like thoſe of the other planets; that the ſplendor of the atmoſphere, ſo near the neighbouring ſun, took off ſo much from the exceſſive luſtre of venus, as to render her appearance much neater, better defined and freer from colours than at any other time and place; and that the eye-glaſs of the teleſcope need not then be ſmoaked. Laſtly, that in Nov. 1691 he ſaw her at noon very near her ſuperior conjunction with the ſun, appearing round and very ſmall *. What has been ſaid of venus is applicable to mercury; whoſe phaſes were firſt diſcovered by *Martinus Hortenſius*.

* Mem. de l' Acad. 1700. p. 288. 4to.

Venus and mercury viſible in the ſun's diſk.

1053. Nothing is more commodious for ſettling the motions of mercury and venus than their appearances upon the ſun's diſk, a delightful ſight to aſtronomers, and long wiſhed for in vain before the invention of teleſcopes. The writer of the life of *Charles* the great relates, that mercury was ſeen in the ſun for 8 days together, 17. Mar. A. D. 807 *. but this

* Mem. de Math. & Phy. Tom. X. p. 20.

* Du Ham. Hiſt. R. Ac. Sci. p. 469.

this no doubt was ſome large ſpot in the ſun; ſince mercury is far too ſmall to be ſeen there with the naked eye. *Kepler*, before he finiſhed his tables, had placed the nodes of mercury's orbit at the beginning of the ſigns of gemini and ſagittary, and thence predicted his appearance in the ſun, 20 May 1607; on which day he obſerved the ſun very diligently, viewing his image caſt upon a paper in a dark room through a ſmall hole in the window-ſhut. Then alſo there happened to be a large ſpot in the ſun, which *Kepler* took for mercury. It was lucky for him that teleſcopes were invented ſoon after, which preſently convinced him, that what he took for mercury was only a ſolar ſpot. Upon this he rectified his tables, and by placing the nodes of mercury about the middle of taurus and ſcorpio, he predicted, that his firſt appearance in the ſun would be on the 7th Novem. 1631.

1054. *Gaſſendus* was then at *Paris* and made due preparation to obſerve it, in the manner he had uſed to obſerve ſpots upon the ſun, by receiving its rays through a teleſcope a little lengthened, in order to collect them into an image caſt upon a white paper *; and in watching for mercury he fell into the contrary miſtake to that of *Kepler*, taking mercury himſelf for a ſolar ſpot, juſt ſprung from the ſun, having found it quite free from them the day before. However he took the poſition of the ſpot once or twice, intending to compare it with that of mercury, which he thought would appear much larger. But in taking its third poſition, and obſerving it moved quicker than ſpots uſually did, he began to ſuſpect it might be mercury himſelf; and being farther aſſured of it by his fourth obſervation, he prepared to obſerve its egreſs from the ſun's diſk, which happened Novemb. 7^{d}. 10^{h}. 20^{m}. *mane Sty. N.* at the diſtance of $33\frac{1}{2}$ degrees from the ſun's vertical point. This was the firſt time that mercury was ever ſeen in the ſun, and the only obſervation then made of that tranſit. Several ſubſequent obſervations of the like tranſits are collected in *du Hamel*'s Hiſt. of the Royal Acad. of Sci. p. 470. edit. 2d, from which I extracted that of *Gaſſendus*.

* Rem. 128.

1055. In the year 1691 Dr. *Halley* publiſhed * a moſt accurate theory for finding the viſible conjunctions of mercury and venus with the ſun, together with a ſpecimen of it, in tables of all the times of their appearance in the ſun's diſk for two centuries, beginning from the time of the invention of teleſcopes; which tables agree with the lateſt and beſt obſervations to a wonderful degree of exactneſs. For the obſerved time of the conjunction of mercury on the 29th of Octob. 1723, was but $4\frac{1}{2}$ minutes ſooner than the prediction, and the latitude of mercury but ſix ſeconds more ſoutherly; the error in longitude being little more than two diameters of this exceeding ſmall planet, and in latitude ſcarce a ſingle diameter: ſo that for the future, aſtronomers may truſt that table of tranſits to a few minutes, as its author has juſtly obſerved *, and not wait with

* Phil. Tranſ. N. 193.

* Phil. Tranſ. N. 386.

with the uncertainty of hours, nay days, as had been lately done. The diameter of mercury was then obſerved with an excellent micrometer applied to the Hugenian teleſcope of 123 feet, and found to be $10''\frac{3}{4}$.

1056. A ſight of venus in the ſun was wiſhed for by aſtronomers with more impatience than that of mercury, as being the propereſt means for adjuſting the ſun's diſtance from us, and ſome other important points in aſtronomy. But *Kepler*, having declared in his Optical Aſtronomy, publiſhed in 1604, and alſo in his epitome of the Copernican Aſtronomy, in 1621, that no ſuch thing could happen in that century, nor the next till the year 1761, all thoughts of it were quite laid aſide. Nevertheleſs upon correcting his tables in the year 1630, he altered his opinion and concluded that venus would be ſeen in the ſun in the year following, and accordingly gave publick notice of it in the ſame advertiſement, wherein he predicted that tranſit of mercury which *Gaſſendus* obſerved. But as to venus, which was impatiently watched by *Gaſſendus* and others, the event did not anſwer.

1057. Notwithſtanding theſe diſappointments, which occaſioned a general deſpair of obtaining a ſight of this rare phænomenon, yet it was accidentally predicted and ſoon after obſerved by our countryman *Jeremias Horrockes*, a youth of an admirable genius, at Hool a ſmall village in Lancaſhire about 15 miles northwards from Liverpool. In the treatiſe he wrote upon this ſubject, (which was publiſhed by *Hevelius* at the end of his *Mercurius in ſole viſus)* he tells us, that meeting with *Lanſbergius*'s tables and finding them much extolled by their author, he thence computed ſeveral diaries of the planetary aſpects, in order to examine their agreement with the heavens, and correct them by his own obſervations; which he did accordingly. Among other aſpects he computed the conjunction of venus with the ſun, by theſe and alſo by *Kepler*'s tables, and found they agreed well enough in the ſame concluſion. This was contrary to *Kepler*'s computation from his own tables, as I ſaid above, but *Horrockes* had corrected them by ſome of his own obſervations. At the approach of the deſired time for obſerving this conjunction, through a teleſcope he projected the ſun's image upon a white paper in a dark room, and after waiting many hours with great impatience, at laſt on the 24th of Novemb. 1639 at 3 hours and $\frac{1}{4}$ paſt 12, O. S. juſt after he came from church, he had the pleaſure to find a large round ſpot near the limb of the ſun's image, as repreſented at number 1. He made three obſervations of its poſitions 1, 2, 3, and as many of its magnitude, and found its diameter was $\frac{1}{25}$ of the ſun's at moſt; the ſpot was perfectly round and blacker than ſolar ſpots, from which it differed alſo in the quickneſs of its motion, along the ſpace 1, 3, which it deſcribed in half an hour. He was deprived of a longer ſight of it by the ſetting of the ſun. According to his draught of this appearance, the interior contact of venus with the ſun's Fig. 656.

ſun's limb is about 61 degrees from its loweſt point. He had given notice of this conjunction to his friend *Will. Crabtree* at Mancheſter, who juſt got a ſight of it before ſun-ſet, and obſerved the diameter of venus to be $\frac{7}{209}$ of the ſun's. Theſe two were the firſt and only perſons that ever ſaw venus in the ſun. Whoever peruſes the little treatiſe abovementioned, from which I have taken this account, muſt needs admire the ſpirit and genius of this young aſtronomer, who died at the age of 22 in 1641. His theory of the moon and other poſthumous works were publiſhed in 4to by Dr. *Wallis* in 1673.

1058. By Dr. *Halley*'s table of the viſible conjunctions of venus in the ſun, it appears that *Horrockes* ſaw the laſt that can happen till the year 1761, when on May the 26th, near ſix in the morning at *London* venus will appear in the ſun's diſk not above four minutes ſouth of its center. This tranſit will continue almoſt 8 houres, from 2 in the morning till almoſt 10, and therefore the ingreſs into the ſun's diſk cannot be ſeen in any part of England. Dr. *Halley* farther aſſures us, from his own obſervation of mercury in the iſland of St. *Helena*, that the beginning and end of the tranſits of theſe planets over the ſun's diſk, may be obſerved to the exactneſs of one ſecond of time. I had the good luck, ſays he, to ſee mercury as he was entering the ſun's limb and made a black notch in it, and was certain of the inſtant of his total ingreſs, by a fine thread of the ſun's light, which immediately broke out from the interior contact of the two limbs and ſtruck my eye inſtantaneouſly; and at the beginning of the egreſs of mercury, the like thread of light was broken and vaniſhed inſtantaneouſly at the other interior contact. If the like obſervations be duly made upon the enſuing tranſit of venus, with good clocks and teleſcopes, in ſeveral diſtant countries, he ſhews in what manner we may thence determine the ſun's parallax and diſtance from the earth, to the exactneſs of a 500th part of the whole *; whereas by the beſt obſervations hitherto made, we are not abſolutely certain of thoſe quantities to leſs than about a ſeventh part of them.

* Phil. Tranſ. N. 348.

Revolution of venus about an axis according to *Caſſini*.

1059. In the year 1666 Signor *Caſſini*, then profeſſor of aſtronomy at Bologna, diſcoverd a bright ſpot in venus, not unlike ſome of thoſe that a teleſcope conſtantly ſhews us in the moon near the full; and made the following obſervations upon it in the year 1667 *. The ſeveral ſemicircles *nes* repreſent the diſk of venus, as appearing about half enlightened, at her greateſt weſtern elongation from the ſun; *n* is the northern, *e* the eaſtern and *s* the ſouthern ſide of it; and *t* the places of the bright ſpot at the times of obſervation noted at the ſide of each ſemicircle. Suppoſing he conſtantly ſaw one and the ſame ſpot, adhering to the ſame part of venus, by a compariſon of theſe obſervations one with another, he concluded with ſome diffidence, that venus revolved about an axis in leſs than a day; at leaſt that the ſpot finiſhed its period either of revolution or libration

* Mem. de l' Acad. 1732 p. 197. 4to. Fig. 657.

on in that time, ſo, as in 23 days to return nearly to the ſame place in the diſk at the ſame time of the firſt and laſt of thoſe days. For by reaſon of the vicinity of venus to the ſun and horizon, he could not obſerve her long enough to ſee a continued motion of the ſpot through a large ſpace; and therefore durſt not pronounce whether its motion were of the revolving or librating kind.

1060. To give one deduction of the ſpot's periodical time, on Apr. 20 the ſun roſe at *Bologna* at $5^h.\ 17^m$, when, by the ſecond obſervation, *st* was equal to $\frac{1}{3}$ of the diameter *sn*; and on Apr. 21 the ſun roſe at $5^h.\ 15^m$, and was $6^o.\ 10'$ high at $5^h.\ 51^m$, when, by the fifth obſervation, *st* was equal to $\frac{1}{2}$ *sn*. Therefore in the interval of $24^h.\ 34^m$, ſuppoſing a continued progreſſive motion of the ſpot, it deſcribed an arch of about 20 degrees, anſwering to $\frac{1}{6}$ of the diameter *sn*, or elſe had finiſhed one or more revolutions beſides that arch. But without finiſhing one revolution its motion could not have appeared ſo evidently as it did between the third and ſixth obſervations. Conſequently as $360^o + 20^o : 360^o :: 24^h.\ 34^m : 23^h.\ 16^m$ the periodical time required. Theſe two obſervations are fitter for this purpoſe than the reſt, becauſe the motion of the ſpot, then neareſt to the center, was apparently quickeſt. Fig. 657.

1061. In the 7th obſervation, May 9, he ſaw the ſpot move exactly parallel to the diameter *sn*, from the ſouth towards the north, for above an hour; in the foregoing obſervations its courſe deviated a little from the north towards the weſt, as appears by the figures. Of this direction of motion there is no other inſtance in the heavens, except in the moon's libration. He likewiſe perceived ſome obſcure ſpots upon the diſk, but their motions were too irregular to conclude any thing from them.

Revolution of venus acording to *Blanchini*.

1062. We hear of no other obſervations of this kind till the year 1726, when Sig^r. *Blanchini*, the Pope's domeſtick prelate, obſerving venus at Rome with *Campani*'s glaſſes, whoſe focal diſtances were from 70 to 100 Roman palmes, diſcovered ſeveral dark ſpots in her diſk, as repreſented in the 1ſt, 2d and 3d plates of his book called *Heſperi & Phoſphori nova Phænomena*.

1063. On Feb. 9. he ſaw two ſpots bounded on one ſide by the ſection that ſeparates the light and dark parts of the diſk. Both were ſhaped like a ſegment of a circle, and that which lay towards the northern horn of the diſk was much ſmaller than the other, that lay towards the ſouthern. Fig. 658.

1064. On Feb. 14, the large ſpot was vaniſhed, but he thought he reconnoitred the ſmall one, now advanced towards the ſouthern horn, and followed by two new ones lying towards the northern horn.

1065. On Feb. 16, he believed he ſaw theſe ſpots again, tho' ſomewhat altered in ſhape and carried ſouthwards.

1066. On Feb. 18, the ſouthermoſt ſpot was quite gone; the other

two.

two were advanced towards the ſouth, and a new one appeared in the north.

1067. On Feb. 20, the ſpots laſt ſeen were all advanced ſtill more ſouthwards.

1068. On Feb. 24, the two ſouthermoſt did not appear; that in the north was carried ſouthwards, and was followed by two new ones in the north. On the 26th theſe three appeared a little more ſouthward.

1069. Laſtly on March 5, 24 days after the firſt obſervation, he believed he ſaw thoſe two ſpots, which he firſt diſcovered on Feb. 9, in the ſame ſituation they then had. They appeared narrower, as they ought, becauſe the phaſe of venus was now become ſharper; and of conſequence the dark part of the diſk had now encroached upon a larger portion of them.

1070. Upon theſe obſervations and many more, made in the following months of May and June 1726, of July, Auguſt and September 1727, and of Jan. 7. 1728, *Blanchini* concludes, that a revolution of venus about her axis was not finiſhed in 23 hours, as *Caſſini* imagined, but in $24\frac{1}{3}$ days; that the north pole of this revolution faced the 20th degree of aquarius, and was elevated 15 degrees above the plane of the ecliptick and that the axis kept parallel to it ſelf, during its revolution about the ſun.

Situation of the axis of venus.

1071. The motion of theſe ſpots was from the northern towards the ſouthern ſide of the viſible diſk, quite contrary to that of *Caſſini*'s ſpot; but this is a neceſſary conſequence of the different ſituation of the axis of venus to the eyes of the two obſervers. The only point to be adjuſted is their different determinations of the periodical time of the ſpots.

Fig. 659.

1072. *Blanchini*, to ſupport his opinion, alledges an obſervation made in the evening of Feb. 26. 1726, with *Campani*'s teleſcope of 88 palmes, in the preſence of ſeveral perſons, who agreed with him in the appearances of the ſpots. He obſerved them near an hour from $5^h.\ 25^m$ to $6^h.\ 15^m$, when venus being got behind the Barbarine palace, obliged him to deſiſt. About three hours after the middle of the foregoing obſervation, *i. e.* at $8^h.\ 40^m$, getting ſight of her again and obſerving her till 9, he found the ſpots nearly in the ſame ſituation in which they appeared at $5^h.\ 45^m$, as was evident by comparing a draught of the ſpots then taken, with their preſent appearance.

1073. Now, ſays he, according to *Caſſini*'s period of 23 hours, in the three hours interval between the two obſervations, the ſpots would have finiſhed above an eighth part of a revolution or above 45 degrees of their parallel circles; and thus the ſpot which at $5^h.\ 25^m$ poſſeſſed the middle of the diſk, at $8^h.\ 45^m$ would have been carried about 50 degrees from it towards the ſouthern horn, and have appeared beyond the place of the ſouthern ſpot, which then would have been nearly out of the diſk; and

the

the northern ſpot would have nearly ſucceeded to the place of the middlemoſt; inſomuch that of the three ſpots, which at $5^h.\ 45^m$ were equally diſtributed over the diſk, two would have been found on its ſouthern ſide and not one on the northern. But all the gentlemen that obſerved with me from $8\frac{1}{2}$ till 9, ſaw plainly that the large ſpot appeared about the middle of the phaſe, and found that the ſame opening of the micrometer nearly meaſured the interval between the top of the ſouthern ſpot and the upper horn, and alſo between the top of the northern ſpot and the under horn, juſt as it did before at $5\frac{1}{2}^h$. It muſt needs be acknowledged then, ſays *Blanchini*, that in thoſe three hours the ſpots did not advance above 2 degrees, the ſpace due to the periodical time of 24 days; and that this ſmall advance did not make a perceptible change in their ſituations.

1074. This is Sigr. *Blanchini*'s concluſion, which M. *Caſſini* the ſon, after due conſideration of the obſervations cannot acquieſce in *. I admit, ſays he, M. *Blanchini*'s obſervations, without the leaſt ſcruple, as himſelf has deſcribed them; namely, that at $5^h.\ 45^m$ they ſaw three ſpots as repreſented at *E*, *F*, *G* in the figure for Feb. 26; and that about three hours after they likewiſe ſaw three ſpots nearly in the ſame ſhapes and ſituations as before. But it muſt be conſidered that the courſe of obſerving was interrupted from $6^h.\ 15^m$ to $8^h.\ 40^m$ by the interpoſition of the Barbarine palace, in which interval, of almoſt $2\frac{1}{2}$ hours, they could not ſee the ſpots nor the motion they might have; which, in three hours time *, might carry them through 47 degrees; ſo that the ſouthern ſpot *E*, in approaching to the ſouthern horn, might at laſt go out of the diſk, while the ſpot *F* might move from the center into the place of *E*, and likewiſe the northern ſpot *G* into the place of *F* at the center, and a new ſpot might ſucceed to the place of *G*; ſo that at $8^h.\ 45^m$ three ſimilar ſpots might appear in the ſame places as before at $5^h.\ 45^m$. For by the rules of perſpective, the ſpot *F* being tranſlated to *E*, from the center towards the limb, would become ſmaller in appearance, like that whoſe place it had taken; and on the contrary the ſpot *G*, in moving from the limb towards the center, would grow larger in appearance; and all this will be farther evident by examining the figure; where drawing lines, from the middle of the ſpots, perpendicular to a line that connects the horns, their intervals will anſwer to arches of about 45 degrees. Therefore ſuppoſing the periodical time of the ſpots to be about 23 hours, the middle of the ſouthern ſpot ought to be gone out of the diſk in three hours time, while the ſpot *F* ſucceeded to the place of *E*, and *G* to that of *F*. As to the new ſpot, ſuppoſed to ſucceed to the place of *G*, there is reaſon enough to believe it, upon conſidering M. *Blanchini*'s figures, where we ſee that in ſeveral ſucceſſive days, ſeveral ſpots ſucceeded one another; and in particular that the ſpot there called *A* ought to appear upon the diſk next after *G*, tho'

at

The obſervations reconciled by *Caſſini* the ſon.

* Mem. de l'Acad. 1732 p. 205. 4to. Fig. 659.

* Art. 1073.

at a little greater diſtance than this period requires. Therefore the obſervation on Feb. 26. 1726, is not deciſive, as the author imagined, againſt the period of 23 hours.

1075. After a more critical compariſon of his father's obſervations with *Blanchini*'s, M. *Caſſini* concludes at laſt, that if we ſuppoſe the periodical time of the revolution of venus to be $23^h.\ 20^m$, it agrees equally with both their obſervations; but if in $24^d.\ 8^h$, as *Blanchini* would have it, we muſt entirely reject his father's obſervations as of no conſequence at all. He adds that M. *Maraldi* and he made a very great number of obſervations upon venus in the moſt favourable days in the year 1729, with tubes of 82 and 114 feet, but could never perceive the leaſt ſpot; ſo that *Blanchini*'s ſpots had either diſappeared or elſe the air at Paris was not ſo clear as at Rome; which might probably be the reaſon why *Caſſini* the father could never perceive any ſpots in her at Paris, even with the ſame teleſcopes that he had uſed in Italy.

CHAPTER III.

Teleſcopical diſcoveries in the Moon.

Moon's ſurface mountanous.

1076. GALILEO's teleſcope though but ſmall in compariſon to ours, was yet ſufficient to give him a juſt idea of the moon's ſurface. He immediately concluded it was not ſmooth like a ſpeculum, as many philoſophers had imagined, but rough like the earth, and diſtinguiſhed with innumerable mountains, caves and valleys. This he collected from the following appearances; that in the new moon the line which connects her horns and paſſes between the bright and dark portions of the viſible hemiſphere, was not an even, regular curve, as it ſhould be upon a ſmooth, ſpherical ſurface, but a line compoſed irregularly of many crooked turnings and windings as repreſented in the figure; that many ſmall bright ſpots appeared in the dark portion, ſtanding out at ſeveral ſmall diſtances from that common boundary of the bright and dark portions; that in a few hours they grew ſenſibly larger and approached nearer to, and at laſt united with, the bright portion; juſt as the rays of the riſing ſun ſhine firſt upon the tops of our high mountains, then deſcend gradually to their baſes, and at laſt into the valleys. On the other hand he obſerved many ſmall ſpots interſperſed all over the bright portion of the diſk, ſome of which had their dark ſides next the ſun, and their oppoſite ſides very bright and circular, which plainly ſhewed them to be round cavities, whoſe ſhadows fell within them; and that ſome of theſe were ſurrounded with ridges of mountains. Laſtly that thoſe larger and leſs luminous tracts, that are viſible to the naked eye, appeared ſmoother in the teleſcope and more depreſſed than the ambient brighter regions;

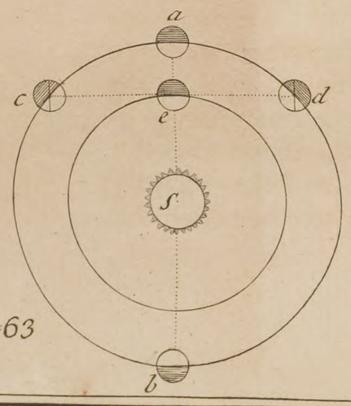
Galileo's nuncius ſidereus.

Fig. 660.

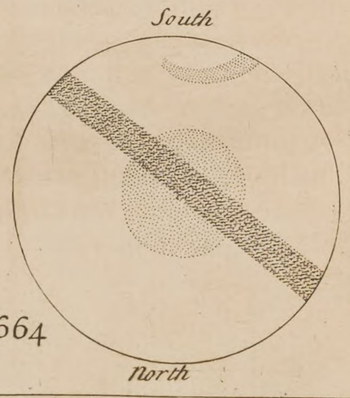


662.

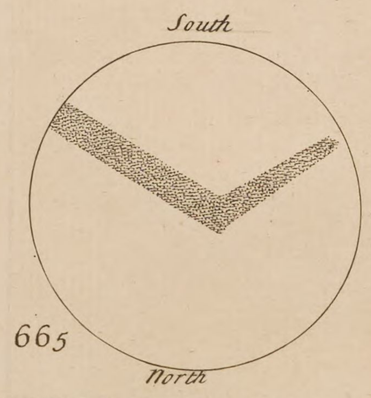




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regions; as was evident by a greater regularity and evenneſs of that part of the boundary of light and ſhade which paſſed through them at certain times, and by its protuberances at both its extremities. Nevertheleſs theſe tracts were not quite free from ſmaller inequalities, eſpecially of light and ſhade.

No ſeas in the moon.

1077. Theſe darker regions may properly enough be compared to the receptacles of our ſeas, emptied of their water. For that they contain none is evident from thoſe permanent bright ſpots obſerved in them by *Galileo*, and becauſe larger teleſcopes plainly diſcover not only ſmall eminencies but cavities within them; which are quite repugnant to the nature of ſeas. The obſcurity of their colour may proceed from a kind of ſoil that reflects leſs light than the other regions do.

Why the moon's limb does not appear mountainous.

1078. The ſurface of the moon being ſo mountainous and irregular as has been deſcribed, *Galileo* conſidered how it came to paſs that the bright circular limb of the diſk did not appear rugged and irregular, as well as the oval boundary between the light and dark parts of it, and explained it in this manner. If the ſurface of the moon had but one row of mountains placed round the limb of the diſk, the bright part of it would appear irregularly indented; but ſince the moon's ſurface is all over mountainous, and ſince the viſible limb muſt be conſidered as a large zone, poſſeſſed by many rows of mountains lying behind one another with reſpect to the obſerver's eye, the mountains in ſome rows, being oppoſite to the valleys in others, will fill up the inequalities in the viſible limb; eſpecially being all ſo remote and uniformly enlightened, that the inequalities of their diſtances from the eye cannot be perceived: juſt as a great number of mountains upon the earth, viewed from a great diſtance, or the waves in the ſea, how great ſoever, do yet compoſe an horizon apparently ſmooth and circular.

Moon's mountains higher than the earth's.

1079. *Galileo* next conſidered the magnitude of the moon's mountains and found them much higher than any of the earth's. He told us before, that when only the tops of them are enlightened by the ſun, they appeared as bright ſpecks in the dark part of the diſk; and he found the apparent diſtance of ſeveral ſuch ſpecks, from the limit between the dark and bright parts of the new moon, was above a 20th part of her apparent diameter; and thence he concluded that the perpendicular height of thoſe mountains was above 4 Italian miles or rather about $5\frac{1}{2}$ Engliſh

1080. *Ricciolus* found the diſtance of the top of St. *Catharine* from the bright part of the moon, to be about a 16th part of her diameter; and thence the perpendicular height of this mountain comes out 9 Engliſh miles, which is three times greater than that of our higheſt mountains.

1081. The method of computation is this. Let *e* be the moon's center, *g c d* a ray of the ſun touching the moon's ſurface in *c*, and the top of a mountain in *d*; join *e c* and *e d*, and let the circle *c a f*, deſcribed with the Fig. 661.

 ſemi-

ſemidiameter *ec*, cut *ed* in *a*; then *ad* is the perpendicular height of the mountain. Now if *ce* be divided into 8 equal parts, we have *cd* equal to 1 of them, by the obſervation of *Ricciolus*; and in the right angled triangle *ecd*, the ſquare of *ce*, which is 64, added to the ſquare of *cd*, which is 1, gives 65 for the ſquare of *de*, whoſe ſquare root *de* = 8.062; hence deducting *ae* = 8 we have *ad* = 0.062, and conſequently *ae* : *ad* :: 8 : 0.062 :: 8000 : 62. But the moon's ſemidiameter is known to contain about 1180 miles engliſh, and as 8000 : 62 :: 1180 : 9, which therefore is the number of miles in the height *a d*.

Maps of the moon.

1082. The fame of *Galileo*'s obſervations ſoon excited many others to repeat them, and to make maps of the moon's ſpots. Among the reſt *Langrenus* the K. of Spain's coſmographer and *Hevelius* conſul of Dantzick were the moſt diligent. To fit their maps for aſtronomical uſes, it was neceſſary to give names to the moſt remarkable ſpots and regions. *Langrenus* called them by the names of the moſt noted mathematicians, philoſophers and patrons of learning. But *Hevelius* pretending great difficulty in a juſt diſtribution of the lands in proportion to the merits of the learned, aboliſhed their received grants and titles, and called them by the geographical names of places on earth, without the leaſt reſemblance in their ſhapes and ſituations. This vanity of his has embarraſſed the lunar regions with a double nomenclature.

1083. That thoſe who are curious in obſerving lunar eclipſes, may note the times when the ſhadow begins to touch the more remarkable ſpots, or biſect them, or wholly cover them, I have given a copy of Monſ. *Caſſini*'s map of the full moon*; in which the poſitions of the ſpots were determined by his own obſervations made at the times of ſeveral eclipſes. He has noted no other than thoſe which appear plaineſt in eclipſes, and to avoid embarraſſing the map with their names, he has only marked them with numbers, referring to the names in the following table.

* Mem. de l' Acad. 1692

Fig. 662.

1. Grimaldus.	13. Capuanus.	25. Menelaus.	37 Snellius & Furnerius.
2. Galileus.	14. Bulialdus.	26. Hermes.	38. Petavius.
3. Ariſtarchus.	15. Eratoſthenes.	27. Poſidonius.	39. Langrenus.
4. Keplerus.	16. Timocharis.	28. Dionyſius.	40. Taruntius.
5. Gaſſendus.	17. Plato.	29. Plinius.	A Mare humorum.
6. Schikardus.	18. Archimedes.	30. Catharina. Cyrill. Theophilus.	B Mare nubium.
7. Harpalus.	19. Inſula ſinus medii.	31. Fracaſtorius.	C Mare imbrium.
8. Heraclides.	20. Pitatus.	32. Promontorium acutum.	D Mare nectaris.
9. Lansbergius	21. Tycho.	33. Meſſala.	E Mare tranquillitatis.
10. Reinoldus.	22. Eudoxus.	34. Promontorium ſomnii.	F Mare ſerenitatis.
11. Copernicus.	23. Ariſtoteles.	35. Proclus.	G Mare fœcunditatis.
12. Helicon.	24. Manilius.	36. Cleomedes.	H Mare criſium.

How ſmall a place may be ſeen in the moon.

1084. Monſ. *de la Hire* demonſtrates that a place in the moon no bigger than Paris, is eaſily diſcernible thro' a teleſcope that magnifies about 100 times; and that Paris placed in the middle of the moon's diſk, would appear as plainly thro' ſuch a teleſcope, as *Mare criſium* does to the naked

ed eye *. Omitting his demonſtration I will conſider this matter in a more general way.

* Mem. de l' Acad. 1706.

1085. A ſpot about 70 Engliſh miles in diameter, in the middle of the moon's diſk, may be juſt diſcerned by the naked eye. For a degree of a great circle on the earth's ſurface contains about 70 miles, and the moon's diſtance from the earth's center contains about 60 ſemidiameters of the earth. Therefore a degree of the moon's orbit, ſuppoſed to be circular, contains 60 times 70 miles, and conſequently a 60th part of that degree, contains 70 miles, and, by ſubtending an angle of a minute at the earth, is therefore viſible to moſt people's eyes *.

* Art. 97.

1086. Hence a teleſcope that magnifies 100 times, will juſt diſcover a ſpot whoſe diameter is $\frac{1}{100}$ part of 70 miles or $\frac{7}{10}$ of a mile; and a teleſcope that magnifies 210 times (as one of Sir *Iſ. Newton*'s form of $3\frac{1}{4}$ feet focal diſtance will do) will juſt diſcover a ſpot whoſe diameter is $\frac{70}{210}$ or $\frac{1}{3}$ of a mile. By which we may form a judgment of the viſibility of ſpots of any given diameters.

No ſenſible alterations in the moon's ſpots.

1087. With ſmaller teleſcopes than thoſe, we might plainly diſcern more minute changes in the moon's ſurface than what happen to the earth's in every ſeaſon of the year, through the alterations in its colour and verdure by heats, colds, ſnows, rains, inundations and the like. But from the time of the invention of teleſcopes, no viſible changes at all, in the colours ſhapes and ſituations of the moon's ſpots, have yet been obſerved: their appearances are always the ſame, allowances being made for their different lights and ſhades in different ages of the moon, for the different goodneſs of teleſcopes, and the clearneſs of our atmoſphere. Indeed as there are no ſeas nor rivers in the moon, (for they could not eſcape our beſt teleſcopes) and of conſequence no clouds, nor rain, nor ſnow, it is hard to conceive, how any great alterations can happen to a globe of dry earth, unleſs by fire.

No atmoſphere in the moon.

1088. Nor is it probable that the moon is ſurrounded with air like ours, though free from vapours. For if ſhe were, her limb would probably appear hazy, and not ſo diſtinctly terminated as we find it to be by its appulſes to fixt ſtars; whoſe rays ſuffer no ſenſible refraction, nor change of colour, nor any gradual decay of their brightneſs, but vaniſh inſtantaneouſly *. Monſ. *de la Hire*, obſerving the moon's appulſe to Aldebaran, through a very good teleſcope of 16 feet, ſaw that ſtar enter very plainly into the bright part of the diſk, till the diſtance of the ſtar's center from the circumference of the diſk became equal to three halves of the ſtar's apparent diameter in that teleſcope; after that it vaniſhed inſtantaneouſly *. And being appriſed that *P. Feüillee* had formerly obſerved the like phænomenon in an appulſe to another ſtar, he took particular notice that Aldebaran advanced very uniformly towards the moon's limb before its immerſion; and therefore could not perſwade himſelf that the moon had

Mem. de l' Acad 1720. p. 141. 4to.

* Mem. de l' Acad. 1699. p. 151. 4to.

any atmoſphere either rarer or denſer than the ambient æther; but is of opinion that this appearance of a ſtar upon the moon's bright diſk, is owing to nothing elſe but the glaring brightneſs of the limb, which cauſes it to appear always bigger than it ſhould do, even through a large teleſcope; and conſequently, that one may ſee the more brillant light of the ſtar directly through that dilated weaker light of the moon's limb; and that the ſtar does not vaniſh till its rays touch the very body of the moon.

The moon's libration.

1089. Another diſcovery that we owe to the teleſcope is, that the hemiſphere of the moon, viſible to us, is not at all times quite the ſame. At one full moon we ſee a ſmall goar or ſegment in the margin of her diſk, that was quite hid at another: ſo that her body appears to us as if it librated to and fro; ſometimes eaſtward or weſtward, at other times northward or ſouthward, and ſometimes in a direction between both. Obſervations upon theſe apparent librations were firſt made by *Galileo*, and afterwards continued by *Hevelius*, and deſcribed in his book *de Motu Lunæ Libratorio*; and the natural cauſe of them may be ſeen in Sir *Iſaac Newton*'s Principia *.

* Lib. III. prop. 17 and 38. Cor.

CHAPTER IV.

Teleſcopical diſcoveries in Mars.

Phaſes of Mars.

1090. THE orbit of Mars ſurrounds the earth's, and therefore he is nearеſt to the earth, when being oppoſite to the ſun, he is ſeen in the meridian at midnight. This achronical ſituation of his, is therefore the moſt advantageous for obſerving the motion of ſpots on his body, his diameter and parallax; all which are above five times greater in his oppoſition to the ſun than in his conjunction. In both theſe poſitions his enlightened hemiſphere is fully expoſed to the earth, as well as to the ſun; but not ſo fully in his quadratures with the ſun, where he appears through a teleſcope a little gibbous, like the moon about three days from the full: which plainly proves that his orbit ſurrounds the earth's at no great diſtance from it. In the figure, *s* is the ſun, *e* the earth, *a* and *b* the places of Mars in oppoſition and conjunction, *c* and *d* his places in the quadratures with the ſun.

Fig. 663.

Dark ſpots in Mars and their periodical revolution.

1091. After *Galileo*'s diſcovery of the phaſes of Mars, ſome other Italians in the year 1636 had an imperfect view of a darkiſh ſpot upon his body; but in the year 1666 Dr. *Hook* at London and Sigr *Caſſini* at Bologna made the firſt diſcovery of ſpots of determinate figures. *Hook* perceived ſome little motions in them, but could not determine whether they revolved or not. But *Caſſini* concluded from their motions, that Mars revolved about an axis of his body in 24 hours and 40 minutes *.

* Phil. Tranſ. N. 14.

1092. In the year 1670 he ſaw the ſame ſpots again, and obſerved their periodical revolution to be the ſame as before*; which M. *Maraldi* confirmed in 1704, and again in 1719; and has given us a curious account of it*, from whence I extract the following particulars. At this time the configuration of the ſpots was quite different from what *Caſſini* ſaw. One of them appeared like a joiner's ſquare, except that the angle was a little obtuſe. Its angular point being pretty diſtinct, was the mark attended to in the following obſervations.

* Du Hamel. Hiſt. Reg. Sci. Acad. p. 97. Ed. 2.
* Mem. de l' Acad. 1720 p. 144. 4to. Fig. 664, 665.

1093. On the 19th and 20th of Auguſt at a quarter paſt eleven at night, this point lay a little eaſtwards from the center of the diſk, and at the ſame times of the ſucceeding nights it was found ſucceſſively more and more eaſtwards; till at laſt it withdrew and vaniſhed under the eaſtern limb of the diſk; and having in like manner traverſed the inviſible hemiſphere, on the 25th and 26th of September it returned, and appeared again in the ſame places of the diſk as before on the 19th and 20th of Auguſt; and therefore ſpent 37 days in theſe apparent retrogradations. This number of days being divided by 36 revolutions of Mars about his axis, performed in that time, gives $\frac{37}{36}$ or $1\frac{1}{36}$ or 1 day and 40 mintues for the time of each revolution, as *Caſſini* had found before: which period was alſo verified by many other obſervations upon this and ſome other ſpots.

1094. Beſides theſe dark ſpots, ſituated in different places of the ſurface of mars, M. *Maraldi* obſerved a bright one near the ſouthern pole, appearing like a polar zone. He obſerved it for ſix months together, and found it ſubject to many changes, appearing ſometimes very bright, at others very faint; and after an entire diſappearance, it revived with the ſame brightneſs as before. At every time of its bright appearance, the diſk of Mars did not appear exactly round; but the bright part of its ſouthern limb, that terminated this ſpot, appeared protuberant in the ſhape of a bright cap, whoſe outward arch was a portion of a larger circle than what terminated the darker parts of the diſk. Its appearance thro' the teleſcope reſembled that of the new moon to the naked eye, when the bright part of the limb appears as a portion of a larger circle than the dark part. The cauſe of which, in both caſes, M. *Maraldi* attributes to a ſtronger impreſſion of the rays of the more luminous part upon the retina of the eye.

Bright ſpots in mars.

1095. The axis of Mars is nearly perpendicular to the plane of his orbit; conſequently its poles are never far from the north and ſouth parts of the limb of his viſible diſk; therefore if the polar zone abovementioned was not all over uniformly bright, thoſe variations of its appearance might reſult from its revolution about the axis of Mars. M. *Maraldi* collected from many obſervations, that a large part of that zone never failed to appear very bright, for ſix months together whenever it lay expoſed to his view; and that the oppoſite part, on the other ſide of the pole,

Situation of his axis.

was

was ſubject to great variations, appearing ſometimes bright, at others faint and contracted in breadth till it totally vaniſhed. Notwithſtanding theſe changes on one ſide of the pole, he takes notice, that the other ſide had continued brighter than the reſt of the diſk, more or leſs, for near 60 years; and that it was the only permanent ſpot upon the whole body of Mars, the dark ones having changed their ſhapes and ſituations and totally vaniſhed in much ſhorter periods of time; which alſo happened to another bright ſpot that he ſaw near the north pole.

Reaſons for an atmoſphere about mars.

1096. By the following obſervations it ſeems as if mars had a very extenſive atmoſphere. *Caſſini* being at Briare in 1672, Oct. 1. at $2^h.45^m$. a. m, obſerved a ſtar in the water of Aquarius, which at the diſtance of ſix minutes from the diſk of Mars, became ſo faint before its occultation, that it could not be ſeen with the naked eye, nor with a 3 foot teleſcope *. The like diminution of its light, after its occultation, was alſo obſerved at *Paris* by M. *Roemer*; who could not ſee that ſtar with a large teleſcope, in a very clear air, till its diſtance from Mars became equal to two thirds of his diameter, and yet ſtars of that magnitude are plainly viſible even in contact with the moon. By a compariſon of ſeveral obſervations then made upon that ſtar, it was alſo judged that it varied its diſtance from the neighbouring ſtars.

* Du Hamel Hiſt. Ac. Reg. Sci. p. 107. Ed. 2.

CHAPTER V.

Teleſcopical diſcoveries in Jupiter.

Galileo's diſcovery of Jupiter's ſatellites.

1097. IN the year 1610 *Galileo* diſcovered the ſatellites of Jupiter, as follows *. At a ſmall diſtance from Jupiter, which in the evening of Jan. 7, he happened to view through a better teleſcope than his firſt, he ſaw three ſmall bright ſtars, which he took to be fixt ſtars, but wondered a little to ſee them placed exactly in a ſtraight line, parallel to the ecliptick, and to outſhine others of the like magnitude. Their ſituation with reſpect to Jupiter was this, *Eaſt* * * O * *Weſt*; two of them lay eaſtwards and the third weſtwards from Jupiter; and the two outermoſt appeared a little bigger than the other. Taking them for fixt ſtars, he did not much mind their diſtances from Jupiter, but happening the next night to look at them again, he ſaw their ſituations quite changed. For now they all lay weſtwards from Jupiter, equidiſtant from one another and cloſer together than the night before, *Eaſt* O * * * *Weſt*. Here again, not taking notice of their approach to one another, he began to wonder how it could be that Jupiter lay weſtwards from two of them the night before, and now eaſtwards from them all; and was afraid leſt, contrary to aſtronomical computation, Jupiter had got before them all by a direct motion. He was there-

* Nuncius ſidereus.

fore

fore very impatient to obſerve him the night after, but was diſappointed by cloudy weather. On the 10th their appearance was thus, *Eaſt* * * ○ *Weſt*; for now he ſaw but two, and both eaſtwards from Jupiter, the third being hid behind the planet, as he imagined; theſe two and the planet were exactly in a ſtraight line parallel to the ecliptick as before. Seeing things were ſo, and that ſuch changes could not be owing to Jupiter's motion, and that the ſtars were conſtantly the ſame, for no other lay near them in the ecliptick, all his ſcruples were changed into admiration; being ſatisfied at laſt that theſe changes of poſition were not owing to Jupiter but to the ſtars themſelves. He therefore reſolved to obſerve them more accurately for the future.

1098. On the 11th he ſaw them in this poſition, *Eaſt* * * ○ *Weſt*; that is, only two ſtars lying eaſtwards from Jupiter, whoſe diſtance from the next to him was triple their diſtance from each other; and the eaſtern ſtar was twice as large as the other; whereas they were almoſt equal the night before.

1099. By this time he was fully convinced that Jupiter had three ſtars revolving about him, as Venus and Mercury do about the ſun; and the ſecond night after he found a fourth: they all lay nearly in a line, in this manner, *Eaſt* * ○ * * * *Weſt*, and appeared equal in magnitude; very ſmall indeed, but exceeding bright, and much more ſplendid than fixt ſtars of that ſize.

1100. Thus he went on for two months together, conſtantly obſerving his new planets, which he called the *Medicean* ſtars, after the name of his patrons, the family of the *Medici*, and concludes his account with the following reflections. Theſe, ſays he, are the obſervations I have hitherto made of the four Medicean planets, of which I was the firſt diſcoverer; and though I cannot yet reduce their periods to calculation, yet I have ſomething to ſay of them worth remarking. Firſt then, ſince they ſometimes go before and ſometimes follow after Jupiter, through certain ſmall ſpaces lying eaſtwards and weſtwards from him, and ſince they always accompany him, both in his direct and retrograde motions, they muſt undoubtedly revolve about him, while he is revolving about the ſun in about 12 years time. In the next place they revolve about Jupiter in unequal circles; becauſe ſays he, I could never ſee two of them in conjunction at their greateſt digreſſions from Jupiter; whereas I have often ſeen two or three, and ſometimes all of them crowded cloſe together near the body of Jupiter. Laſtly, thoſe that revolve in the ſmaller circles, have ſhorter periodical times; becauſe thoſe that appeared nearer to his body were often obſerved to paſs from one ſide of it to the other in a day's time. But the outermoſt of all ſeemed to compleat its revolution in about ſixteen days. Hence, ſays he, we are ſupplyed with a noble argument for the Copernican Syſtem, by removing a difficulty from ſome people's

ple's minds, who, admitting the revolution of the planets about the ſun, are yet ſo diſſatisfied with the moon's revolution about the earth, while both are moving round the ſun, as to think it impoſſible; and conſequently a ſufficient argument for rejecting that ſyſtem. But now we have ocular demonſtration, not only of one but four planets revolving about Jupiter, like the moon about the earth, while all of them are moving round the ſun in larger orbits. Our author concludes at laſt by attempting at a reaſon, why the Medicean ſtars appear at different times of different magnitudes; which we ſhall ſee, by and by, is moſt probably owing to very large, dark ſpots upon their bodies, and to their turning about certain axes of their own, as our moon does about hers.

Satel.	Periodical times.				Diſtances.
1.	1d.	18h.	27′.	34″	5, 667
2.	3.	13.	13.	42	9, 017
3.	7.	3.	42.	36	14, 384
4.	16.	16.	32.	09	25, 299

1101. By the lateſt and moſt exact obſervations, the periodical times, and diſtances of theſe ſatellites from the center of Jupiter, meaſured by his ſemidiameters, are as follow; and the analogy among them is, that the ſquares of their periodical times, are as the cubes of their reſpective diſtances from the center of Jupiter, as in the primary planets.

The earlieſt obſervations of Jupiter's belts and ſpots.

1102. About twenty years after *Galileo*'s diſcovery of theſe ſatellites, we are told that *Fontana*, *Zupus* and *Bartolus* three Neapolitans were the firſt diſcoverers and obſervers of Jupiter's belts. By that time it ſeems the teleſcope had received ſome improvement, particularly by a convex eyeglaſs firſt applyed by *Rheita* inſtead of the concave. For *Gaſſendus*, with *Galileo*'s own teleſcope, could perceive nothing of them. Theſe belts were nearly parallel to the ecliptick, in ſome years quite ſtraight, in others a little incurvated, ſometimes upwards and ſometimes downwards, which curvity *Ricciolus* ſuppoſed to ariſe from a ſlow libration of the planet, or elſe from a ſmall inclination of the axis of the belts to the plane of the ecliptick. About the year 1643 the ſame perſons diſcovered the appearance of two ſatellites, or more probably of their ſhadows, paſſing over the diſk of Jupiter; and alſo of two very large roundiſh ſpots.

Two ſorts of ſpots ſeen upon Jupiter. Mem. de l' Acad. Tom. X. p. 513.

1103. After farther improvements of the teleſcope, M. *Caſſini* in the year 1665 publiſhed a theory of two ſorts of ſpots viſible at certain times upon the diſk of Jupiter. One ſort he ſhewed to be the ſhadows of the ſatellites, paſſing over the diſk, at ſuch times as the ſatellites themſelves paſſed between the ſun and Jupiter, and made there a ſolar eclipſe; ſuch as the moon makes here in paſſing between the ſun and the earth. Theſe ſpots are therefore diſtinguiſhable from all others by the following properties; that they fall preciſely upon that part of Jupiter, where ſome

ſatellite



666

East i West

d c f

667 South North

668 South North

h g

East j West

e f

b

c a

S

669 South North

West

670 South North.

East

b k l a

S

671

c d

672

673

674

675

676

677

678

679

ſatellite would be ſeen by a ſpectator placed at the ſun; that they move from the eaſtern towards the weſtern limb of the diſk with an uniform velocity equal to that of the ſatellite it ſelf; that while the earth is approaching towards the line that connects the ſun and Jupiter, the ſatellite, which caſts the ſhadow, is ſeen eaſtwards from Jupiter, and weſtwards from him after the earth has paſſed by that line; and laſtly that the apparent diſtance of the ſatellite from the ſhadow, is found proportionable to the angle which a line connecting Jupiter and the ſatellite, ſubtends at the earth. What has here been ſaid will appear plainer by applying it to fig. 666, where *s* is the ſun, *j* Jupiter, *abc* the earth's annual orbit, and *def* that of the ſatellite, both deſcribed according to the alphabetical order of thoſe letters. Fig. 666.

1104. The other ſort of ſpots has no relation to the ſatellites, but ſeems to be of the nature of the belts; for in paſſing from the eaſtern to the weſtern limb, they move ſloweſt at the extremities of their courſe, and quickeſt in the middle, where they appear broadeſt; which ſhews them to be thin ſubſtances in or near the ſurface of Jupiter, as was ſaid above in relation to ſpots in the ſun.

1105. M. *Caſſini* has given us a draught of a remarkable round ſpot of this ſort adjoining to the ſouthermoſt of the belts that then appeared. Its neareſt diſtance from the center of the diſk was about one third of Jupiter's diameter, and its own diameter about one tenth of his. *Caſſini* began to obſerve it in 1665 and by repeated obſervations found the periodical time of its revolution to be $9^h. 56^m$; and taking a certain time of its arrival to the middle of its parallel for an epocha, he calculated tables of its motion, and found them conſtantly agreeable to his ſubſequent obſervations, till Jupiter became immerſed in the beams of the ſun; but after his emergence from them, the ſpot was ſo waſted that it could hardly be ſeen; after which it ſoon vaniſhed. Fig 667.

1106. But in the year 1672 he ſaw it again, as he judged by its ſize, ſhape, poſition and adherence to the ſame ſouthern belt as before; and by comparing his preſent obſervations with thoſe he had made ſix years before, he concluded, that the mean period of the ſpot's revolution to the middle of its parallel, was not greater than $9^h. 55^m. 58''$, nor leſs than $9^h. 55^m. 51''$. In the evening of Mar. 1. 1672 at $7^h. 30^m$ he ſaw this ſpot in the middle of the diſk, or rather of its parallel, and next morning at $5^h. 26^m$ he ſaw its return to the ſame place; which was the firſt obſervation he could ever make of an immediate return.

1107. The ſame ſpot continued viſible till the year 1674, when it wholly diſappeared. In 1677 it appeared again in the ſame parallel to Jupiter's equator, but was ſoon extinct, and never ſeen again till Mar. 1685. From this time M. *Caſſini* continued to obſerve it till Octob. 1687. After that it never appeared again till in 1690, and was viſible but a Mem. de l' Acad. 1708. p. 235 4to.

very ſmall time, perhaps by reaſon of great changes which happened that year in Jupiter's belts. It appeared again in 1692, and diſappeared in 1693 with part of the contiguous belt. In 1694 it revived with the ſame belt, and continued viſible till Jupiter entered the beams of the ſun; but was never ſeen after till in 1708 though frequently watched.

1108. M. *Maraldi* having conſidered theſe viciſſitudes of appearance and diſappearance, concluded they had no regular relation to Jupiter's diſtances from the ſun, as ſome changes upon the earth have to the variation of ſeaſons; that they rather had a dependence upon the contiguous belt; that this belt had ſometimes appeared interrupted, and that the breach widened more and more, till the whole belt vaniſhed together with the contiguous ſpot; that the ſpot had never appeared without the belt, though the belt had without the ſpot; and laſtly that the ſpot was probably ſome effuſion of the matter of the belt into a determinate place of Jupiter's body; its latitude from his equator being conſtantly the ſame, and its longitude too, becauſe it returned to the middle of the diſk at the times calculated by the tables of its motion, conſtructed from the obſervations made upon its firſt appearance. In the year 1713 M. *Maraldi* ſaw this ancient ſpot again, and made ſome more obſervations upon it*, confirming what has been ſaid.

* Mem. de l' Acad. 1714.

Changes in Jupiter's belts and ſpots Mem. de l' Acad 1692. Fig. 668. 669. 670.

1109. In the largeſt of Jupiter's northern belts, lying next his center, in Oct. 1691, *Caſſini* ſaw two bright ſpots almoſt as broad as that belt, as in fig. 668; and at the end of that month, two more oppoſite to one another. They revolved in $9^h. 51^m$. He alſo perceived that this broader belt grew narrower, while thoſe on each ſide of it grew broader, till in December following they all became nearly of the ſame breadth, as if the intermediate belt diſcharged it ſelf into the collateral ones, which he thought not improbable from the appearance of ſome dark parts between them like tracts of communication, as expreſſed in fig. 669 and 670; that the largeſt of the northern and ſouthern belts were frequently interrupted, and that their broken ends generally revolved in the ſame time with the ſpots in thoſe belts; that at certain times in Oct. 1691, ſeven or eight dark belts appeared very near to one another, moſt of them upon the ſouthern part of the diſk; that ſome late ſpots near Jupiter's equator revolved quicker than the old ones, their periodical times being but $9^h. 50^m$; and in general, that all ſpots near the equator revolved quicker than thoſe that lay farther from it; which he thought might be owing to the ſun's greater heat upon the equator; whoſe plane is parallel to that of the belts and to the courſe of the ſpots, and differs but little from the plane of Jupiter's motion round the ſun; that ſeveral ſpots which appeared round at firſt, grew oblong by degrees, in a direction parallel to the belts, and ſometimes ſplit themſelves into two or three rounder ſpots. Laſtly, that he never ſaw ſo many new ſpots as about

Sept.

Sept. 1690, perhaps, ſays he, becauſe Jupiter was then the neareſt poſſible to the ſun and the earth too, which ſituation gave an advantage to the view, and returns but once in 83 years.

Spheriodical figure of Jupiter.

1110. We have juſt taken notice that Jupiter's axis is nearly perpendicular to the plane of his orbit, ſo that the difference of ſeaſons there is very ſmall. In 1691 *Caſſini* made obſervations upon the ſpheroidical figure of Jupiter, and found the ratio of his axis to his equatorial diameter as 14 to 15. In 1719 Mr. *Pound* found it as 12 to 13 with an excellent micrometer applyed to the Hugenian teleſcope of 123 feet, as Sir *Iſaac Newton* informs us, who deduced the latter ratio from the conſideration of its cauſes *, that is, from Jupiter's denſity and periodical time of revolving round his axis, compared with thoſe of the earth.

* Princip. ph.loſ. p. 415.

Spots upon Jupiter's ſatellites Mem de l' Acad. 1707 p. 289. 4to. Fig. 666.

1111. In the year 1707 Mar. 26. M. *Maraldi* perceived a round black ſpot upon Jupiter's diſk; which at 6^h. 50^m. p. m. had paſſed by the middle of it, and in approaching towards the weſtern limb, did not alter its ſize, ſhape or velocity; by which and the ſlowneſs of its motion he concluded that it did not adhere to the body of Jupiter. Neither was it the ſhadow of a ſatellite upon the diſk, though round and black, as they uſually appear. For, of the three innermoſt ſatellites, which he then ſaw, the firſt and third he knew were in the remoter halves of their orbits; and though the ſecond was in its nearer half, yet it had paſſed by its conjunction with the ſun and Jupiter. As to the fourth, he knew alſo by the tables and ſoon after by obſervations, that it then was at *d* between the earth at *c* and Jupiter at *j*, and conſequently that its ſhadow was projected ſo far eaſtwards from Jupiter, that it could not arrive at the place of the ſpot in leſs than 7 hours. Beſides, had the ſpot been a ſhadow of this fourth ſatellite, the ſatellite it ſelf would have been ſeen weſtwards from Jupiter, at the diſtance of about two of his diameters.

1112. Hence he was perſwaded that the dark ſpot was in the ſatellite it ſelf, and was confirmed in his opinion by the following arguments; that its motion was agreeable to the known motion of the ſatellite; that in going out of the diſk it grazed upon the weſtern end of one of the belts, in the very place where he ſaw the ſatellite it ſelf emerge from the diſk ſome minutes after the vaniſhing of the ſpot; that this interval of time might ariſe from the difficulty of diſcerning the ſpot to the very edge of the diſk, or, perhaps, from the ſituation of the ſpot upon the body of the ſatellite; that if the ſpot lay on the weſtern ſide of the ſatellite, it ought to diſappear before the brighter eaſtern ſide could emerge from the diſk of Jupiter; that the ſpot began to emerge from the diſk at 7^h. 49^m, and the ſatellite at 8^h. 6^m, when it appeared very ſmall; that this interval of 17 minutes, agreed well enough with the time that the diameter of this ſatellite ſhould take up in emerging from the diſk; ſo that the ſpace between the antecedent limb of the ſpot and the ſubſequent limb

of the ſatellite anſwered to the whole diameter of the ſatellite. The obſervations were made with a 34 foot tube.

1113. On April 4. he obſerved a like tranſit of the third ſatellite, in the form of a ſpot, over the diſk of Jupiter. But at the next conjunction, on the 11th, though he ſaw the ingreſs and egreſs of the ſame ſatellite, yet during its paſſage, in three hours and a half, he could not perceive any ſpot at all; and therefore concluded, that the ſpot had vaniſhed ſome time between this and the foregoing tranſit; and adds, that a viſible tranſit of a ſatellite in the form of a ſpot on the diſk is a thing that happens very rarely. He alſo obſerves, that the ſatellites appear ſometimes like little bright ſpots upon the margin of Jupiter's diſk, and diſappear near the middle of it, where we can only diſtinguiſh them when they happen to have large dark ſpots upon the hemiſphere next us; and that *Caſſini* the Father had obſerved the like phænomena in every one of theſe ſatellites.

1114. It is highly probable, ſays he, that the ſatellites turn about certain axes of their own, and have permanent ſpots like thoſe in the moon, or variable ones like thoſe in the ſun and primary planets; whoſe changes of magnitude and poſition may occaſion their viſible and inviſible tranſits over the diſk of Jupiter. For the apparent magnitudes and degrees of brightneſs of the ſame ſatellites are very different at different times; the fourth appears generally the ſmalleſt of all, but ſometimes the largeſt, and, though its ſhadow on Jupiter's diſk ought to be rather leſs in diameter than the ſatellite it ſelf, and be ſtill diminiſhed a little by the apparent incroachment of the ambient light, yet it appears larger than the ſatellite ſeen at the ſame time on the outſide of the diſk. The third ſatellite is alſo variable in apparent magnitude, being generally larger than the reſt, though ſometimes equal to them and ſometimes leſs. The like happens to the other two.

1115. But none of them appear large enough out of the diſk, to diſtinguiſh their ſpotted parts from the bright ones, as it happens alſo to certain fixt ſtars, which yet increaſe and decreaſe in apparent brightneſs: and while the ſatellites are entering into the ſhadow of Jupiter, we plainly perceive a gradual decay of their brightneſs, but cannot diſcern the phaſes of the eclipſe even with the beſt teleſcopes. M. *Maraldi* concludes at laſt that *Caſſini*'s hypotheſis of ſpots upon the ſatellites is farther confirmed by the outermoſt of ſaturn's, which, for many years after *Caſſini* had firſt diſcovered it, could never be ſeen on the eaſtern ſide of ſaturn, till it became viſible even through the ſame teleſcopes in Sept. 1705, and and continued ſo till Jan. 1706.

Tranſit of the body and ſhade of Jupiters 4th. ſatellite over his disk. Phil. Tranſ. Jones's abr. Vol 4, p. 307.

1116. In 1719 Feb. 16^{d}. 9^{h}. 45^{m}, Mr. *Pound*, through the Hugenian teleſcope of 123 feet, ſaw the outermoſt ſatellite of Jupiter in the middle of his diſk, and its ſhadow near the eaſtern limb; the ſatellite appeared

to

to him almoſt as black as its ſhadow, but ſomewhat leſs and a little more northerly. For, at the ſame time he ſaw the three innermoſt ſatellites to the eaſt of Jupiter, and the times, 9^h. 45^m and 11^h. 45^m, at which the outermoſt and its ſhadow arrived at the middle of the diſk, were agreeable to the times, found by calculation, in which they ought to be there. He adds, that at other times he had ſeen the firſt and ſecond ſatellites, appearing not as dark ſpots, but as bright ones, ſomewhat different from the brightneſs of Jupiter, for ſome little time after they entered his diſk, but as they approached nearer the middle he loſt ſight of them; that he had frequently ſeen the ſame ſatellites appearing brighter at ſome times than at others; that when one of them ſhined with its utmoſt ſplendor the light of the other was conſiderably diminiſhed; that from theſe obſervations it was very probable at leaſt, not only that the ſatellites revolve upon their proper axes, but alſo that ſome parts of their ſurfaces do very faintly, if at all, reflect the ſolar rays to us.

Tables of the eclipſes of Jupiter's ſatellites.

1117. Tables of the motions of Jupiter's ſatellites were firſt publiſhed by Sig. *Caſſini*, and were afterwards epitomiſed, corrected, and reduced to our ſtyle and meridian by Dr. *Halley* * and laſt of all by Mr. *Pound* *; who applyed himſelf particularly to rectify the motion of the innermoſt ſatellite, and to facilitate the calculation of its eclipſes in the ſhadow of Jupiter, as affording the beſt and moſt frequent opportunitys of determining the longitudes of places at land, by clocks and teleſcopes of a convenient length, thirteen of theſe eclipſes happening every twenty three days

* Phil. Tranſ. abrid. Vol. 1. p. 409.

* Phil. Tranſ. abrid. Vol. 4. p. 308.

Applyed to find the longitude of places.

1118. The application of this moſt uſeful invention, to the purpoſes of geography and navigation was firſt ſuggeſted by *Galileo* and firſt put in practice by the members of the Royal Academy of Sciences, ſent by order of *Lewis* the 14th into many diſtant parts of the world; and if duly proſecuted, might contribute more to the improvement and perfection of geography and navigation in a few years, than all other methods can do in as many ages. And till the geography of the ſea-coaſt be duly ſettled, the very beſt methods of determining the place of a ſhip at ſea, at her approach towards land, where the danger is greateſt, will be ſtill defective.

Velocity of light.

1119. From the conſideration of the eclipſes of theſe ſatellites Monſ. *Roemer* made a noble diſcovery of a method for determining the velocity of light; by which it was concluded and confirmed by long experience that light is ſhot from the ſun to the earth in 7 or 8 minutes of time; a diſtance ſo great, that a cannon ball cannot deſcribe it in leſs than 25 years, at the rate of 500 feet in every ſecond of time.

1120. *Roemer*'s method is this. Let *s* be the ſun, *abc* the annual orbit of the earth, *j* Jupiter, *efgh* the orbit of the innermoſt ſatellite, the propereſt for this enquiry by reaſon of the quickneſs of its revolution; and Fig. 671.

let

let this ſatellite enter the ſhadow of Jupiter at *g* and emerge from it at *h*. Now ſuppoſing the earth at *b*, ſometime before the laſt quadrature, let an emerſion of this ſatellite be obſerved at *b*; then if the earth continued in the ſame place, we ſhould ſee the next emerſion at the end of 42 hours and a half, this being ſuppoſed the exact time in which this ſatellite revolves to the ſhadow. Likewiſe if the earth continued at the ſame place *b* during any number of revolutions, ſuppoſe 30, we ſhould ſee an emerſion at the expiration of 30 times 42 $\frac{1}{2}$ hours. But the earth in that time being really transferred from *b* to *c*, farther from Jupiter, it follows, if light requires time for its propagation, that this emerſion will be perceived later at *c* than it would have been at *b*, and that to 30 times 42 $\frac{1}{2}$ hours, we muſt add the time which the light takes up in deſcribing the ſpace *k c*, the difference of *b h* and *h c*.

1121. On the contrary towards the other quadrature, whilſt the earth in going from *d* to *a*, is approaching towards Jupiter, the immerſions of the ſatellite at *g* ſhould be perceived ſooner at *a* than they would be, had the earth remained at *d*.

1122. Now theſe retardations of the emerſions in going from Jupiter and accelerations of the immerſions in going towards him, have been often found to amount to above ten minutes; and from the moſt accurate conſideration of them it is concluded that light deſcribes a line *k c* equal to the ſemidiameter of the earth's annual orbit, nearly in half a quarter of an hour. The motion of Jupiter in his orbit during the earth's paſſage from *b* to *c* and from *d* to *a*, is conſidered in that concluſion; and it is at laſt agreed by aſtronomers that theſe equations, of the times of theſe eclipſes, cannot be accounted for either by any inequality in the motion of the ſatellite, or by any excentricity or inclination of its orbit; and laſtly that the three other ſatellites require the ſame equations of the times of their eclipſes *.

* Phil. Tranſ. Vol. 1. abrid. p. 422.

CHAPTER VI.

Teleſcopical diſcoveries in Saturn.

The firſt teleſcopick obſervations upon Saturn.

1123. GALILEO was the firſt perſon that obſerved any thing extraordinary in Saturn, which at that time he took for three globes, a larger between two ſmaller. They appeared almoſt to touch one another, and their centers lay exactly in a ſtraight line, which he judged nearly parallel to our equator. In the year 1610 he advertiſed this diſcovery by the letters of this ſentence tranſpoſed; *Altiſſimum Planetam tergeminum obſervavi*; and afterwards added this caution, that unleſs the teleſcope was good and magnified at leaſt 30 times in diameter, Saturn would not appear through it as three diſtinct globes, but as one, only

only lengthened like an olive. He obſerved much the ſame appearance for above two years together, till in 1612 he was amazed to find the middle globe left quite alone. But ſometime after that, the collateral globes appeared to him again, and being viewed by different perſons, through teleſcopes of different goodneſs, they ſeemed to ſome to ſtick to the middle globe, to others to be ſeparated from it, and in proceſs of time to put on various ſhapes, ſometimes round, ſometimes oblong like acorns, ſometimes ſemicircular, then lunar, with horns pointing towards the globe in the middle, and growing by degrees ſo long and ſo wide as to encompaſs it, as it were with an oval ring.

1124. Theſe ſtrange appearances were obſerved for forty years together, and many hypotheſes were invented to ſolve them, as by *Hevelius* in his book *de Saturni nativa facie*, by *Roberval*, *Hodierna*, *Gallet* and others, an account of which is given us by *Hugenius* in his *Syſtema Saturnium* publiſhed in 1659. This judicious perſon, finding no ſatisfaction in what had been offered to the publick, and judging the chief impediment to lye in the ſmalneſs and other imperfections of the beſt teleſcopes then made uſe of, reſolved in the firſt place to improve the art of grinding object-glaſſes; in which he ſoon ſucceeded ſo far as to make ſome teleſcopes that magnified two or three times more than ordinary. With theſe he diſcovered a ring about Saturn, which he ſhewed was the true cauſe of all thoſe ſurpriſing phænomena, and beſides that a ſatellite, revolving round Saturn every 16 days, or thereabouts, in the plane of the ring produced.

Hugenius's obſervations upon Saturn.

1125. In March 1655, he ſaw Saturn through a 12 foot tube, as repreſented in fig. 672, with two Arms oppoſitely extended from the globe. They appeared a little thicker towards their extremities than near the diſk, and kept this ſhape till Saturn became immerſed in the beams of the ſun. But when he had loſt theſe arms in 1656 and recovered them again ſome time after, they appeared through the 12 foot tube, in the ſame ſhape as before, but through another tube of 23 feet, which he had prepared in that interval of time, they appeared ſlenderer towards their extremities, as in fig. 673. He therefore concluded they would have appeared of this truer ſhape before, had they been ſeen through the longer tube. A darkiſh zone or line paſſing ſtraight over the diſk, between the upper ſides of the Arms was viſible even through the 12 foot tube.

Fig 672.

Fig. 673.

1126 In Jan. 16. 1656, after his return from his travels, he ſaw Saturn quite round as in fig. 674, but others had obſerved this phaſe before about the latter end of November; which continued round when Saturn entered the ſun's beams a ſecond time. In every obſervation of this phaſe, that darkiſh line paſſed through the center of the diſk, and was conſtantly directed to the ſatellite, and was alſo in the direction of the diurnal motion of the planet, and conſequently parallel to our equator.

Fig. 674.

1127.

1127. In Octob. 1656, Saturn began to appear again, with his arms
Fig. 675. as in fig, 675, of the ſame ſhape as in the year before, but more diſtinctly defined through better teleſcopes. The darkiſh line now joined the underſides of each arm, and was ſtill parallel to the diurnal motion of the planet, and likewiſe ever after.

1128. On Novem. 26. The arms grew a little broader and appeared not ſo bright near the body as at their extremities.

1129. In 1657. Decem. 17. As ſoon as Saturn had emerged from the ſun, both the arms were divided into two parts by a dark ſpace next the
Fig. 676. body, and appeared like *Anſæ* as repreſented in fig. 676. The darkiſh zone was now carried farther downwards; and the ſame appearance continued till Saturn's heliacal ſetting.

1130. In 1658. Novem. 10. After his heliacal riſing, the *Anſæ* ſeemed wider than before, but were not diſtinct by reaſon of the vapours near the horizon. But on Feb. 6. they appeared very diſtinct, as repreſented
Fig. 677. in fig. 677, which phaſe continued till the latter end of March, when the author left off obſerving. I have omitted an account of his other obſervations, as relating only to the motion of the ſatellite.

Dimenſions of Saturn and his ring.

1131. In the year 1656. before he had gone through all theſe obſervations, he advertized his diſcoveries in Saturn in the letters of this ſentence tranſpoſed; *Annulo cingitur, tenui, plano, nuſquam cohærente, ad eclipticam inclinato*; and afterwards he deſcribed the ſhape of the ring as
Fig. 678. in fig. 678, where he makes the ſpace between the globe and the ring equal to, or rather bigger than, the breadth of the ring; and the greateſt diameter of the ring in proportion to the diameter of the globe as 9 to 4. But the truer proportion is 7 to 3, as Mr. *Pound* determined it by an excellent micrometer applyed to the *Hugenian* glaſs of 123 feet. That there is empty ſpace between the ring and the Body is evident not only from its colour and other optical appearances, but alſo from what Mr. *Whiſton* relates in his Memoirs of Dr. *Clarke*'s life; viz. that the Doctor's father once ſaw a fixt ſtar through the dark ſpace of one of the *Anſæ*.

The ring ſeems to be double.

1132. In the year 1675, after Saturn had emerged from the ſun's rays, Sig. *Caſſini* ſaw him, in the morning twilight, with a darkiſh belt upon his globe parallel to the long axis of his ring, as uſual. But what was moſt remarkable, the broad ſide of the ring was biſected quite round by a dark elliptical line, dividing it as it were into two rings, of which the inner one appeared brighter than the outer, with nearly the like difference in brightneſs as between that of ſilver poliſhed and unpoliſhed; which though never obſerved before, was ſeen many times after with tubes of 34 and 20 feet, and more evidently in the twilight or moon light, than in a darker ſky*.

* Phil. Tranſ. Abr. Vol. 1. p. 367. Item Recueil d' Obſer. de l' Acad. Tom. X. p. 582.

1133. Mr *Hadley* with his 5 foot Reflecter took notice*, that the upper part of the ring ſeemed to be narrower than the lower or inner part next

* Phil. Tranſ. Abridg. Vol. 6. p. 153.

the

the body, and that the dark line which ſeparated them was ſtronger next the body and fainter towards the upper edge of the ring as in fig. 679. Fig. 679. Within the ring he alſo diſcerned two belts, one of which croſſed Saturn cloſe to its inner edge, and ſeemed like the ſhade of the ring upon the body of Saturn; but when he conſidered the ſituation of the ſun in reſpect to the ring and Saturn, he found that belt could not ariſe from ſuch a cauſe. The edge of the ſhadow of Saturn caſt upon his ring was alſo vible, as repreſented in the figure,

1134. Agreeable to theſe are Mr. *Pound*'s obſervationes made with the Hugenian glaſs *; who beſides the ring on the ſouth ſide, ſaw a zone on the north ſide, not ſo far from the center as the ring, and not unlike the ſmalleſt of Jupiter's belts.

* Phil. Tr. Abr. Vol. 4. p. 321.

1135. *Hugenius* was of opinion at firſt, that the plane of the ring was conſtantly almoſt parallel to the plane of the earth's equator, and conſequently inclined to the ecliptick in an angle of about 23 ½ degrees. Becauſe by ſeveral methods, mentioned in his *Syſtema Saturnium* p. 51, he found that the long axis of the ellipſis, which the ring reſembles, was at all times nearly parallel to the plane of our equator. But in 1668 Meſſrs. *Roëmer*, *Picard* and himſelf meaſured diverſe ways the inclination of the greater diameter of the oval to our equator, and found it 9 degrees, and thence concluded its inclination to the ecliptick was about 31 degrees, eſpecially becauſe in the beginning of July 1664 Sig. *Campani* had obſerved, that the ellipſis of the ring appeared broader than was expected, its long axis being double the ſhort one *; for as the long axis is to the ſhort one, ſo is the radius to the ſine of the ring's inclination to a line connecting its center with the eye of the obſerver; which inclination was therefore 30 degrees, and Saturn's latitude being then about one degree, gave 31 degrees for the ring's inclination to the ecliptick.

Inclination of the ring to the ecliptick.

* Phil. Tr. N. 45. p. 900.

1136. But unleſs *Campani*'s teleſcopes were then very excellent, or at leaſt much better than *Huygen*'s, no certain concluſion can be drawn from his obſervation upon the ſhape of the ellipſis. For at the mean diſtance of Saturn from the ſun or earth, the long axis of his ring, according to *Hugenius* *, appears under an angle of 64 ſeconds, but according to Mr. *Pound*, of but 42 ſeconds *; and there is much the ſame difference of 22″ in their meaſures of Jupiter's apparent diameter, ariſing from the different goodneſs of their teleſcopes, the better of the two ſhewing the object diſtincter and freer from a falſe border of dilated light, and conſequently under a ſmaller angle. Now this border being ſeen of the ſame breadth quite round the ring, makes the ellipſis appear broader than it ſhould do in proportion to its length, and conſequently would give too great an inclination of its plane to the viſual rays. But what to allow for that border is uncertain.

* Syſtem Sat. p. 77.

* Newton's Princip. p. 392.

Saturn's phaſes explained.

1137. *Hugenius* explains the phaſes of Saturn, during his whole revolution about the ſun, by the following principles. Firſt, that the plane of the ring keeps conſtantly parallel to it ſelf. Secondly, that the convex edge of the ring reflects too few of the ſun's rays to render it viſible; the foundation of which ſhall be conſidered by and by.

Fig. 680.

1138. Admitting theſe two principles, let *abcd* be Saturn's orbit, *s* the ſun in its focus, *asc* a line drawn parallel to the plane of the ring.

1139. Firſt then, whenever Saturn arrives at *a* or *c*, he will appear round, (as if he had no ring,) to an eye in any place of the earth's orbit *txuy*. For though the diſtance between the planes of the ring, produced through the ſun, be ſo ſmall that the outward ſegments of the ſun's body will ſend ſome rays upon thoſe planes, yet by falling too obliquely they will be ſcattered too thinly to render them viſible. And though they fall more directly upon the outward edge of the ring, yet in fact it is inviſible, either becauſe it reflects too few of thoſe rays, according to the principle premiſed, or becauſe its thickneſs ſubtends too ſmall an angle at our eyes as Dr. *Jurin* has obſerved in his curious *Eſſay on diſtinct and indiſtinct viſion*, at the end of the Remarks upon this Book.

1140. Secondly, for the reaſons laſt mentioned, the ring is alſo inviſible whenever its plane produced paſſes through the earth: as when Saturn is at *e* and the earth at *t* or *u* in the line *etu* parallel to *asc*.

1141. Thirdly, the ring continues inviſible all the time that its plane produced paſſes between the ſun and the earth: as at *e*, where its dark plane is expoſed to the eye placed any where in the arch *txu* oppoſite to the ſun, and its light plane is turned from the eye; the edge being conſtantly inviſible as before.

1142. Draw *bsd* perpendicular to *asc*, and while Saturn is moving from *a* to *b*, or *c* to *d*, a ſpectator at the ſun would ſee the elliptical figure of the ring grow broader continually, till it arrives at *b* or *d*, where it would appear broadeſt; the plane of it being then inclined to the viſual rays in the greateſt angle.

1143. And to a ſpectator at the earth, the ellipſis will appear broadeſt, when a plane paſſing through the centers of the Earth and Saturn, and ſtanding upright on the plane of his orbit, becomes perpendicular to the line *asc*: for then it is alſo perpendicular to the plane of the ring, above which the eye is then elevated nearly as much as poſſible.

1144. The figures in Saturn's orbit, repreſent the ring in its parallel poſitions, the ſhaded half being below the plane of the orbit and the other above it; and the collateral figures ſhew the correſponding phaſes of Saturn at thoſe places, as ſeen from the ſun, or even from the earth. For the radius of the earth's orbit being between $\frac{1}{9}$ and $\frac{1}{10}$ of Saturn's, can ſcarce

ever

ever ſubtend a greater angle at Saturn than 6 degrees: ſo that the elliptical ſhapes of the ring, ſeen from the ſun, or from the earth, are not much different.

1145. *Hugenius* grounds the neceſſity of admitting his ſecond principle upon the appearance of that darkiſh belt, which he conſtantly ſaw upon that part of Saturn's body where the outward edge of the ring was optically projected*. Firſt becauſe the edge was as fully expoſed to the ſun as the plane of the ring. And ſecondly becauſe that dark belt could not be the ſhadow of the ring caſt upon the body. For in all his obſervations, his eye happened to be more elevated than the ſun above the enlightened plane of the ring; and conſequently could ſee nothing of the ſhadow of its outward edge. He therefore concluded that in theſe caſes the dark liſt that he ſaw was only the optical projection of the outward edge; which being thick enough to be viſible upon a light ground, would be alſo viſible upon a dark one, or againſt the ſky, if it reflected light enough; and conſequently that the Arms vaniſhed, in the caſes above mentioned, for want of a reflective power of the edge. In the round phaſe indeed, the dark plane of the ring was expoſed to his eye in a very oblique angle; and its optical projection upon Saturn together with that of the edge it ſelf, made up the whole darkiſh liſt ſeen as in fig. 674. In fig. 680 the numbers 1, 2, 3, 4, 5 ſhew the places of Saturn, and 1, 2, 3, 4, 5 the correſponding places of the Earth, when Saturn appeared to him as repreſented in fig. 673, 74, 75, 76, 77 reſpectively. And the angle $5\,5\,z$, (for example) which the viſual ray makes with $5\,z$, the ring's interſection with the plane of the orbit, being greater than the angle $s\,5\,z$, which the ſun's rays make with it, ſhews that the eye was more elevated than the ſun above the enlightened plane of the ring. The ſhort axis of the ellipſis ſeen from the ſun, is as the angle $s\,5\,z$, but ſeen from the earth, as the angle $5\,5\,z$ very nearly, becauſe the earth deviates very little from the plane of Saturn's orbit.

The grounds of the ſecond principle.

* Art. 1125.

1146. The longitude of the line $a\,s\,c$, to which the interſection of the plane of Saturn's ring and orbit is conſtantly parallel, was determined by *Hugenius* to be in 20°. 30′ of Virgo and Piſces about the year 1656. But by ſome better obſervations made in 1685, M. *Maraldi* placed it ſcarce ſo forward as 19°. 55′ of thoſe ſigns; and by comparing an obſervation of *Caſſini*'s made in 1671, with another of his own in 1715, he ſettled it more exactly in 19°. 45′ of thoſe ſigns *.

The longitude of the nodes of the ring. Fig. 680.

* Mem de l' Acad. 1715.

1147. He determined theſe places of the nodes of the ring, by biſecting the interval between Saturn's heliocentrick places, given by the obſerved times when he loſt his Arms and regained them, or when they ſeemed to be of the ſame ſize before and after their diſappearance; or elſe by biſecting the interval between one heliocentrick place and the point oppoſite to the other, when the ring was obſerved in oppoſite parts of the orbit.

1148. By obſervations in the former caſe he found that interval to be 3 degrees, and in the latter but little more than one degree; which ſhews that leſs than half a degree's elevation of the ſun's central rays above the plane of the ring, is enough to make it viſible to an eye ſufficiently raiſed above that plane, and aſſiſted with as good glaſſes as were uſed by *Caſſini* and *Maraldi* at the Royal Obſevatory.

Times of the round phaſes how found.

1149. Therefore ſince Saturn, in this part of the zodiack, deſcribes a degree in a month, it follows that his ring is inviſible for want of the ſun's rays, no more than 15 days before, or 15 days after, its arrival at 19°. 45′ of Virgo or Piſces. The times when this round phaſe ſhall happen may therefore be readily found by the tables of Saturn's motion, or even by an Ephemeris; and ſo may the times when the plane of the ring produced will paſs through the earth, by ſeeking when Saturn's geocentrick place ſhall be in 19°. 45′ of Virgo or Piſces; and likewiſe the times when the ring will appear moſt open, *viz.* when its geocentrick place is in 19°. 45′ of Gemini and Sagittarius, that is, 90 degrees from its nodes.

And which plane of the ring is enlightened.

1150. It may not be amiſs to obſerve, that the ſun enlightens the northern plane of the ring during Saturn's heliocentrick motion from 19°. 45′ of Virgo to the oppoſite point, and the ſouthern plane during its paſſage through the other ſix ſigns.

A ſuſpicion that the ring turns round an axis. Mem. de l' Acad. 1715.

1151. In Oct. 1714 when the earth was very near to the plane of the Ring produced, and was moving towards it, M. *Maraldi* obſerved, while the Arms were decreaſing night after night, both in length and breadth, that the eaſtern Arm appeared a little larger than the other for three or four nights, and yet vaniſhed firſt. For after two nights interruption by clouds, he ſaw the weſtern Arm alone, but never after, becauſe the next night was cloudy, and on the night after that (viz. the 14th of Octob.) Saturn appeared quite round and continued ſo ſome months.

1152. This inequality of the arms gave him grounds to ſuſpect, that ſome parts of the ring or even the whole were not bounded by planes exactly parallel; and that the larger Arm did not really diſapear firſt, but in the interval between the obſervations, was transferred into the place of the ſmaller, by a circular motion of the ring about an axis perpendicular to its planes. But whether the globe of Saturn turns round an axis or not, it does not appear by any obſervations that I have yet ſeen.

Diſcovery of 4 ſatellites more.

1153. Every one knows that Saturn has five ſatellites; that which *Hugenius* diſcovered in 1655, is the outermoſt but one; all the reſt were diſcovered by Signor *Caſſini*: the outermoſt of all, and alſo the middlemoſt, or the next within the Hugenian, in 1671, and the two inner ones in 1686, which he diſcovered with tubes of 100 and 136 feet, but afterwards could ſee all five with a 34 foot tube. He called them *Sidera Lodoicea* in honour of *Louis le Grand*, in whoſe Reign and Obſervatory they were firſt diſcovered.

In

1154. In 1659 M. *Huygens* publiſhed, in his *Syſtema Saturnium* a table of the mean motion of his own ſatellite, which Dr. *Halley* found by his own obſervations in 1682 to be conſiderably run out; and therefore preſented the publick with a new table computed from correcter Elements *. He alſo reformed *Caſſini*'s tables of the mean motions of all theſe ſatellites *, and about the year 1720 he publiſhed them a ſecond time, farther corrected from Mr. *Pound*'s obſervations made with the Hugenian teleſcope *. Here he takes notice that the four inner ſatellites deſcribe their orbits very nearly in the plane of the ring produced, which he affirms to be parallel to our equator as to ſenſe, *quicquid in contrarium proferant nonnulli*; and that the orbit of the 5th ſatellite is ſituated a little wide of the reſt, according to ſome late obſervations by M. *Caſſini* the ſon. Tables of their motions.

* Phil. Tranſ. abrid. Vol. 1. p. 370.
* Ibid p. 376.
* Ibid Vol. 4. p. 223.

1155. The diſtance of the Hugenian ſatellite from Saturn's center, has been commonly obſerved to contain 8 ſemidiameters of the ring, but Mr. *Pound* with his micrometer applyed to the Hugenian teleſcope, found it to be 8,7 *. Hence the diſtances of all the reſt, in the table annext, were deduced from their periodical times, according to the known relation among them, (that the cubes of the diſtances are as the ſquares of the periodical times,) and are found agreeable to the obſerved diſtances. So much for Saturn.

Satel.	Periodical times				Diſtances.
1.	1d.	21h.	18′.	27″.	2, 10
2.	2.	17.	41.	22.	2, 69
3.	4.	12.	25.	12.	3, 75
4.	15.	22.	41.	14.	8, 70
5.	79.	07.	48.	00.	25, 35

* Newt. Princip. p. 392.

1156. The following proportions of the mean diſtances of the earth and planets from the ſun, were alſo deduced from their periodical times; according to the law juſt mentioned, diſcovered by *Kepler* and demonſtrated by Sir *Iſaac Newton*.

Periodical times in days and decimal parts of a day.

Saturn.	*Jupiter.*	*Mars.*	*the Earth*	*Venus*	*Mercury.*
10752,275,	4332,514.	686,9785.	365,2565.	224,6176.	87,9692.

Mean diſtances from the ſun.

954006.	520046.	152369.	100000.	72333.	38710.

1157. By comparing theſe proportions of the diſtances of the ſun and planets, with the proportions of their apparent diameters meaſured by a micrometer, we have the proportions of their real diameters, as follow.

The

Real diameters of the Sun and Planets.

The Sun.	*Saturn.*	*Jupiter.*	*Mars.*	*the Earth.*	*Venus.*	*Mercury.*	*Moon.*
10000.	790.	996.	57.	109.	112.	40.	30.

1158. For, the Sun's apparent diameter at his mean distance from us is $32'.12'' = 1932$.

1159. And the Earth's apparent diameter, supposed to be seen from the sun at the same mean distance, is $21''$, as being double the sun's horizontal parallax of $10''\frac{1}{2}$, which by many repeated observations Dr. *Halley* found to be not greater than $12''$ nor less than $9''$ *. Therefore the sun's real diameter is to the earth's as $1932'' : 21'' :: 10000 : 109$.

* Art. 875.

1160. Sir *Isaac Newton* collected * from Mr. *Pound's* observations made with a micrometer applyed to the Hugenian telescope of 123 feet, that an observer at the sun would see Saturn at his mean distance, under an angle of $16''$, and Jupiter under an angle of $37''$; and consequently would see Saturn, if brought down to the earth's mean distance, under an angle of $\frac{954006}{100000} \times 16'' = 152,64096''$. Therefore the Sun's real diameter is to Saturn's, as $1932'' : 152,64096'' :: 10000 : 790$.

* Philos. Princip p. 392.

* Art. 60.

1161. By the like reduction the Sun's real diameter is to Jupiter's, as $1932'' : 192,417'' :: 10000 : 996$.

1162. When Mars was nearest to the earth, *Hugenius* found his apparent diameter did not exceed $30''$, and that the distance of Mars from the earth, was then to the sun's mean distance from us, as 15 to 41 *. Consequently Mars removed to the sun at that distance, would have appeared under an angle of $\frac{15}{41} \times 30'' = 10,9756''$. Therefore the sun's real diameter is to that of Mars, as $1932'' : 10,9756'' :: 10000 : 57$.

* Systema Saturnium p. 79.

1163. Dr. *Halley* collected * from the appearance of Venus and Mercury in the sun's disk *, that Venus seen from the sun at her mean distance, would appear under an angle of $30''$, and Mercury at his mean distance, under an angle of $20''$. Consequently Venus removed to the Earth's mean distance from the sun, would appear under an angle of $\frac{72333}{100000} \times 30'' = 21,6999''$. Therefore the Sun's real diameter is to that of Venus, as $1932'' : 21,6999'' :: 10000 : 112$.

* Phil. Transf. N. 348. abrid. by Jones Vol. 4. p 213.

* Art. 1054. &c.

1164. By the like reduction the Sun's real diameter is to Mercury's as $1932'' : 7,742'' :: 10000 : 40$.

1165. The apparent diameters of the planets, as seen from the sun at their mean distances from him, were collected from their observed diameters in this manner. Let the sun be at *s*, the earth at *e*, the planet at *p*, when its diameter was observed at the distance *pe*. By astronomical tables the angles of the triangle *sep*, and consequently the proportions of its sides are given; and also that of *sp* to *sm*, supposed to be the planet's mean distance from the sun; and from these proportions those of its apparent diameters at the distances *ep*, *sp*, *sm*, are deduced by art. 60.

Fig. 681.

1166.

1166. From the ſun's horizontal parallax of $10''\frac{1}{2}$, it follows that his diſtance from us is 9962 diameters of the earth. Becauſe in a right angled triangle, in which the earth's ſemidiameter ſubtends an angle of $10''\frac{1}{2}$ at the ſun's center, the diſtance between the centers of the ſun and earth, is to the earth's ſemidiameter, as the radius to the tangent of $10''\frac{1}{2}$, that is, as 19924 to 1. Diſtances of the Planets from the Sun.

1167. Hence the real diſtances of all the planets from the ſun, are given by the table of their proportions in Art. 1156.

1168. The moon's apparent mean ſemidiameter is $15'.38'' = 938''$, and at that time her mean horizontal parallax is $57'.12'' = 3432''$. Therefore the earth's real ſemidiameter is to the moon's, as $3432'' : 938'' ::$ 109 : 30 almoſt, which expreſſes her real diameter with reſpect to thoſe of the ſun and planets in Art. 1157. Moon's real diameter and diſtance.

1169. Hence the moon's mean diſtance from us is a little above 60 of the earth's ſemidiameters. Becauſe the radius is to the tangent of $57'.12''$ as $60\frac{1}{19}$ to 1 nearly.

CHAPTER VII.

Teleſcopical diſcoveries in the Fixt Stars.

1170. THAT the fixt ſtars have no ſenſible parallax, or, which is the ſame thing, that the earth's annual orbit (whoſe diameter a cannon ball could not deſcribe in leſs than 50 years,) would appear of no ſenſible magnitude through a teleſcope placed at a fixt ſtar, is ſuch an amazing concluſion as could not be believed, were it not ſupported by undeniable evidence. But as this is the caſe, it is no longer a wonder why the beſt teleſcopes don't at all magnify the apparent diameters of the fixt ſtars, though they diſcover vaſt multitudes that are quite imperceptible to the naked eye; and the more of them as the aperture is more enlarged to take in more light, and the eye-glaſs made flatter to render it diſtinct*. The Milky Way, which had puzzled the ancient philoſophers for many ages, was found at laſt to be nothing elſe but a prodigious number of very minute ſtars, ſo cloſe to one another that the naked eye can only perceive a whitiſh mixture of their faint lights. This was *Galileo*'s diſcovery, who found alſo that thoſe faint ſtars, which Aſtronomers call *Nebuloſæ*, appeared through his teleſcope to be ſmall cluſters of very minute ſtars. Multitudes of Teleſcopick Stars. * Art. 360.

1171. *Hugenius* in the year 1656, looking by chance through a large teleſcope, at three ſmall ſtars very cloſe to one another in the middle of Orion's ſword, ſaw ſeveral more as uſual. But the three little ſtars very near one another (marked θ by *Bayer*), together with four more, ſhone out as it were through a whitiſh cloud, much brighter than the ambient ſky: which being very black and ſerene cauſed that lucid part to appear like Lucid ſpots among the fixt Stars. Fig. 682.

like an aperture, that gave a proſpect into a brighter region. He viewed it many times, and found it continued in the very ſame place, and of the ſame ſhape as the figure repreſents, and called it *Portentum cui certe ſimile aliud nuſquam apud reliquas fixas potuit animadvertere* *.

* Syſtema Saturnium p. 8.

1172. But in the Philoſophical Tranſactions *, there is an account of a later diſcovery of five more ſuch lucid ſpots, though leſs conſiderable than this of *Hugenius*; the middle of which, we are there told, is at preſent in ♊. 19°. 00′. with ſouth latitude 28°. 45′; and that it ſends forth a radiant beam into the ſouth eaſt, as another in the girdle of *Andromeda* ſeems to do into the north eaſt. It is alſo there remarked, that "though theſe ſpots are in appearance but ſmall, and moſt of them but a few minutes in diameter, yet ſince they are among the fixt ſtars, as having no annual parallax, they cannot fail to occupy ſpaces immenſely great, and perhaps not leſs than our whole ſolar ſyſtem; in all which ſpaces it ſhould ſeem, that there is a perpetual uninterrupted day".

* N. 347. Jones's abr. Vol 4. p. 224.

New Stars.

1173. It is to the Author of theſe reflections, if I miſtake not, that we owe another curious account of what is principally remarkable in the new ſtars that have appeared and diſappeared for 150 years laſt paſt*. I will mention but one or two. " That in the chair of *Caſſiopeia* was not ſeen by *Cornelius Gemma* on the eighth of November 1572, who ſays, he that night conſidered that part of the Heaven in a very ſerene ſky, and ſaw it not: but that the next night, Novemb. 9, it appeared with a ſplendour exceeding all the fixt ſtars and ſcarce leſs bright than Venus. This was not ſeen by *Tycho Brahe* before the 11th of the ſame month; but from thence he aſſures us, that it gradually decreaſed and died away, ſo as in March 1574, after 16 months, to be no longer viſible; and at this day not the leaſt ſigns of it remain. The place thereof in the ſphere of the fixt ſtars, by the accurate obſervations of the ſame *Tycho*. was of. 9°. 17′. from the firſt ſtar of Aries, with 53°. 45′ north latitude". To this account Sir *Iſaac Newton* adds *, that in November, when it firſt appeared, it ſeemed equal to Venus in brightneſs, in December to Jupiter, in January 1573 leſs than Jupiter but bigger than Sirius, and equal to him in February and March; in April and May equal to the ſtars of the ſecond magnitude, in June, July and Auguſt to thoſe of the third, in September, October and November to thoſe of the fourth, in December and January 1574 to thoſe of the fifth, in February to thoſe of the ſixth, and in March it vaniſhed. That its colour was at firſt clear, white and ſplendid, afterwards yellow, and in March 1573 red and fiery like Mars or Aldebaran, in May of a pale livid colour like that of Saturn, which grew fainter and fainter till it vaniſhed.

* Phil. Tranſ. N. 346. Jones's Abr. Vol. 1. p. 222

* Philoſ. Princip. p. 526.

1174. " That ſuch another ſtar was ſeen and obſerved by the ſcholars of *Kepler* to begin to appear on Sept. 30. *St. Vet. Anno* 1604, which was not to be ſeen the day before; but it broke out at once with a luſtre greater

greater than that of Jupiter; and like the former, died away gradually, and in much about the ſame time diſappeared totally, there remaining no footſteps thereof in January 160$\frac{5}{6}$. This ſtar was near the ecliptick, following the right leg of *Serpentarius*; and by the obſervations of *Kepler* and others, was in 7^{s}. 20°. 00′. from the firſt ſtar of *Aries*, with north latitude 1°. 56′."

1175. Laſtly, that the ſudden eruption of ſuch another ſtar, ſhining out more than uſual, engaged *Hipparchus* to make the firſt catalogue of the fixt ſtars; that poſterity might know what changes might happen among them.

1176. We obſerved above*, from Sir *Iſaac Newton*, that thoſe comets which approach ſo near the ſun as to paſs through his atmoſphere, may be ſo much reſiſted and retarded after ſeveral revolutions, as at laſt to fall down upon the ſun with a vaſt force. And from thence he conjectures, that the ſtars we have mentioned, which ſuddenly ſhine out with very great ſplendor and then decay gradually till they vaniſh out of ſight, may now and then be ſtirred up and blaze out again by the ſhock of a comet falling down upon them. But thoſe other new ſtars, which appear and diſappear periodically, which increaſe by very ſlow degrees, and ſeldom exceed the ſtars of the third magnitude (ſeveral of which may be ſeen in the hiſtory I mentioned) he takes to be of another ſort, or at leaſt in another ſtate; which revolving about their axes, like the ſun, may expoſe their light and dark parts to us ſucceſſively. For the fixt ſtars are undoubtedly ſelf-ſhining bodys of the ſame kind as the ſun, and therefore equally ſubject to large dark ſpots or cruſts upon their ſurfaces. Becauſe the light of the ſun propagated to thoſe vaſt diſtances, and reflected back from opake bodys of no ſenſible apparent magnitudes, would be too much rarified to affect our ſenſes; as *Galileo* collected from the fainter lights of the remoter planets from the ſun, compared to the luſtre of the fixt ſtars.

The Origine of new Stars.
* Art. 1049.

1177. After ſeveral attempts by Dr. *Hook*, Mr. *Flamſtead** and others*, to determine the annual parallax of the fixt ſtars, the Honourable *Samuel Molyneux* Eſq; in the year 1725, erected at *Kew* a very accurate inſtrument, in order if poſſible to arrive at ſome degree of certainty in this difficult inquiry: in the proſecution of which he followed Dr. *Hook* in ſome reſpects, as in taking the zenith diſtances of the brighteſt ſtar in the Dragon's head at the times of its tranſits over the meridian, and alſo in the form of his inſtrument, conſtructed almoſt upon the ſame principles with the Doctor's, but executed to a degree of exactneſs vaſtly greater, and chiefly owing to the care and contrivance of Mr. *George Graham*.

An inquiry into the annual parallax of the fixt ſtars.
* Walliſii opera Vol. 3. p. 701,
* Phil. Tranſ. N. 364. Abridg. Vol. 6. p. 165.

1178. The Rev. Mr. *Bradley* Profeſſor of Aſtronomy at Oxford, who all along aſſiſted Mr. *Molyneux* in the proſecution of this noble deſign,

has obliged the publick with a very accurate hiſtory of it, in a letter to Dr. *Halley**; containing not only an account of ſeveral new and ſurpriſing phænomena that attended the obſervations, (which he therefore continued and repeated after Mr. *Molyneux*'s deceaſe,) but alſo a compleat diſcovery of the true cauſe of them; which at laſt enabled him to ſettle the point in queſtion, and to draw from it ſome admirable conſequences relating to the propagation of light. As I look upon theſe diſcoveries to be ſome of the fineſt that we have had ſince the invention of teleſcopes, I will endeavour to give the ſubſtance of them in as clear a manner as I can.

*Phil. Tranſ. N 406. Abridg Vol. 6. p 167.

Some account of the obſervations and phænomena.

1179. The reſult of the obſervations upon the bright ſtar in the dragon's head, marked γ by *Bayer*, was this.

Beginning from December 3. 1725, its diſtance from the zenith being taken ſeveral days, at the time of its *tranſit* over the meridian, there appeared no material difference in the obſervations.

1180. On Decem. 17. it paſſed a little more ſoutherly from the zenith than before, and ſtill more on the 20th; which was matter of ſurpriſe, both becauſe no ſenſible alteration of parallax could ſo ſoon be expected in this ſtar at that time of the year, and becauſe it was the contrary way to what it would have been, had it proceeded from an annual parallax.

1181. About the beginning of March 1726, the ſtar was found to be 20″ more ſoutherly than at the time of the firſt obſervation, and ſeemed to have arrived at its utmoſt limit ſouthwards.

1182. By the middle of April it appeared to be returning back again towards the north, and about the beginning of June it paſſed at the ſame diſtance from the zenith as it had done in December when it was firſt obſerved.

1183. From that time it continued to move northwards till September following, when it again became ſtationary, being then near 20″ more northerly than in June, and no leſs than 39″ more northerly than it was in March.

1184. From September it returned towards the ſouth, till it arrived in December at the ſame ſituation it was in at that time twelve months, allowing for the difference of declination on account of the preceſſion of the equinox.

1185. By the like obſervations made upon a ſmall ſtar almoſt oppoſite in right aſcenſion to γ *Draconis*, and at about the ſame diſtance from the north pole of the equator, it appeared to change its declination 19″, that is about half as much as γ *Draconis* did in the ſame time. Which plainly proved, as Mr *Bradley* obſerves, that theſe apparent changes were not owing to a nutation of the earth's axis, ſince the changes on this account would have been nearly equal in theſe ſtars, as lying near the ſolſticial colure.

Upon

1186. Upon comparing the obſervations with each other it was diſcovered in both theſe ſtars, that the apparent difference of declination, reckoned from the limits above mentioned, was always nearly proportionable to the verſed ſine of the ſun's diſtance from the equinoctial points.

1187. And that the whole difference of declination in theſe ſtars, was as the ſine of the latitude of each reſpectively.

Mr *Bradley*'s hypotheſis to ſolve theſe phænomena.

1188. After a year's obſervations upon many other ſtars, in different parts of the heavens, made with a new inſtrument ſet up at *Wanſted* in 1727, Mr. *Bradley* found out ſome other properties of their apparent motions; and after examining and rejecting two or three hypotheſes, by which he attempted to ſolve them, at laſt he conjectured that all theſe phænomena proceeded from the progreſſive motion of light and the earth's annual motion in her orbit. For he perceived, that if light was propagated in time, the apparent place of a fixt object would not be the ſame when the eye is at reſt, as when it is moving in any other direction than that of a line paſſing through the eye and object; and that when the eye is moving in different directions, the apparent place of the object would be different. I will firſt deduce ſome conſequences from this hypotheſis and then compare them with the phænomena.

Some conſequences from the hypotheſis Fig. 683.

1189. If an eye moves uniformly in a ſtraight line from *a* to *b* in the time that the light of a fixt ſtar deſcends uniformly in a ſtraight line from *c* to *b*, the ſtar will appear in a direction conſtantly parallel to *ac*.

For conceiving the eye to carry the line *ac* parallel to itſelf, its interſection with the fixt line *bc* will move uniformly* from *c* to *b*, and will therefore accompany a particle of light deſcending uniformly from *c* to *b*; and becauſe this interſection is a moving point, not only in the fixt line *bc*, but alſo in the moving line *ac*, it is plain that the particle which accompanies the interſection *c*, moves relatively in the moving line *ac*. In like manner a particle of every other ray, parallel to *cb*, which the moving line *ac* ſucceſſively meets with, moves alſo in the moving line *ac*; and thus a ſucceſſion of theſe particles, moving along *ac*, conſtitute a viſual ray in whoſe direction the ſtar appears.

* Euclid VI. 2.

1190. Hence ſuppoſing the earth's center *b* to move uniformly in a circular orbit ♈ ♉ *Ab*, round the ſun in its center *B*; if in a line *BC* drawn towards a fixt ſtar, ſuppoſed infinitely diſtant, you take a diſtance *BC* in proportion to *Bb* or *BA*, as the velocity of light to the velocity of the earth's center, an obſerver upon the earth at *b*, will conſtantly ſee that ſtar in a direction very nearly parallel to a line *AC*, connecting the point *C* with a point *A* in the orbit conſtantly 90 degrees behind the earth. Fig. 684.

For drawing *ba* and *bc* parallel to *BA* and *BC* reſpectively, and tending the ſame ways from *b* and *B*, and alſo any line *ac* parallel to *AC*; by the ſimilar triangles *bca*, *BCA*, we have *bc* : *ba* :: *BC* : *BA*, as the velocity of light to the velocity of the earth. Conſequently if an eye

be ſuppoſed to move along the tangent *ab* with this latter velocity, it will ſee the ſtar in a direction conſtantly parallel to *ac** or *AC*. But the eye in the orbit moves with that velocity, and paſſes by the point *b* in the direction of that tangent, and therefore at that paſſage it ſaw the ſtar in the ſame direction in which the other eye in the tangent ſees it conſtantly. The earth's diurnal motion alters this concluſion ſo little that I need not here conſider it.

* Art. 1189.

1191. Therefore the apparent parallax of the ſtar to an obſerver at *b*, is conſtantly meaſured by the angle *ACB*, if the point *A* be always 90 degrees behind the earth at *b*, and conſequently 90 degrees before the ſun's apparent place ⊙ in the ecliptick.

1192. Hence the apparent latitude of any ſtar, ſuppoſed infinitely diſtant, will be leaſt of all when the ſun's place in the ecliptick is 90 degrees forwarder than the ſtar's; and from that time it will increaſe for half a year, and then decreaſe for the next half year, and its increment reckoned from theſe limits will be conſtantly as the verſed ſine of the ſun's longitude reckoned from his place before mentioned.

Fig. 685.

For drawing *CD* perpendicular to the plane of the orbit, join *AD*, and draw *DB* cutting the orbit in *L* and *M*. The point *L*, neareſt to *D*, is the ſtar's place in the ecliptick, and the point ⊙, oppoſite to *b*, is the ſun's place therein. Now when the point ⊙ was at *N*, 90 degrees forwarder than *L*, the point *A*, being always 90 degrees forwarder than ⊙ *, was at *M*, the fartheſt point from the perpendicular *CD*; and conſequently the ſtar's apparent latitude, always meaſured by the angle *CAD*, was then the leaſt poſſible.

* Art. 1191.

Draw *AE* perpendicular to *LM*, and joining *CL*, *CE*, *CM*, draw *MF* perpendicular to *CE* produced. Then conceiving the point *A* to move in the perpendicular *AE* towards *E*, the angle *CAD* will approach to a *maximum* *CED*, and therefore will increaſe very little, eſpecially as the angular approach *ACE* is exceeding ſmall. Therefore inſtead of the apparent latitude *CAD* we may take *CED*, and conſequently the angle *ECM* for the increment of the leaſt latitude *CMD*: now this ſmall angle *ECM* is as its ſine *MF* or (becauſe the ratio of *MF* to *ME* varies very little) as *ME* the verſed ſine of the arch *MA* equal to *N* ⊙, the ſun's longitude from *N*, 90 degrees forwarder than the ſtar's place *L*.

1193. When a ſtar is ſituated any where in the ſolſticial colure, the increments or decrements of its latitude and declination are the very ſame quantities; and therefore if the ſtar be ſuppoſed infinitely diſtant, and its longitude be in the beginning of Capricorn, with north declination, its apparent declination will be the leaſt at the time of the vernal equinox, and the greateſt at the autumnal; and its increments and decrements reckoned from theſe limits, will be proportionable to the verſed

ſine

ſine of the ſun's longitude reckoned from the equinoctial points: which agrees with the phænomena in Art. 86.

1194. The whole apparent parallax LPM of a ſtar in the pole of the ecliptick, is to the whole apparent parallax LCM in the latitude of any other ſtar, as the radius to the ſine of the latitude CBL. For ſince BP equals BC, drawing BF and BG perpendiculars to CL and CM, the ſmall angle BPL is to BCL as BL to BF, that is, as the radius to the ſine of the angle BLF or of the latitude CBL*. Again, the ſmall angle BPM is to BCM as BM to BG, that is, as the radius to the ſine of BMG or of the latitude CBL, as before. Therefore the whole angle LPM is to LCM as the radius to the ſine of the latitude CBL. Fig. 686.

* Art. 204.

Hence, from the obſerved parallax in latitude, or in declination of ſuch ſtars as lye in or near the ſolſtitial colure, we have the parallax that would belong to a ſtar in the pole of the ecliptick, which is plainly the greateſt of all. Thus in γ *Draconis* whoſe latitude $CBL = 74°.\ 58'.\ 20''$, the obſerved parallax LCM was $39''$*, and thence the greateſt parallax LPM comes out $40'',4$. Likewiſe in the little ſtar above mentioned, whoſe diſtance from the north pole of the equator is $38°.\ 28'.\ 35''$*, and conſequently its latitude a little above $28°.\ 02'.\ 25''$, as being almoſt oppoſite in right aſcenſion to γ *Draconis*, the obſerved parallactick angle LCM was $19''$*, and thence LPM comes out $40'',4$ as before.

* Art. 1183.

* The 36th. Star *Camelopardi Hevelii* in *Flamſtead's* Catalogue.

1195. Mr. *Bradley* having applyed his obſervations upon the parallax in declination of ſtars in any ſituation whatever, to his theory farther purſued, aſſures us they all conſpire to prove, that the greateſt parallax LPM is about 40 or 41 ſeconds, and thinks the *medium* $40''\frac{1}{2}$ cannot differ ſo much as one ſecond from the truth.

* Art. 1185. The greateſt apparent parallax.

1196. Hence the velocity of ſtar-light comes out 10210 greater than the velocity of the earth's mean motion round the ſun. For the former velocity is to the latter as BP to BL or BM*, that is as the radius to the tangent of BPL or $BPM = 20''\frac{1}{4}$ as above determined.

The Velocity of Star-light.

* Art. 1189.

1197. From what has been ſaid Mr. *Bradley* infers. 1. That the lights of all thoſe ſtars arrive at the earth with equal velocities. 2. That unleſs their diſtances from us are all equal, (which for other reaſons beſides that of their different luſtre, is highly improbable) their lights are propagated uniformly to all diſtances from them. 3. That the velocity of ſtar-light is ſuch as carries it through a ſpace equal to the ſun's diſtance from us in $8'.\ 13''$, (this time being to the time in which the earth might deſcribe that diſtance, with the velocity of her mean motion round the ſun, as 1 to 10210, and this latter time, to half a year, as the diameter of a circle to its circumference.) 4. That the time ſo determined can ſcarce differ 5 or 10 ſeconds from the truth, which is ſuch a degree of exactneſs as can never be expected from the eclipſes of Jupiter's ſatellites. 5. That as this determination of the velocity of ſtar-light, comes out a *medium* among

Some properties of Light.

mong ſeveral determinations of the velocity of the ſun's light reflected from thoſe ſatellites, we may reaſonably conclude that the velocities of theſe lights are equal. And laſtly, ſince it is highly probable that the velocity of the ſun's emitted light is alſo equal to that of ſtar-light *, it is equally probable that its velocity is not altered by reflection into the ſame medium.

* Art. 1176.

1198. From Art. 1190, 1194. &c, it follows plainly, that a ſtar placed in the pole of the ecliptick would appear in a years time, to deſcribe about the pole a little circle whoſe apparent ſemidiameter is $20'' \frac{1}{4}$; and that any other ſtar will appear to deſcribe, about its true place, an ellipſis whoſe long axis is at right angles to the circle of longitude paſſing through the ſtar's true place, and equal to the diameter of the little circle juſt mentioned, and whoſe ſhort axis is to the long one, as the ſine of the ſtar's latitude to the radius.

Real parallax of the Stars inſenſible.

1199. Upon this theory farther purſued Mr. *Bradley* proceeds ſynthetically, by aſſuming the *maximum* of apparent parallax as determined above, and calculating tables of the differences in declination of *γ Draconis* ſituated near the ſolſtitial colure, and of *y Urſæ Majoris* nearer to the equinoctial than the ſolſtitial colure; and by comparing the tables with his obſervations, he found they agreed together throughout the year, ſometimes in the very ſame number of ſeconds, and that in 50 or 60 obſervations of each ſtar, they never differed ſo much as two ſeconds; allowing for the variation of declination cauſed by the regreſſion of the equinoctial points: which amounts to a phyſical demonſtration of the truth of his theory, and does in conſequence afford a very ſatisfactory anſwer to the point in queſtion, concerning the real parallax and diſtance of the fixt ſtars. As to which he believes he may venture to ſay, that the real parallax in either of the ſtars above mentioned does not amount to $2''$, being of opinion that if it were $1''$ he ſhould have perceived it in the great number of obſervations that he made eſpecially upon *γ Draconis*; which agreeing with the theory, without allowing any thing for a real parallax *, nearly as well when the ſun was in conjunction with, as in oppoſition to this ſtar, it ſeemed to him very probable, that its real parallax is not ſo great as one ſingle ſecond; and conſequently that it is above 400000 times farther from us than the ſun.

* Art. 1189. 1190. &c.

Some account of Mr. *Molyneux*'s parallactick teleſcope.

1200. As the agreement of the obſervations with one another and with the theory, to the degree of exactneſs here mentioned, may ſeem incredible to perſons unacquainted with the ſtructure of the inſtruments here made uſe of; having by me a memorandum that I took of Mr. *Molyneux*'s when I ſaw it at *Kew*, I will endeavour to give the reader a general Idea of it; ſuch at leaſt as may ſatisfy his curioſity till Mr. *Bradley* ſhall be pleaſed to favour us with a fuller deſcription both of this and his own.

1201. The teleſcope *ab* was ſuſpended in a vertical poſition by two poliſhed

poliſhed cylinders *c*, *d*, fixt near the top of the tube, ſo that their common axis, if produced through the tube, would paſs at right angles to its axis, through a point near the center of the object glaſs. When the teleſcope was turned upon theſe cylinders, its axis of viſion moved like a pendulum in the plane of the meridian; while a fine long wire *abe*, whoſe loop was put over one of the cylinders, hung down by the ſide of the tube, being gently ſtretched by a plumbet immerſed in a veſſel of water, deſigned to retard its vibrations. The lower end of this wire played gently againſt the ſmooth ſide of a ſlender braſs plate *fg*, fixt tranſverſely to the ſide of the tube, ſo as to point northwards and ſouthwards; and in the middle of this plate was punched a very fine round hole at *b*, rather broader than the thickneſs of the wire *ab*. The teleſcope was gradually moved upon its axis of ſuſpenſion by the preſſure of a long ſkrew *hi*, in a direction parallel to *fg*; the ſkrew worked in a hole of a plate *kl*, fixt to the wall of the houſe, and the tube was made to bear againſt the end of the ſkrew by a weight *m* fixt to the end of a ſtring *mno* paſſing over a pulley *n*, and having its other end tyed to a hook, fixt at *o* in the ſide of the tube. The oppoſite end of the long ſkrew *hi* was fixt, like an axis, in the center *i* of a braſs wheel *pqr*, whoſe circumference was divided into a convenient number of equal parts, while an index *rs* fixt to the board *kl*, pointed to the diviſions of the wheel. Fig. 687.

1202. Things being thus prepared, while the wheel *pqr* was gently turned, the part of the wire near *b*, which played againſt the tranſverſe plate *fg*, was viewed through a double microſcope *tu*, till the little hole in this plate appeared to be biſected by the wire. The teleſcope being thus rectified immediately before the beginning of every obſervation of the tranſit of γ *Draconis*, (which paſſes very near the zenith of *Kew*,) and the diviſion of the wheel over againſt the index *rs* being then noted, the wheel was turned again, till the interſection of the wires in the focus was brought to touch the ſtar at the inſtant of its tranſit. Then by the number of the revolutions of the wheel and the parts of a revolution that had paſſed by the fixt index *rs*, the angular motion of the axis of the teleſcope was eaſily collected, from a proper table of minutes and ſeconds anſwering to thoſe revolutions. Now the differences of theſe angles found at different obſervations, are the differences of the ſtar's declination. And the inſtrument being rectified every time as above, it is eaſy to underſtand that theſe angular differences could not be altered by any warping, ſhrinking or ſwelling of the materials of the inſtrument.

1203. Mr. *Bradley*'s inſtrument has the addition of a divided arch of $12\frac{1}{2}$ degrees in the place of the plate *fg*; and though its radius is but $12\frac{1}{2}$ feet, which is but half that of Mr. *Molyneux*'s, yet he is ſatisfied, from all the trials he has made, that when it is carefully rectified as above, its ſituation may ſecurely be depended on to half a ſecond.

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THE
AUTHOR'S REMARKS
UPON THE WHOLE WORK.

TO ſhorten the way to the principal concluſions in the foregoing work, I forbore to mention many things not unworthy of notice; but being not neceſſary to the train of reaſoning, for greater perſpicuity I choſe to ſeparate them from it; and to put them together under the title of Remarks, to be peruſed or paſſed by at the pleaſure of the reader.

Having begun a new numeration of the additional figures for the Remarks, I quote them thus, Fig. 1. 2. &c. and the former figures thus, FIG. 1. 2. &c. which I omit to quote whenever the Remark relates to the figure belonging to the article it ſelf.

UPON BOOK I.

Chapter 1. *Concerning Light.*

Upon ART. 4.

1. By ſome authors the angle *ACP* is called the angle of inclination, and *ECQ* the refracted angle, and their difference *ECG* the angle of refraction; which for greater diſtinction may be called the angle of deviation.

Upon ART. 13.

2. *Hugenius* gives us the following hiſtory of the diſcovery of this law[a]. "The refraction of rays at the ſurfaces of tranſparent bodies, is a thing taken notice of by the ancients. For *Ariſtotle* has a problem concerning the apparent curvity of an oar in water; and *Archimedes* is ſaid to have written a book about the appearance of a ring or circle under water; in which no doubt he conſidered the refraction of the rays and the fallacy of ſight thence ariſing. *Alhazen* the Arabian and *Vitellio* tell us that the Angles of incidence and refraction are in a given ratio, and thought they had pretty well proved it by experiments; but this proportion being found erroneous in large angles, the moderns began to examine the matter more ſtrictly. *Kepler* among the reſt made ſeveral experiments about it, but miſſed his aim[b]; nevertheleſs his conjectures and attempts became uſeful to others. After the invention of teleſcopes, the ſubject of this enquiry, being thought more valuable than before, was farther purſued; and *Willebrordus Snellius* after many troubleſome experiments and attempts, at laſt found out the truth; but ſtill he did not throughly comprehend his own invention.

3. It was this. Suppoſing the ſurface of water to be *AB*, and an object under it at *D*, which to the eye at *F* appeared as it were in the line *FC*; he produced this *FC* till it met in *G* with he perpendicular *DA* to the ſurface *AB*. Then he affirmed that the image of the object *D* appeared at *G*; and that *CD* was to *CG* in a certain given ratio, as of 4 to 3, if the fluid was water. This is very true and agrees perfectly with our 5th law. Becauſe, by the known properties of the triangle *CDG**, the ſide *CD* is to *CG*, as the ſine of the angle *DGC* or *AGC* or *HCF*, to the ſine of *CDG* or *DCE*; which are the angles of incidence and refraction. * Art. 221.

4. But yet *Snellius* never imagined that this was the ratio of the ſines. For being prepoſſeſſed with a notion that all depended upon the apparent image; even of the perpendicular *HC* he thought there was a refraction, or as he falſly calls it a decurtation or ſhortening of the viſual ray: being led into the miſtake by obſerving that the whole bottom of a veſſel, filled with water, ſeemed to riſe up when viewed from above. But the true cauſe of this appearance is to be gathered from the tendency of the rays to both the eyes. [Here our author himſelf has made a ſlip as well as *Snellius*. For the bottom of the veſſel ſeems as much elevated to one eye as to both, and the true cauſe of its elevation is explained in Art. 139, 145 and 146.] I have ſeen the whole book which *Snellius* wrote upon this ſubject, though not yet publiſhed; and am told that *Des Cartes* ſaw it too; and perhaps it was from hence he found that the true meaſure of refractions was to be taken from the ſines of the angles: and this he applyed very ſucceſsfuliy to the explication of the rain-bow, and the determination of proper figures of glaſſes to refract rays to a given point. So far *Hugenius*.

Upon Chapter 2. Concerning Glaſſes.

Upon ART. 42.

5. I forgot to obſerve that FIG. 59 and 74 repreſent the refractions of a ray paſſing through a ſphere placed within a medium denſer than it ſelf.

Upon ART. 48.

6. line 1. read thus On the contrary, if rays be returned directly back from the focus *F* in convex glaſſes, and towards it in concaves, the &c. See the end of that article.

Upon ART. 60.

7. Read the title of this article thus. Small angles ſubtended by the ſame perpendicular, are reciprocally as its diſtances from the angular points.

Upon Chapter 3. Concerning the eye and manner of viſion.

Upon ART. 85.

Variations of figure in perfect eyes.

8. If an object be viewed diſtinctly and ſucceſſively at three different diſtances from the eye; the firſt of which may be the leaſt diſtance at which it can be viewed diſtinctly, the ſecond double the firſt, and the third infinite; it is remarkable that as great alterations in the figure of the eye are neceſſary for ſeeing the object diſtinctly at the firſt and ſecond diſtances, whoſe difference is but ſmall, as at the ſecond and third, whoſe difference is infinite. For let *BCDE* be the axis of the eye infinitely produced; *BC*, *BD*, *BE*, the three diſtances of the object from the cornea *AB*; and *CA*, *DA*, *EA*, three rays falling upon any given point of the cornea; whereof *EA* is parallel to the axis.

Fig. 2.

9. Now to procure diſtinct viſion of the points *C*, *D*, *E*; it is plain that every one of the rays *CA*, *DA*, *EA* muſt be ſucceſſively refracted to the ſame point *F* upon the retina, where it is cut by the eye's axis. At firſt let us ſuppoſe the point *F* to be given, or the length of the axis *BF* to be immutable; and then the quantity of the refraction of each ray muſt be varied. And becauſe the diſtance *CD* is ſuppoſed equal to *CB* or *CA*, the angle *CAD* is equal to *CDA* and conſequently to *DAE*. Therefore conceiving each ray to come back again from the fixt point *F*, to the points *C*, *D*, *E* ſucceſſively; the whole quantity of its refractions muſt firſt be leſſened by the angle *CAD*, and then by the equal angle *DAE*; and ſo the changes of the figures of the refracting ſurfaces muſt be much the ſame when the object is removed from *C* to *D*, as when it is removed from *D* to *E*.

Fig. 3.

10. Secondly, let all the refracting ſurfaces be ſuppoſed to keep their figures immutably; and let *F* be their principal focus, that is, of rays falling parallel upon the cornea after refraction through them all; and likewiſe *G* their other principal focus, that is, of rays falling parallel upon the backſide of the chryſtalline and refracted through them all. I find by computation that *BG* is but 5 or 6 tenths of an inch, and therefore if we make *GC* equal to *CD*, reckoning the diſtances of the object from *G* inſtead of *B*, the preſent caſe will not be much different from the former. Now let a pencil flowing from *C* be refracted towards *c*, and another from *D* towards *d*; and by art. 373 we have *Fc* reciprocally as *GC*; (as if the refractions were only made through a lens [a];) that is, *Fc* : *Fd* : : *GD* : *GC* : : 2 : 1; that is, the variations *cd* and *dF* of the places of the retina are equal to each other while the diſtance *GC* varies from ſingle to double and then to infinite.

[a] Art. 240.

11. Laſtly if we ſuppoſe diſtinct viſion to be ſucceſſively procured by a variation partly of the place of the retina, and partly of the figures of the refracting ſurfaces, it is eaſy to apprehend that the variations of all theſe parts taken together muſt ſtill be equal in both the caſes abovementioned. As to Mr. *Huygens*'s opinion that the cryſtalline may approach towards the cornea for ſeeing near objects diſtinctly; I formerly computed, from ſuch meaſures of the eye as I then had, that if the cryſtalline could move ſo far as to touch the cornea, even this motion would be too little for the purpoſe aforeſaid. But even this is obſtructed by the Uvea, which lyes much nearer to the cryſtalline than to the cornea, according to M. *Petit*; who finds alſo that the ſurface of the uvea in human eyes, is not ſpherical but plane. See Mem. de l' Acad. 1728. p. 206. 4to.

In ſhort-ſighted eyes.

12. Hence if ſhort ſighted perſons can read a ſmall print diſtinctly at two different diſtances, whereof the larger is but double the leſſer, which I believe moſt of them can do; it follows that as great alterations of figures are made in their eyes as in perfect eyes, that can ſee diſtinctly at all intermediate diſtances between infinity and the larger of thoſe two. And this is the reaſon that a ſhort ſighted perſon can ſee diſtinctly at all diſtances with one ſingle concave of a proper figure; which otherwiſe muſt have been differently figured for different diſtances.

13. It follows then that the cauſe of ſhort ſightedneſs, is not a want of power to vary the figure of the eye, and the quantity of refractions; but

but that this whole quantity is always too great for the diſtance of the retina from the cornea.

Dr. Pemberton's opinion upon theſe variations in the eye.

14. The ingenious and learned Dr. *Pemberton* upon taking his degree at *Leyden* has printed a diſſertation upon this ſubject; and in the dedication of it to Dr. *Mead*, has declared his opinion to be this. That the cryſtalline changes its figure in ſuch a manner, that as one ſurface becomes rounder, the other becomes flatter; but which of the two ſurfaces becomes rounder for viewing near objects, and then flatter for remote ones, he ſays, he had not then diſcovered. But I could never ſatisfy my ſelf of the certainty of the phænomenon he builds upon. For though the experiment was carefully made in the manner he directs, both by my ſelf and one or two of my friends, yet none of us met with the expected ſucceſs. Poſſibly I might have repeated it again, had I not ſince been appriſed that the Doctor's concluſion could not ſtand good, although the experiment were to ſucceed ever ſo well; inaſmuch as the whole tenour of his reaſoning depends upon this fundamental ſuppoſition: viz. That in viewing an indiſtinct object the pupil of the eye is always contracted to the leaſt ſize of which it is capable. For the contraction of the pupil does not depend upon the indiſtinctneſs of the object only, but alſo upon the degree of light.

Mr. *de la Hire's* opinion. Diſſertation ſur les differens accidens de la Vüe. Mem. a Pariſ. 1694. Fig. 4.

15. Mr. *de la Hire* is the only writer I have ever ſeen who maintains, that for viewing objects diſtinctly at different diſtances from us, no variations are ever made in any part of the eye except in the ſize of the pupil. The experiment he builds his concluſion upon may be made in the following manner.

16. Let two very ſmall holes, *a*, *b*, be made with a pin in a card or in paper, ſo near to each other, that being held cloſe to the eye, the rays that come through both, may enter the pupil. Then let a ſmall black ſpot *c* upon a white paper, be viewed through them; and if the experimenter be ſhort ſighted, let the paper be placed at ſuch a diſtance from his eye, as he uſually ſees an object at with moſt diſtinctneſs and moſt eaſe, as ſuppoſe at the diſtance of ſix inches; and in looking through the holes the ſpot will appear diſtinct and ſingle. Then let the paper be removed to a greater diſtance, ſuppoſe of ten inches, ſuch as that the ſame eye may be able to ſee an object without any apparent indiſtinctneſs. Then let the ſpot be attentively viewed by the naked eye, in order to make ſuch a change in its conformation, as is uſually ſuppoſed neceſſary to ſee an object diſtinctly at ſuch an increaſed diſtance. Now, the eye being ſuppoſed to have taken the neceſſary conformation for ſeeing that ſpot diſtinctly at that diſtance, it may in conſequence of this ſuppoſition be expected, that upon clapping the card before the eye and looking through the two holes, the ſpot ſhould appear ſingle, as it did at the former diſtance of ſix inches. But experience ſhews the contrary: for it appears double like two diſtinct ſpots, *d*, *e*; whoſe interval *de* is ſo much the greater as the diſtance of the paper from the eye is greater, as in fig. 5. Fig. 5.

17. If the experimenter be long ſighted, let the paper, likewiſe, be firſt placed at ſuch a diſtance from the eye, as he uſually ſees an object at with moſt diſtinctneſs and moſt eaſe, as ſuppoſe at the diſtance of fifteen inches; and in looking through the holes the ſpot will appear diſtinct and ſingle. Then let the paper be brought nearer, ſuppoſe to the diſtance of ſeven inches, at which diſtance the ſame eye is able to ſee the object without any apparent indiſtinctneſs. And let the ſpot be viewed attentively by the naked eye, in order to make ſuch a change in its conformation, as is commonly ſuppoſed neceſſary in order to ſee an object diſtinctly upon ſo leſſening the diſtance. Now, the eye being ſuppoſed to have taken the neceſſary conformation for ſeeing the ſpot diſtinctly at that diſtance, it may, in conſequence of this ſuppoſition, be expected, that upon clapping the card before the eye and looking through the two holes, the ſpot ſhould appear ſingle, as it did at the former diſtance of fifteen inches. But experience ſhews the contrary: for it appears double like two diſtinct ſpots, *d*, *e*, whoſe interval *de* is ſo much the greater as the diſtance of the paper from the eye is leſs, as in fig. 7. Fig. 6. Fig. 7.

18. Now the ſpot appears ſingle in the firſt caſe of each of theſe two experiments, becauſe the rays *ca*, *cb*, fig. 4 and 6, are united at a ſingle point *f* exactly upon the retina. But when the paper is remoter, fig. 5, *ca* and *cb* diverge leſs than before, and therefore are reunited at *f* before they arrive at the retina; then croſſing each other they fall upon it in two diſtinct points *g*, *h*, which occaſion the double appearance at *d* and *e*. For if the holes be moved upwards, the upper ſpot firſt diſappears at *d*; becauſe the upper ray *ca*, firſt miſſes the pupil. And when the paper is brought nearer, as in fig 7, the rays *ca*, *cb* diverge more than before, and therefore tend to reunite in the point *f* behind the retina, upon which they fall in two diſtinct points *g*, *h*, which occaſion the double appearance at *d* and *e*. For if the holes be moved upwards, the under ſpot firſt diſappears at *d*, becauſe the upper ray *ca* firſt miſſes the pupil.

19. We come now to conſider the conſequence which Mr. *de la Hire* draws from theſe experiments. His argument runs thus. It is commonly believed, that an eye which is ſo formed, as naturally to unite the rays upon the retina, when the object is at ſix inches diſtance, can make ſuch a change in its conformation as ſtill to unite them

them exactly upon the retina, when the object is removed to a greater distance, as that of ten inches. If this opinion were true, the eye of the observer in the second case of the first experiment*, must have made such a change in its conformation. But the experiment shews that this eye was not in such a conformation as to unite the rays exactly upon the retina; for upon clapping the card before it, the appearance was of two distinct spots, not of one only, as it ought to have been, if the eye had had the supposed conformation. And just after the same manner he reasons upon the second experiment.

*Rem. 16.

20. In order to make this reasoning conclusive, Mr. *de la Hire* ought to have proved, that whatever conformation the eye had, in viewing the spot without the holes, the same conformation must necessarily have continued, when the spot was seen through the holes.

21. But we take the contrary to be highly probable. For when the spot was viewed at the distance of six inches, the eye was then in its natural conformation. It received the rays in the same manner as an artificial eye of the same dimensions might have done, without any the least strain, *nisus* or endeavour. But when the spot was viewed at the distance of ten inches, it must at the first instant have appeared indistinct, and in order to remedy that indistinctness, the eye may be supposed to have exerted some force in order to change its conformation so as to suit it self to that distance. If so, this forced conformation will continue while the occasion remains, and no longer. While the eye is viewing the object at ten inches distance, if it happens in the least to relax and unbend it self, a sense of indistinctness will immediately begin to arise, which will serve as a monitor to return exactly to the necessary conformation; but the moment the eye ceases to view the object at that distance, it will probably depart from this forced conformation, and return to its natural conformation suited to the object at six inches distance. Therefore when the card is clapped before the eye, as it must necessarily then lose sight of the spot, before it comes to see the spot through the two holes, it may then probably depart from the forced conformation, and return to its natural state, or near it; the consequence of which is, that the rays will not now unite upon the retina, but will therefore exhibit the appearance of two spots.

22. I might here observe that Mr. *de la Hire* himself must necessarily admit one alteration in the eye at this instant of time, namely the dilatation of the pupil. Why then may not the conformation of the coats and humours as well be supposed to change at the same time.

History of the causes of vision.

Upon ART. 87.

23. Great disputes have been among the ancient philosophers, whether vision be caused by reception of rays into the eye, or by emission of them from the eye to the object. At last the latter opinion was generally received and followed by *Euclid*, *Ptolemy*, *Alhazen* and other ancient opticians; who thought it more agreeable to nature that certain Emanations, by them called visual rays, should flow from an animate substance to an inanimate one, rather than on the contary.

24. *J. Baptista Porta* is said to be the first discoverer of those pictures of external objects that appear upon the wall of a dark room, and are made by rays coming through a small hole in the oppofite wall. This he describes at large in his book *de Magia Naturali* printed in 1560, and shews by what methods the distinctness of these pictures may be promoted: and then concludes that he had not only decided the grand dispute about the reception and emission of rays, but had also found out the true cause of vision. For, says he, the image is let in through the pupil, and is painted upon the surface of the crystalline humour, which answers to the wall in the dark room, as the pupil does to the hole in the window-shutter. And here he followed the opinion of *Vitellio* and others, who imagined we began to perceive, when the crystalline was enlightened, but that the perception was not compleat till it was propagated from thence and united as it were in the optick nerve; that vision was then distinct when the rays fell perpendicularly upon the crystalline; and confused, when coming from collateral objects, they fell obliquely upon it.

25, These were the received notions about vision till about the year 1600, when *Kepler* made the grand discovery*, and shewed by his geometry in what manner the rays were refracted through all the humours of the eye, and formed a distinct picture upon the retina; in like manner as pictures are formed by a glass globe full of water. He also discovered the constitution of defective eyes; that is, how the pictures became confused, and shewed in what manner they were rendered distinct by spectacles and concave glasses, whose effects had been so much admired for two or three hundred years, and had so long perplexed the greatest wits to account for them.

* Paralip mena ad V tellionem

Inverted pictures i the eye co sidered.

26. The vulgar account of objects appearing erect notwithstanding the inversion of the pictures upon the retina, is also *Kepler*'s; who tells us that the mind perceiving an impulse of the ray *P p* (in FIG. 153) on the lower part of the retina, considers this ray as directed from an higher point of the object; and likewise perceives the impulse of the ray *R r* upon the higher part of the retina, to be directed from the lower part of the object. *Des Cartes* illustrates this solution, by conceiving a blind man to hold in his

his hands two ſticks croſſing each other, and to puſh the top and bottom of an upright object, with their extremities; and obſerves that this man will judge that to be the upper part of the object, which he puſhes with the ſtick held in the lower hand, and that to be the lower part of the object which he touches with the ſtick in his upper hand. But we ſhall ſhew hereafter that all this is quite foreign to the purpoſe.

ts in the s. 27. People in growing old, are often troubled with the appearance of dark, irregular ſpots continually flying before their eyes, like flies; eſpecially in looking at bright objects, ſuch as white paper or the ſky-light. M. *de la Hire*'s account of them is this. They are of two ſorts, ſome permanent, which in fixing the eye upon a point of an object, appear always fixt in the ſame ſituation to that point; others ſeem to fly about, and to change their ſituation though the eye be fixt. The ſhapes of both ſorts are changeable; the firſt ſort are commonly like a dark ſpot upon a white ground, the ſecond ſort appear like the knots of a deal board; ſome parts of them being very clear and ſurrounded with dark threads; they are alſo attended with long fillets of irregular ſhapes, which are bright in the middle and terminated on each ſide by parallel black threads. Sometimes after ſhaking the head ſuddenly and fixing the eye upon an object, they appear to deſcend gradually.

28. The ſpots that appear fixt in reſpect to the axis of the eye, for that reaſon muſt be cauſed by ſome diſorder in a correſponding fixt part of the retina, or in ſome part of the vitreous humour lying pretty cloſe to the retina. For an opacity of the coats or humors in any part remoter from the retina, by intercepting ſome part of the rays of every pencil, could only cauſe an uniform obſcurity or faintneſs of light in every place of the retina, and not a total defect of
rt. 281. it in any particular place[a]. An inſtance of this kind may be ſeen in the Phil. Tranſ. N°. 384, of a woman who in looking with the left eye only, at three ſhort words in print, could ſee the extreams but not the middlemoſt; and in looking with the right eye only, at the middle between four ſhort words, could ſee but three of them; one of the two middlemoſt being covered by a dark round ſpot; but in looking with both eyes ſhe could ſee them all.

29. M. de la *Hire* attributes the cauſe of theſe permanent ſpots to ſmall drops of extravaſated blood upon the retina. But he finds it more difficult to account for the volatile ſpots. When the rays of the ſun are tranſmitted through a piece of bad glaſs, and fall upon white paper, the ſhadows of the little ſands, veins and irregularities in it, appear not unlike theſe ſpots. He therefore imagines the aqueous humor is ſometimes troubled with ſome little mothery, ropy ſubſtance; ſome parts of which, by the figures of their little ſurfaces or by refractive powers, different from the humor it ſelf, may caſt their diſtinct images upon the retina He ſuppoſes them in the aqueous humor rather than the vitreous, becauſe of its greater fluidity for a freedom of deſcent, and becauſe they will then appear to deſcend; as being ſituated before the pupil or at leaſt before the place of interſection of the pencils. But if this heterogeneous mother be in the vitreous humor, it muſt be lighter than this humor, ſo that after a ſudden ſhake of the head, it may firſt deſcend a little, and then aſcend gradually, to cauſe the gradual apparent deſcent abovementioned. Theſe ſpots are obſerved to change their figures, ſometimes in two or three hours, at other times not in two or three days; and to appear more numerous at one time than another. From *de la Hire*'s *differens accidens de la vuë*.

Opacity of the cornea. 30. It ſometimes happens that the cornea grows opake; which I am told may be cured by glaſs ground to an impalpable powder and blown every day into the eye; provided the opacity does not quite go through the thickneſs of the cornea.

Opacity of the cryſtalline. 31. An opacity of the cryſtalline is called a Cataract and is cured by couching; that is by piercing the ſide of the cornea with a ſharp needle and by forcing the cryſtalline out of its place into one ſide of the eye. And then the patient is obliged to uſe a very convex glaſs, to ſupply the want of the cryſtalline.

Part of the retina inſenſible of light. 32. Monſr. L' Abbe *Mariotte* is the author of a curious experiment, ſhewing that any object, whoſe picture falls upon the baſe of the optick nerve, where it enters the bottom of the eye, is not perceived while the other eye is ſhut. He deſcribes this experiment in a letter to M. *Piquet*, as follows[b]. "Having often obſerved in diſſections of men as well as brutes, that the optick nerve does never anſwer juſt to the middle of the bottom of the eye, that is to the place where the picture of the objects we look directly upon is made; and that in man it is ſomewhat higher and on the ſide towards the noſe; to make therefore the rays of an object to fall upon the optick nerve of my eye, and to to find the conſequence thereof, I made this experiment.

b Phil. Tranſ. N°. 35.

33. I faſtened on an obſcure wall, about the height of my eye, a ſmall round paper to ſerve me for a fixt point of viſion; and I faſtened ſuch another on the ſide thereof towards my right hand, at the diſtance of about two feet, but ſomewhat lower than the firſt, to the end that it might ſtrike the optick nerve of my right eye, while I kept my left ſhut. Then I placed my ſelf over againſt the firſt paper, and drew back by little and little, keeping my right eye fixt and very ſteady upon the ſame; and being about

ten

ten feet diſtant, the ſecond paper totally diſappeared.

34. That this cannot be imputed to the oblique poſition of the ſecond paper to my eye, is hence evident, that I can ſee other objects farther extant on the ſide of it. So that one would believe the ſecond paper was taken away by a ſlight, if one did not ſoon find it again by the leaſt ſtirring of ones eye. This experiment I made often, varying it by different diſtances, and by removing or approaching the papers one to another proportionably. I made it alſo with my left eye by keeping my right ſhut, the ſecond paper being faſtened on the left ſide of my point of viſion. So that by the ſituation of the parts of the eye, it cannot be doubted but that this defect of viſion is upon the optick nerve; and the ſame experiment ſucceeds in other perſons though not exactly at the ſame diſtances.

A remarkable inference from it.

35. This experiment hath given me cauſe to doubt, whether viſion be indeed performed in the retina, (as is the common opinion,) or rather in that other membrane, which at the bottom of the eye is ſeen th ough the retina and is called the Choroïdes. For if viſion were made in the retina, it ſeems that then it ſhould be made wherever the retina is; and ſince the retina covers the whole nerve, as well as the reſt of the bottom of the eye, there appears no reaſon to me, why there ſhould be no viſion in the place of the optick nerve where it is. On the contrary if it be in the choroïdes that viſion is made, it ſeems evident that the reaſon why there is none in the nerve, is that the choroïdes parts from the edges of it, and covers not the middle thereof, as it does the reſt of the bottom of the eye.

Diſputed. a Phil. Tranſ. N°. 59.

36. *Piquet*'s objections to this inference are ſpecified in *Mariotte*'s reply [a], from which I have extracted what follows. "You ſay in your firſt objection that if the ſclerotis and choroïdes be taken away from a freſh eye, and the retina be left diſtended on the vitreous humor, one ſhall not be able to ſee well through this membrane; whence you conclude it is not tranſparent enough to let ſo much light paſs upon the choroïdes as is ſufficient for viſion. In my opinion there is reaſon to doubt of this conſequence. There being a great difference between the retina of a dead animal and a living one. Different diſpoſitions change the qualities of things. Fat, which is traſparent when melted, grows opake when cold. The tunica cornea of an eye being held ſome hours in ones hand, in a hot air grows thick and a little after opacous. Take the eye of an ox new killed, while it is hot, and cut it in two, in ſuch manner that a good part of the vitreous humor may remain extended upon the retina; then you ſhall ſee diſtinctly the colours of the choroïdes, the baſe of the optick nerve, the trunk of the little veſſels that proceed from thence, and their diſpoſition through the thickneſs of the retina; with ſo much perſpecuity that you cannot even diſcern whether there be a retina beyond the vitreous humor or no. The tranſparency of the retina will alſo be evident by this other experiment. Place by night a lighted candle near your eyes, and cauſe a dog, diſtant from the candle 8 or 10 paces, to look upon you; then you ſhall ſee in his eyes a light ſufficiently bright. Which I hold to proceed from the reflection of the candle; whoſe image is painted on the choroïdes of the dog; which having much whiteneſs and luſtre, cauſes this ſtrong reflection. For if it proceeded from the cryſtalline or the retina [or the far ſurface of the vitreous] the ſame appearance would be ſeen in the eyes of men, birds and other animals, who have the choroides black; which is not found ſo by us. It is therefore manifeſt by this experiment that the rays do paſs with much force as far as the choroïdes. The like experiment may be made with cats, in whoſe eyes this light appears blueiſh, which ſhews that it proceedeth from their choroïdes which hath much of this colour. But this colour or any other, which may be in the choroïdes, brings no confuſion to the ſenſe of ſeeing. For the ſenſes receive no impreſſion from their own organs.

37. Black bodies grow hotter in the ſun and take fire ſooner than white ones; and therefore the light acts more upon them. And this is the cauſe why men and birds ſee better and more diſtinctly than moſt other animals. For their choroïdes being black and by conſequence very ſenſible of light, they contract much their pupil or ſight-hole of the eye, which makes the rays which paſs there from each point of the object, to be all very near the axis of the cryſtalline and to reunite more exactly in a point [a], than in the eyes of moſt other animals which have their choroïdes white towards the axis of the eye and by conſequence leſs ſenſible of light: who in recompence can very much dilate the pupil of their eyes when they ſtand in need of a great light. But then their ſight is not ſo diſtinct, becauſe the rays which fall towards the edges of the cryſtalline, do interſect the axis too near the cryſtalline in their refractions [b]. a Art. 81. b Art. 75.

38. It is true that to ſupply this defect in ſome ſort, they have a little cryſtalline in the middle of the great one; and this little cryſtalline being of a more ſpiſs conſiſtence than the great one, its refraction is alſo more ſtrong [c]; and makes the rays, which come from a ſingle point in the axis produced, and paſs near the center of the cryſtalline, to be more refracted than if there had been but one cryſtalline; and brings their interſections with the axis nearer to the interſections of the rays that fell more remote [d]. c Art. 5. d Art. 73.

39. Fiſhes alſo have a double cryſtalline. For other-

otherwiſe their ſight would be more confuſed than that of other animals who live in the air. For their cryſtalline being ſpherical, if it was homogeneal, the rays of a pencil would cut the axis in points at greater diſtances from one another, than if it had been lenticular, and its convexities were of greater ſpheres.

40. I come now to a more particular proof of the want of viſion on the baſe of the optick nerve. It is agreed that in this experiment moſt men loſe ſight of an entire circle of white paper, whoſe diameter is about the 9th or 10th part of its diſtance from the eye. Now the viſual triangle whoſe baſe is the diameter of this circle and vertex is near the center of the eye, is nearly milar to the triangle, whoſe baſe is the diameter of the picture of that circle upon the retina and vertex the ſame center, where the extream rays interſect each other: the which center being 6 or 7 lines from the baſis of the optick
n. 90. nerve [e], whoſe breadth is about $\frac{1}{4}$ of a line, this baſis will alſo be about the 9th or 10th part of its diſtance from the ſaid center in the eye. And ſo the picture of the white paper will preciſely cover the baſe of the optick nerve: and becauſe the paper then wholly diſappears, it follows that the baſis of the optick nerve is inſenſible of light. Whence I conclude more accurately than before, that the choroïdes is the principal organ of ſight, and that the retina is not; ſeeing the retina is placed in that baſis, and is there apparently diſpoſed in like manner as the reſt of it upon the bottom of the eye.

41. You alledge that the trunks of the veſſels which proceed from the baſis of the nerve, may be the cauſe of this defect of ſight. Yet you cannot deny but that they are very ſmall; and that it is very hard to diſcern the little holes through which they paſs, when the nerve is cut off above its inſertion into the eye. Now becauſe they come out of the baſis by two diſtinct little holes, the diameter of each of which does not take up above an eigth part of the diameter of the baſis, it follows, if the reſt of the baſis was ſenſible of light, that we ſhould not loſe ſight of a paper of 2 inches diameter at moſt, at 10 feet diſtance; and ſometimes in fixing one eye upon a little piece of paper, two other very little ones, ſeparated one from the other, would diſappear; which is contrary to experience: for the default of viſion is continued.

42. To confirm my opinion the more, I will add ſome other obſervations. The firſt is that the pupil dilates it ſelf in the ſhade and contracts in a great light: and it is hard to account for this unvoluntary motion, but by the hypotheſis that the choroïdes is ſenſible of light. For then it is eaſy to conceive that being hurt by too much light, it may dilate or contract its fibres, which have one continuity with thoſe of the fore part of the uvea: and thus it can contract its aperture; and when it is not hurt, relax it again. Whereas if the retina be ſuppoſed the organ of ſight, it will be very difficult to explain how this contraction is made. The author's other arguments are taken from the ſtructure of the eyes of birds.

43. So far Mr. *Mariotte*; whoſe experiment I have tryed in a chamber from which all ſenſible light was excluded, except what came into it through a key-hole; and this alſo diſappeared totally when it fell upon the baſe of the optick nerve; which ſhews it to be totally inſenſible of light. Yet in looking at objects of an uniform colour with one eye, we are not ſenſible of any ſuch defect or dark round ſpot, as the woman was, whom I mentioned in the 28th remark. This defect of ſenſation having been conſtantly ſupplyed by the other eye, is now ſupplyed by the imagination only.

44. M. *Picard* has deviſed a way by which an object is loſt keeping both eyes open [a]. "Faſten againſt a wall a round white paper of the bigneſs of an inch or two; and on the ſides of this paper put two marks, one on the right, the other on the left ſide, each about two feet diſtant from the paper, and ſomewhat higher. Then place your ſelf directly before the paper at the diſtance of nine or ten feet, and put the end of your finger over againſt both your eyes, ſo that it may hide from the right eye the left mark, and from the left eye the right mark. If you remain firm in that poſture, and look ſteadily with both eyes on the end of your finger, the paper which is not at all covered thereby, will totally diſappear. Which is the more ſurpriſing, becauſe without this particular encounter of the optick nerves, where no viſion is made, the paper will appear double; as you will find when the finger is not rightly placed. See FIG. 190.

[a] Phil. Tranſ. N°. 35.

45. As to the diſpute whether the retina or the choroïde be the principal organ of ſight. I will ſubjoin a remark taken from M. *de la Hire*'s diſſertation abovementioned. "To clear up this difficulty in ſome meaſure, we muſt conſider what relates to our other organs of ſenſe; and I think by this compariſon it will be evident enough, that the retina is the principal organ of ſight, although a certain part of it be not ſuſceptible of immediate impreſſions from outward objects. I ſay then the retina is the principal organ of ſight, as being an expanſion of the optick nerve; becauſe we muſt not ſeek for ſenſations any where elſe but in the nerves. But yet this organ may receive the impreſſions of light from an intermediate organ, which receives them immediately from the object it ſelf. For this is the caſe of the organs of our other ſenſes. Whence it is evident that this intermediate organ is the choroïde, becauſe it touches and ſupports the retina

The diſpute adjuſted.

tina every where but at the baſe of the nerve; and being of a dark colour it is more capable of being agitated by the impreſſions of light, than if it was white or tranſparent. Nature acts in the ſame manner in the organ of hearing. For the labyrinth has a proper diſpoſition to receive the tremblings of the air, and to communicate them to the ramifications of the auditory nerve expanded within it. The caſe is the ſame in the other ſenſes, as M. du *Verney* has obſerved in the 96th page *de l' organe de l' oüie*. The conſtitution of the nerves is too tender and delicate to be expoſed to the naked impreſſions of external bodies; and therefore the coats of the nerves receive theſe impreſſions, and communicate them to the nerves themſelves in a proper manner for exciting ſenſations.

Beams of light ſhooting from candles.

46. This Author accounts for thoſe beams of light which appear to ſhoot upwards and downwards from a lighted candle, as follows. The circumſtances of them are theſe: while you look at the candle if you bend your head downwards, you will only ſee the upper beams, and in lifting your head upwards you will only ſee the under ones; and to ſee both the upper and under beams at once, you muſt hold your head upright and almoſt cloſe your eye-lids. To ſhew the cauſe of theſe appearances, it is to be obſerved that the eye is always moiſtened with a tenaceous ſort of water, which is gathered more plentifully to the edges of the eye-lids than any where elſe, becauſe they rub the cornea. This water ſticking to each eye-lid, is there formed into a concave ſurface, like that of water adjoining to the ſides of a drinking glaſs: and then the rays which flow from the candle *B*, in paſſing through this concave at the upper eye-lid *H*, (which when the head is bowed downwards, is below the upper margine of the pupil) are there refracted upwards from the thinner towards the thicker part of the water[a]; and therefore in entering the pupil they diverge from one another[b], and fall upon the upper parts of the retina, as if they had come from a long beam *B N* below the candle *B*. But if the head be bowed too low, the projection of the upper eye-lid and eye-laſh, may hinder the rays from falling upon this watery ſurface; and then the beam will diſappear, though the candle may ſtill be viſible, by rays paſſing as uſual through the lower part of the pupil; which is alſo agreeable to experience. In this poſition of the head no beam can appear above the candle, becauſe the lower eye-lid *I* is below the pupil.

Fig. 8.

a Art. 40.

b Art. 41, 42.

47. But when the head is lifted up, the eye being fixt upon the candle, the under eye-lid is alſo liſted above the lower margine of the pupil; and then the rays which paſs through this watery concave at *I* are bent downwards and are directed through the pupil towards the lower part of the retina; and cauſe the appearance of a beam above the candle. But no beam can appear below it becauſe the upper eye-lid is now above the pupil.

48. Therefore holding the head upright, and cloſing the eye-lids till their interval be leſs than the diameter of the pupil, it is plain that the beams will ſhoot both upwards and downwards. And theſe explications are confirmed by intercepting the rays that fall upon theſe watery concaves at *H* or *I*, with any obſtacle *P* held cloſe to the eye. For when the beam *B N* appears below the candle if the obſtacle be raiſed gradually from below towards the pupil, it will have no effect; but if from above the obſtacle be brought downwards, the beam below the candle will diſappear before the candle it ſelf does.

49. Though M. *Rohault*'s ſentiments upon this ſubject cannot be ſupported[c], yet I do not deny but ſome rays are reflected from the thickneſs of the eye-lids into the pupil, in ſome poſitions of the eye and candle: but theſe rays cauſe a different appearance from what I have been ſpeaking of. As ſoon as I had invented this ſolution I intended to have printed it, but I found ſoon after that M. *Briggs* an Engliſh phyſician, had in general given the ſame ſolution in his *Opthalmography*. So far *de la Hire*.

c Syſtem Nat. Phil Part. I. Chap. 35

50. The manner in which theſe gentlemen have explained theſe appearances, would have been very juſt, had the extremities of theſe beams appeared coloured. But ſince they do not, they cannot be cauſed by ſuch great refractions[d], though they may by inflections of the rays[e] upwards and downwards at the edges of the upper and under eye-lid, when over againſt the pupil.

Their true cauſe.

d Art. 17

e Art. 18

Upon ART. 88, 89.

51. In order to determine the propereſt glaſſes for defective eyes, the limits of confuſed and diſtinct viſion, or the diſtances of thoſe places from the eye, where an object begins to appear confuſed, may be found by meaſuring the leaſt diſtance from which a long-ſighted perſon can ſee a pretty large print diſtinctly and read it readily: and likewiſe by meaſuring the greateſt and the leaſt diſtances, from which a ſhort-ſighted perſon can ſee a ſmall print diſtinctly and read it readily: or ſtill more exactly by placing the end of a long ruler cloſe to the eye, or rather a little under it, and by obſerving the greateſt and leaſt diſtances at which the lines drawn lengthways along the ruler, begin to appear confuſed. I ſhall call thoſe glaſſes the propereſt for defective eyes, which are the leaſt concave or the leaſt convex of any that will anſwer the purpoſe of diſtinct viſion; for a reaſon to be mentioned hereafter.

Glaſſes for defective eyes determined.

52. Let *E q* be the leaſt diſtance from which any ſmall objects appear diſtinct to the eye of a long-

Fig. 9.

long-sighted person; and EQ the least distance from which he wants to see them distinctly. Towards q take QF to QE as QE to Qq, and EF will be the focal distance of a convex lens, which being put close to his eye, will make him see an object distinctly at any place between Q and F, and possibly beyond F. For the rays that flow from Q will emerge from the glass and will enter the eye as if they had come directly from q, to the naked eye [a]; and supposing Q to recede from the eye, q will also recede from it to infinity through places where the naked eye can see distinctly. And therefore the refracted rays, diverging as from these places, will also produce distinct vision of the object Q as far as to F; and still farther if the person can see distinctly by converging rays.

a Art. 239.

53. Therefore if he wants to see distinctly from no less a distance than half Eq, that is, only as near again as with his naked eye, a convex lens whose focal distance is Eq will be the propereſt; and will make him see an object distinctly at any distance not less than half Eq. For supposing Qq and QE to be equal, the point F will fall upon q by the foregoing proportion.

Glasses for short-sighted persons. Fig. 10.

54. Let EF be the greatest distance from which an object at F appears distinct to the eye of a short-sighted person; and it will also be the focal distance of a concave lens, which being put close to his eye at E, will be the propereſt for seeing remote objects distinctly. Because the rays of a pencil, which come from any remote object, and consequently fall parallel upon the lens, will emerge from it to the eye, as if they had come directly to the naked eye from an object at F. And consequently the picture of the remote object formed upon the retina by rays refracted through the lens, will be as distinct as the picture of an object at F seen by unrefracted rays.

Fig. 11.

55. Let Eq be the least distance from which the same person can see an object distinctly with his naked eye; then say as QF to QE so QE to Qq, and placing Qq towards F, the point q will be the nearest point, which he will be able to see distinctly through the lens abovementioned. For by art. 239, the rays of a pencil which fall upon the lens converging towards Q, will after refraction converge to q; and on the contrary, the rays which flow from q will emerge from the lens diverging from Q; and supposing the point q to recede from the eye, the point Q will also recede from it to such places where the naked eye can see distinctly. But if the point q approaches towards the eye, the point Q will also approach towards it, to such places where the naked eye cannot see distinctly, by the supposition.

56. Consequently if QF, the space between the limits of confused vision, be not less than QE, that one glass whose focal distance is EF, will make all objects appear to him distinct which are any where placed beyond F, the reach of his naked eye. For in this case Qq cannot be greater than QF, as is manifest by the proportion above.

57. But if he wants a pair of concave spectacles to read or write with, let the distance Eq be no greater than what is convenient for that purpose; and let QF be the limits of confused vision as before; and towards q take FG to FE as FE to Fq, and a concave lens whose focal distance is EG will be the propereſt for this purpose. For by art. 239, the rays of a pencil which fall on this glass converging towards F will converge to q after refraction, and on the contrary; and therefore he will see an object distinctly as far off as q; and also nearer than F, if QF be but half of EF. For supposing rays to fall on the lens converging towards Q, say as QG to QE so QE to QH, and the refracted rays will converge to H and consequently H will be the nearest point that can be seen distinctly through this glass. But if Q bisects EF it is manifest that QH is less than QF; because QG, QF, QH are now continual proportionals.

Fig. 12.

Directions for the choice of convex and concave spectacles.

58. Thus any person may be fitted with the propereſt glasses though he lives at a distance from the shops where they are sold, by sending the workman their focal distances computed by the foregoing rules. But if choice of glasses be at hand, he may be better fitted by trial; observing only to use those glasses which are the least concave or the least convex of any that will fit the eye. These are the glasses which I have computed and called the propereſt. For since they cannot be put quite close to the eye, the less any glass is concave, the less it diminishes the pictures of objects upon the retina. It will also accustom the eye to that conformation of its coats and humors, which is proper for seeing objects as far off as it can; and consequently may prevent the eye from growing more and more short-sighted. On the other hand, the less any glass is convex, the less it magnifies the pictures of objects upon the retina; and also obliges the eye to that conformation, which is requisite for seeing objects as near as it can. Both which may prevent the eye in some measure from growing more and more long-sighted. For when the picture upon the retina is very large, it need not be quite so distinct, as when it is smaller, to convey an idea of the same number of parts of an object; and consequently the eye will be more at liberty to recede from that conformation, which is proper for the glass; and to relapse into that to which it inclines, and which is only proper for seeing remote objects.

Directions to prevent short-sightedness.

59. It is generally observed that those persons whose eyes are mostly employed upon remote

mote objects, as countrymen, ſportſmen, ſailers, travellers and the like, want ſpectacles the ſooneſt; and on the other hand, that the greateſt numbers of ſhort-ſighted perſons, are found amongſt ſcholars and mechanicks, who are daily converſant with books and other near objects; ſo that the eye it ſeems, like every thing elſe, is inclined to keep to that conformation, to which it is moſt accuſtomed.

60. Out of ſo great a number of ſhort-ſighted perſons as are daily to be met with, it is probable there are but few that were born ſo, even of ſhort-ſighted parents. For it generally grows upon young people at the age of 20 or 25, and therefore may poſſibly be prevented by uſing their eyes while young to all ſorts of conformations; that is by often looking through glaſſes of all ſorts of figures, and by reading, writing or working with ſpectacles of ſeveral degrees of convexity. For whatever be the powers by which the eye conforms it ſelf to diſtinct viſion, they may poſſibly grow weak and loſe their extent one way or other, for want of variety of exerciſe. I have heard of perſons who have had ſo great a latitude of ſight, as to be able to ſee diſtinctly through glaſſes of all manner of ſhapes; and poſſibly moſt children may have the ſame faculty, and by practice may preſerve it. To imagine that ſuch an exerciſe of the eyes can any way injure them, ſeems to me to be an opinion without any foundation; provided due care be taken to avoid looking at objects that are too bright.

Short ſightedneſs comes by accidents.

61. Dr. *Briggs* mentions a perſon above ſeventy years old who had uſed ſpectacles for ten years, and yet by catching cold by reading in the winter time too near a window, he ſuddenly became ſo ſhort ſighted that he could not diſtinguiſh objects 3 feet off; and after his cold and defluxion was cured, he continued to read the ſmalleſt prints without ſpectacles for many years[a]. I know a young gentleman who became ſhort-ſighted immediately after coming out of a cold bath, in which he did not totally immerſe himſelf; and ever ſince he has uſed a concave glaſs for many years. It is commonly thought that ſhort-ſightedneſs wears off in old age, but I queſtion whether this be matter of fact or an hypotheſis only.

a Opthalmographia p. 34.

Properties of ſhort-ſightedneſs.

62. It is remarkable of ſhort ſighted perſons, that they write a ſmall hand and love a ſmall print, becauſe they can ſee more of it at a view; that they do not look at the perſon they converſe with, becauſe they cannot ſee the motion of his eyes and features, and therefore are attentive to his words only; that they ſee more diſtinctly and ſomewhat farther off by a ſtrong light than a weak one; becauſe a ſtrong light cauſes a contraction of the pupil, and conſequently of the pencils both here, and at the retina; which leſſens their mixture and conſequently the apparent confuſion; and therefore to ſee more diſtinctly they almoſt cloſe their eye-lids; for which reaſon they were anciently called *Myopes*.

63. To theſe perſons a lighted candle at a great diſtance appears very large and round, becauſe the retina cuts the pencils at a good diſtance from the image; and conſequently this ſection partakes of the figure of the pupil as well as of the image of the candle. On the other hand dark objects appear ſmaller to them, becauſe the contiguous pencils that come from the contiguous brighter objects, are diffuſed into the pictures of the dark ones.

Spectacles for diving.

64. *Hugenius* has determined the proper convexity of ſpectacles for the uſe of Divers in the Sea, and has found that if the convexities of the glaſſes be made equal on both ſides, they muſt be the ſame as the convexity of the cornea; whoſe diameter is about $\frac{1}{5}$ of an inch. "It is certain ſays he[b], that neither fiſh out of water, nor other animals within water, can ſee any object diſtinctly. Divers ſee things in water much in the ſame manner as an old man ſees things through a very concave glaſs put near to his eye. For ſince it is found by experiments, that the aqueous humor, lying next to the cornea, has nearly the ſame refractive power as water; it follows that when the eye is under water, there can be no refraction of the rays at their firſt entrance into it. For though the cornea may have a different refractive power from water, yet being but thin and terminated by parallel ſurfaces, adjoining to fluids equally refractive, it will tranſmit all the rays in a manner ſtraight through it[c]. Therefore parallel rays which, by the refraction at the cornea, when out of water, were made to fall converging upon the cryſtalline, will now fall parallel upon it: and therefore the cryſtalline will not be able to collect them to a point upon the retina, but only to direct them to a point beyond it: and ſo the viſion will be confuſed. On the other hand the refractions at the cornea of a fiſh out of water will be very great, whereas under water there is none at all or much ſmaller; and ſo the rays will croſs one another before they fall upon the retina, and cauſe a confuſed appearance.

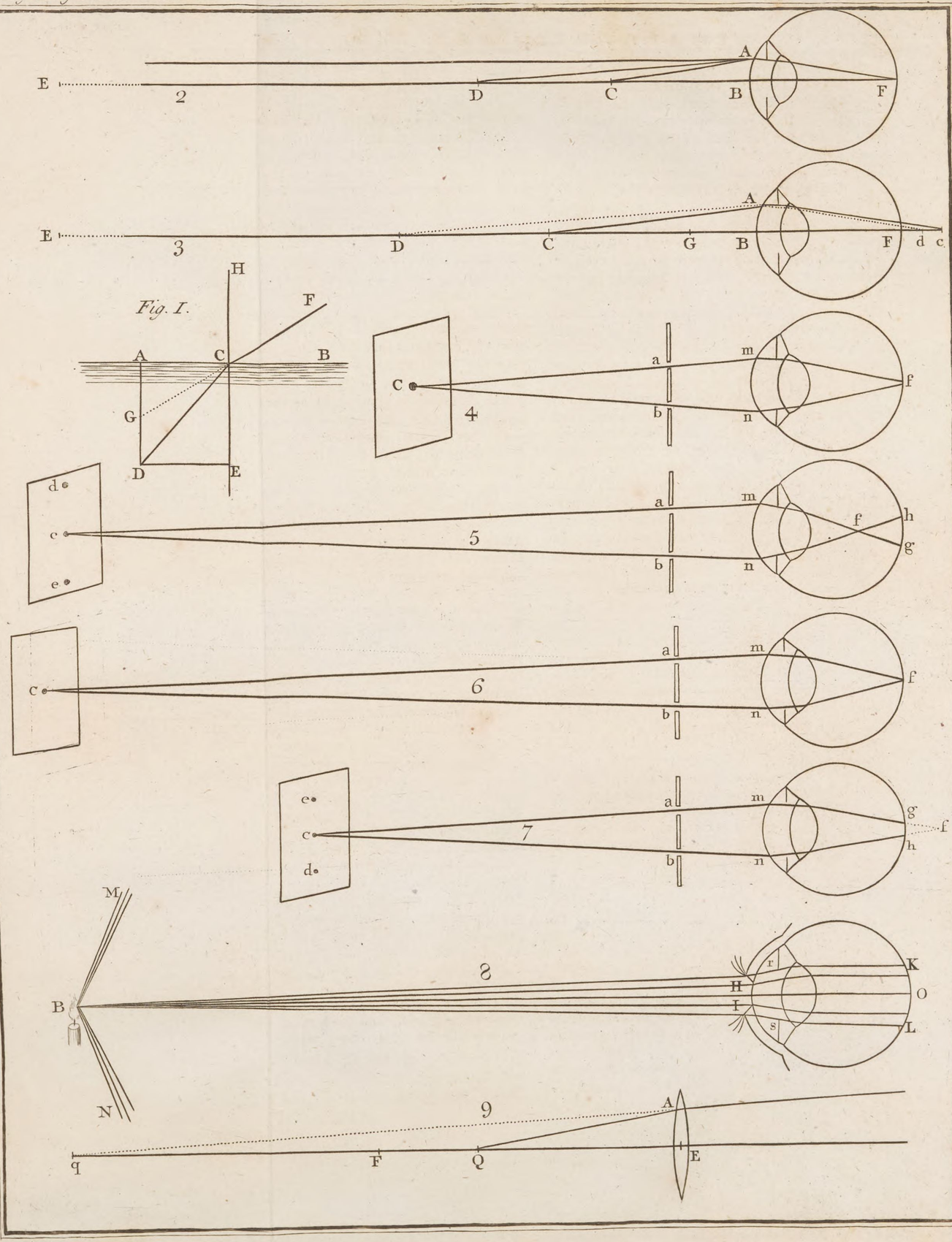
b Dioptric. pag. 118.

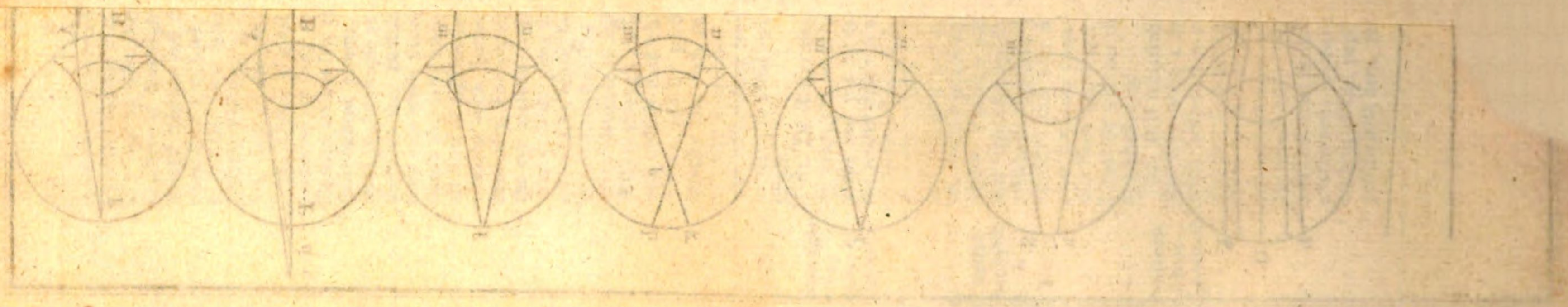
c Art. 37.

65. But to correct the confuſion in human eyes under water, it is requiſite to find the convexity of a lens, which being applyed to the eye, ſhall tranſmit the rays to the cryſtalline, with the ſame degree of convergency, as they uſually have when the eye is out of water. The ſine of incidence is to the ſine of refraction out of water into glaſs as 9 to 8, and the ſurface of the cornea is a portion of a ſphere whoſe diameter is $\frac{3}{5}$ths of an inch of our Rheinland foot or of the old Roman foot. Let AC be a ſection of this ſurface through its center B; and the ſine of

Fig. 13.

inci-


E
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Fig. I.
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B
N
8
r
K
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O
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9
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E



incidence be to the ſine of refraction, out of air into the aqueous humor, as 4 to 3. Therefore taking *BD* triple of the ſemidiameter *BA*, it is certain that parallel rays in air, will be refracted by the aqueous humor, towards the point *D* *. But when the eye is under water this refraction at *AC* will be nothing; therefore a convex lens muſt now be applyed to *AC* which ſhall collect parallel rays to the ſame point *D*. Let *EAF* be this lens, having one ſide plane, and the other convex towards the eye, and let its ſemidiameter be *AH*. Therefore ſince parallel rays are collected to *D*, we have *HD* to *DA* as 9 to 8, that is, in the ratio of refraction of glaſs under water [a], and disjointly *HA* to *AD* as 1 to 8. But we had *AD* to *AB* as 4 to 1 or as 8 to 2; therefore by a regular equality of ratio's [b], we have *HA* to *AB* as 1 to 2. But *AB* is $\frac{1}{10}$ inch and therefore *AH* is $\frac{1}{20}$ inch; which was to be found. Now if inſtead of this plano-convex lens you would have a lens of equal convexities, they muſt each be the ſame as that of the cornea [c]; that is they muſt be portions of a ſpherical ſurface whoſe diameter is $\frac{1}{5}$ inch. So far Mr. *Huygens*.

* Art. 214.

a Art. 225.

b Euclid. V. 22.

b Art. 235.

A perfection in the eyes of fiſhes.

66. It is a common opinion that the want of refraction of the viſual rays at the cornea of the eye of a fiſh, is compenſated by the ſphericity of the cryſtalline humor; ſo that the diſtance of the retina from the cornea need be no greater than in other animals whoſe cryſtallines are lenticular. But this is a miſtake. For conceiving a lens to be made of two equal ſmall ſegments of the ſpherical cryſtalline, its focal diſtance would be ſhorter than that of the whole ſphere, meaſured from its remoter ſurface, by three quarters of its diameter, whatever be the refractive power of the ambient medium. This may be eaſily collected from Art. 227 and 232. Accordingly it is obſerved that fiſh have larger eyes than land animals in proportion to their bodies. There may be this advantage indeed in a ſpherical cryſtalline, that the eye will take in more objects at one view, provided the cornea be ſufficiently protuberant and the pupil large, as is uſual in fiſh's eyes. The reaſon is that the rays from collateral objects fall perpendicularly upon the ſides of a ſpherical cryſtalline, and obliquely upon a lenticular one. Conſequently the pictures of collateral objects upon a retina concentrick to a ſpherical cryſtalline, will be as diſtinct as thoſe of objects placed directly before the eye [c]. By this means animals whoſe eyes are placed on each ſide of their heads, will have the advantage of ſeeing quite round them almoſt at one view; which is a great perfection in viſion and preſervative of their lives; and in fiſhes it may compenſate for their want of hearing.

c See Fig. 124, 153.

The antiquity of ſpectacles.

67. Mr. *William Molyneux* has given us the fulleſt account of the antiquity of ſpectacles, and places the invention about the year 1300 *. "Were there no other uſe of dioptricks than that of ſpectacles for the help of defective eyes; I ſhould think the advantage that mankind receives thereby, inferior to no other benefit whatſoever, not abſolutely requiſite to the ſupport of life. For as the ſight is the moſt noble and extenſive of all our ſenſes; as we make the moſt frequent and conſtant uſe of our eyes in all the actions and concerns of human life; ſurely that inſtrument that relieves the eyes when decayed, and ſupplies their defects, rendering them uſeful, when otherwiſe almoſt uſeleſs, muſt needs of all others, be eſteemed of the greateſt advantage. How melancholy is the condition of him who only enjoys the ſight of what is immediately about him? with what diſadvantage is he ingaged in moſt of the concerns of human life? Reading is to him troubleſome, war more than ordinary dangerous, trade and commerce toilſome and unpleaſant. And ſo likewiſe on the other hand, how forlorn would the latter part of moſt mens lives prove, unleſs ſpectacles were at hand to help our eyes, and a little formed piece of glaſs ſupplyed the decays of nature? The curious mechanick, engaged in any minute work, could no longer follow his trade, than till the 50th or 60th year of his age; the ſcholar no longer converſe with his books, or with an abſent friend in a letter. All after would be melancholy idleneſs, or he muſt content himſelf to uſe another man's eyes for every line. Thus forlorn was the ſtate of moſt old men, and many young, before this admirable invention; which on this very account can never be prized too highly

* Dioptric. p. 207. and 251.

Optick-glaſſes unknown to the ancients.

68. That the ancients had no knowledge of optick-glaſſes, is moſt evident from their univerſal ſilence in this matter: their moſt learned and inquiſitive philoſophers making no mention, or the leaſt hint thereof, in their writings. And doubtleſs a contrivance of that univerſal uſe, beneficial to all old men, both in reading and writing, could never have been ſo concealed, as that not the leaſt footſteps thereof ſhould remain to poſterity. The only relief they had for their decayed ſights were certain *Collyria* or eye-ſalves; and when theſe failed them, they were left almoſt in the dark for minute and cloſe objects.

69. We hear indeed mighty ſtories of *Archimedes* burning the ſhips of *Marcellus*, at a great diſtance from the walls of *Syracuſe*. But whether the matter of fact be true or falſe (as I am very inclinable to believe it falſe) yet there is no mention of his performing this admirable effect by optick-glaſſes. Perhaps, if there were any ſuch thing done at all; it was performed by concave ſpeculums: and no one denies the an-

cients the knowledge of catoptricks. For *Archimedes* himſelf writ a book, as it is ſaid, *De Speculis Uſtoriis Parabolicis*; but it has never yet ſeen the light.

70. And yet there are in the world a ſort of men, ſo devoted to the paſt ages, that they will not allow any improvemens of arts in the modern generation, unknown to the ages ſome centuries before us. Of this claſs was he, (whoever he was) that rather than the ancients ſhould be ignorant of optick-glaſſes, would forge a paſſage in *Plautus*, which really is not at all to be found in him, for confirmation of his opinion.

Pretended paſſage in *Plautus*.

71. *Pancirollus* (who ſurely was too candid a perſon to be the firſt author of this fiction) in the ſecond book *De Rebus Inventis, Tit.* 15, quotes this paſſage from *Plautus, Cedo Vitrum, neceſſe eſt Conſpicilio uti*; which, ſays he, cannot poſſibly be meant of any other thing but of the glaſſes which we call ſpectacles. And his commentator *Salmuth* takes ſome pains to cite *Chriſtianus Becmannus* (I ſuppoſe in his *Oratio de Barbarie & Superſtitione ſuperiorum Temporum*) for clearing this paſſage of *Plautus*: but yet he is ſo hard preſſed with it, that by no art, but by main ſtrength he breaks through it, and ſays, that notwithſtanding that paſſage, yet certainly optick glaſſes are a modern invention.

a Voſſius de Scient. Math Cap. 26. S. 10.

72. Whereas had he been aware, that that quotation from *Plautus* is a mere fiction a; and that no ſuch paſſage can be found in all his writings; he might eaſily have avoided its force, without all that ſtir. For ſo we ſhall find it anſwered in the *Lettere Memorabili del Abbate Michele Giuſtiani, Parte Terza, Let.* 16.

Paſſage in *Pliny*.

73. Another place cited for the antiquity of optick-glaſſes, is that of *Pliny*, Lib. 7. Cap. 53. *Hiſt. Nat.* wherein we find the word *Specillum*. To this paſſage we have this anſwer in the forementioned letters of *Giuſtiani*; that *Specillum* cannot poſſibly be here meant of a ſpectacle glaſs, ſeeing we find the expreſſion, *Inungit Specillum*; which, ſays he, cannot be underſtood of ſpectacles, which we rather wipe and cleanſe, than anoint and greaſe. But this conſtruction of the learned authors is much forced and unnatural: for the plain ſenſe of that paſſage in *Pliny* is this. *Pliny* in that chapter is giving inſtances of the ſudden deaths of many men; and telling how they were ſeized, whilſt they were doing ſo or ſo, and wholly thoughtleſs of that fatal moment. Amongſt many other examples, he has this; *Super omnes C. Julius medicus dum inungit, Specillum per oculum trahens.* The meaning whereof is no more, than that the phyſician *C. Julius* was on a ſudden ſeized by death, whilſt he was applying an unguent to his patient's eye, and drawing his probe (called *Specillum*) through it. Whereas to join *inungit* and *Specillum*, ſpoils the grammatical ſenſe of the whole, and renders it unintelligible.

74. It is evident therefore, that from neither of theſe paſſages can we draw any argument for the antiquity of optick-glaſſes. Wherefore ſeeing we muſt neceſſarily allow this invention due to the modern age of the world; our next enquiry ſhall be, where firſt to fix it. But herein we ſhall find but faint traces to direct us.

Menage's opinion.

75. *Monſieur Menage* a learned and ingenious Frenchman, in his *Origini della Lingua Italiana, Genev.*, 1685, commenting on the word *Occhiali del Galilæi*, diſcourſes there of the time of the invention of ſpectacles: and after relating the known ſtory of *Frier Jordan*, of which more anon, he has this notable paſſage; that *Monſieur du Cange* had told the author (*Monſieur Menage*) of a greek poem, the manuſcript whereof is now in the French King's Library, wherein the poet, who lived An. 1150, jeſting on the phyſicians of thoſe times, ſays of them to this purport in french, *Qu'ils tatent le poux, & qu'ils regardent les excremens du malade avec une Verre.* That they obſerve the excrements of their patients with a glaſs. But Monſieur *Menage* is of opinion, that this was a tranſparent glaſs, whelmed over the veſſel, more for the relief of their noſe againſt the ſtench, than of their eyes.

Frier *Spina's* pretenſe.

76. But however we may doubt of ſpectacles being ſo ancient as 1150, we may be certain that about the thirteenth century, they were commonly known and uſed. For (beſides what we ſhall ſay hereafter of our country-man *Frier Bacon*) the moſt learned Monſieur *Spoon* in his *Recherches Curieuſes d'Antiquitè, Diſſert.* 16. inſerts a letter of Signior *Redi* to *Paulus Falconerius*, concerning the time when ſpectacles were invented; and this he fixes between 1280 and 1311, from the teſtimony of a manuſcript chronicle in latin, in the library of the Friers Preachers of St. *Katherine* at *Piſa*, Fol. 16. Wherein it is ſaid, that *Frater Alexander de Spina, Vir modeſtus & bonus, quæcunque vidit aut audivit facta, ſcivit & facere. Ocularia ab aliquo* primo *facta, & communicare nolente, ipſe fecit & communicavit corde hilari & volente.* And this *Alexander de Spina* was a native of *Piſa*, and dyed there, An. 1313.

Another authority.

77. Signior *Redi* has in his library a manuſcript written An. 1299. *Di Governo della Famiglia de Scandro di Pipozzo.* In which there is this paſſage; *Mi truovo coſi gravoſo di anni, che non arei valenza di leggere e ſcrivere ſenza vetri appellati Okiali, truovati novellamente per commoditè delli pouveri veki, quando affiebolano del vedere.* Thus in engliſh, I find myſelf ſo preſſed by age, that I can neither read or write without thoſe glaſſes they call ſpectacles,

lately

lately invented, to the great advantage of poor old men, when their ſight grows weak.

78. The Italian Dictionary *de la Cruſca*, on the word *Occhiale*, makes this remark, that *Frier Jordan de Rivalto*, who dyed at *Piſa*, An. 1311, in a book of ſermons which he writ An. 1305, tells his auditory in one of them, that it is not twenty years ſince the art of making ſpectacles was found out, and is indeed one of the beſt and moſt neceſſary inventions in the world.

79. About the ſame time *viz.* 1305. *Bernard Gordon* a famous Phyſician of *Montpelier*, in his *Lilium Medicinæ*, thus commends a certain eye-ſalve; *Et eſt tantæ Virtutis, quod decrepitum faceret legere Literas minutas abſque Ocularibus.* And An. 1633, *Guido de Chauliac*, in his book entituled *Grand Chirurgery*, after propoſing ſeveral *Collyria*, ſaith; if theſe or the like will not do, you muſt make uſe of ſpectacles.

80. From all which we may be pretty certain, that ſpectacles were well known in the 13th century, and not much before. But who the happy man was, that firſt hit upon this lucky thought, may yet be queſtioned. It is true indeed, if we credit the forementioned Chronicle of the Convent at *Piſa*, Frier *Spina* makes as fair a challenge to the invention, as the firſt author, who refuſed to communicate it. But I am apt to believe, that, whoever this cloſe man was that would not impart to *Spina*, he was a Frier; and that theſe monkiſh men, and *Jordan* amongſt the reſt, had this invention whiſpered amongſt themſelves, before it was publick; and that they all had the firſt hint thereof from our Country-man *Frier Roger Bacon.*

81. That this learned *Frier Bacon* who dyed An. 1292, and lyes buried at *Oxford*, did perfectly well underſtand all ſorts of optick glaſſes, ſhall be plainly made out, from the natural and eaſy ſenſe of his own words, in his book of Perſpective: whereby we ſhall find, that he not only underſtood the effects of ſingle convex and concave glaſſes; but knew likewiſe the way of combining them, ſo as to compoſe ſome ſuch inſtrument as our teleſcope. This perhaps will be looked upon as a great paradox, and as great partiality in an engliſh author to his countryman; eſpecially conſidering, how univerſally the contrary has prevailed; the votes of moſt learned men having conferred the honour of this invention on other pretenders. But if from the unconſtrained words of his books, we plainly make out this aſſertion, I hope the attempt may not be counted unreaſonable or partial.

82. And firſt in his book of perſpective Part III. Diſtinction 2. Ch. 3. he has theſe words; *Si vero Corpora non ſunt plana* (having treated of them before) *per quæ viſus videt, ſed ſphærica; tunc eſt magna diverſitas, nam vel Concavitas Corporis eſt verſus oculum, vel Convexitas, &c.* By which it is manifeſt, he knew what a concave and convex glaſs was. Moreover, in the ſame place Diſtinct. ultim. he proceeds thus; *De Viſione fractâ majora ſunt; nam de facili patet, maxima poſſe apparere minima, & è contra; & longè diſtantia videbuntur propinquiſſimè, & è converſò. Sic etiam faceremus Solem & Lunam & Stellas deſcendere ſecundum Apparentiam hic inferius, &c.* Thus in engliſh; greater wonders than all theſe are performed by refracted viſion; for thereby it is eaſily made appear, that the greateſt object may be repreſented as very little, and contrarily; and ſo likewiſe, the moſt diſtant objects as juſt at hand, and contrarily. Hereby alſo we may bring the ſun and moon and ſtars down here below in appearance, &c. This I think is ſo expreſs in the point, that it leaves no room to doubt, but that he had ſome admirable ſecret in optick-glaſſes. Add to this what he has in his epiſtle *ad Pariſienſem*, of the Secrets of Art and Nature, Cap. 5. *Poſſunt etiam ſic figurari Perſpicua, ut longiſſimè poſita appareant propinquiſſima, & e contrario. Ita quod ex incredibili diſtantia legeremus literas minutiſſimas, & numeraremus res quantumcunque parvas, & ſtellas faceremus apparere quò vellemus.* Glaſſes or diaphonous bodies, ſays he, may be ſo formed, that the moſt remote objects may appear as juſt at hand, and contrarily; ſo that we may read the ſmalleſt letters at an incredible diſtance, and may number things though never ſo ſmall, and may make the ſtars appear as near as we pleaſe.

83. And that theſe things may not ſeem incredible of this great man; who, in that dark, ignorant age could be maſter of theſe admirable inventions; I ſhall refer the reader, for a more compleat account of him, to *Ant. a Wood Hiſt. & Antiquit. Univerſit. Oxonienſis*, Lib. 1. Pag. 136. and to Dr. *Plott*'s *Nat. Hiſt. of Oxfordſhire*, Cap. 9. Sect, 2, 3. &c. and Sect. 39, 40, 41. Where we may find, how he was perſecuted by the ignorant malicious Friers of his order, as practiſing magick and necromancy: for which they caſt him into priſon, and there detained him for a long time, ſome ſay to his death, in the 78th year of his age.

A more ample paſſage from Frier *Bacon.*

84. Mr. *Molyneux*'s quotations from Frier *Bacon* being imperfect, by reaſon of his abſence from books, as he informs us, I will here ſupply this defect. This author having deſcribed ſeveral canons, as he calls them, for ſhewing the viſual angle under which an object appears by refractions through a plane and a ſpherical ſurface, and alſo the place of its image; proceeds immediately to apply them to the ſolution of ſeveral appearances. As why an oar appears crooked in water; why a piece of money in the bottom of a baſon, becomes viſible by pouring in water, when before that it could not be ſeen

over

over the ſide of the baſon; why the ſun and moon appear ſometimes larger than ordinary through denſe vapours in the horizon. And at laſt he adds [a], "*Si vero homo aſpiciat literas & alias res minutas per medium cryſtalli, vel vitri, vel alterius perſpicui, ſuppoſiti* [i. e. *ſuperimpoſiti*] *literis; & ſit portio minor ſphæræ, cujus convexitas ſit verſus oculum; & oculus ſit in aere; longe melius videbit literas, & apparebunt ei majores. Nam ſecundum veritatem canonis quinti de ſphærico medio infra quod eſt res, & citra ejus centrum, & cujus convexitas eſt verſus oculum; omnia concordant ad magnitudinem: quia angulus major eſt ſub quo videtur, & imago eſt major, & locus imaginis eſt propinquior, quia res eſt inter oculum & centrum: & ideo hoc inſtrumentum eſt utile ſenibus & habentibus oculos debiles. Nam literam quantumcunque parvam poſſunt videre in ſufficienti magnitudine. Si vero ſit portio major ſphæræ vel medietas, tunc ſecundum canonem ſextum accidit majoritas anguli, & majoritas imaginis, ſed propinquitas deeſt, quia locus imaginis eſt ultra rem, eo quod centrum ſphæræ eſt inter oculum & rem viſam; & ideo non ita valet hoc inſtrumentum ſicut ſi eſſet minor portio ſphæræ. Et inſtrumenta planorum corporum cryſtallinorum ſecundum primum canonem de planis, & ſphæricorum concavorum ſecundum primum canonem & ſecundum de ſphæricis, poſſunt facere hoc idem. Sed inter omnia portio minor ſphæræ, cujus convexitas eſt verſus oculum, evidentius oſtendit magnitudinem, propter tres cauſas ſimul aggregatas, ut notavi.* "If the letters of a book or any minute objects, be viewed through a leſſer ſegment of a ſphere of glaſs or cryſtal, whoſe plane baſe is laid upon them, they will appear far better and larger. Becauſe by the 5th canon about a ſpherical medium whoſe convexity is towards the eye, and the object is placed below it, and between the convexity and its center; all things concur to magnify it. For the angle under which it is ſeen is greater, and its image is alſo greater, and nearer to the eye than the object it ſelf; becauſe the object is between the center and the eye. And therefore this inſtrument is uſeful to old men and to thoſe that have weak eyes. For they may ſee the ſmalleſt letters ſufficiently magnified. But if the medium be the larger ſegment of a ſphere or but half of one, then by the 6th canon, the apparent viſual angle will be greater than the true, and the image alſo greater than the object; but the place of it will be beyond the object; becauſe the center of the ſphere is between the eye and the object. And therefore this inſtrument is not ſo powerful in magnifying as a leſſer ſegment of a ſphere. Alſo inſtruments made of cryſtal bodies with plane ſurfaces, by the firſt canon about planes, and with concave ſurfaces, by the firſt and ſecond canons about ſpherical ſurfaces, will perform the ſame thing. But the leſſer of two ſegments of a ſphere magnifies more manifeſtly than any of them all, by reaſon of the concurrence of all the three cauſes, as I ſaid before.

[a] Rog. Bacon Opus Majus Lond. 1733 pag. 352.

85. This is a tranſlation of the whole paſſage, and in his figures the center of the convexity *op* is *d*; the object is *fg*; the incident rays *fo*, *gp*; the refracted ones *oa*, *pa* to the eye at *a*; the image at *bc*, terminated by the lines *df*, *dg* produced till they cut the viſual rays produced in *b* and *c*. Fig. 14.

86. To find an author ſpeaking of a ſmall ſegment of a ſphere of glaſs, of its magnifying the letters of a book, of its being a proper inſtrument for helping decayed ſight, and to ſay he was not poſſeſſed either of the theory or the uſe of ſpectacles, may appear to be a paradox; but I hope to ſatisfy my readers it is not a miſtake. Firſt then our author plainly propoſed to lay the flat baſe of his ſegment upon the letters. For the word *Suppoſiti* muſt have been a contraction in his writing of the word *Superpoſiti* or rather *Superimpoſiti*; as appears by the ſequel and by the canon he quotes. Beſides, he ſays not a word of holding the ſegment at a diſtance from the letters; nor could he indeed becauſe he has not treated of a double refraction at both its ſurfaces, without which he could conclude nothing at all about its effects when raiſed from the book; for he argues from nothing but theory throughout the whole chapter. His pretenſe examined.

87. In the next place I obſerve that he is quite miſtaken in aſſerting twice together, that the leſſer ſegment of a ſphere magnifies the letters more than the larger. The contrary to which is true; for I ſhall prove by and by, that when the thickneſs of the ſegment is very ſmall, it magnifies the letters inſenſibly; and that as its thickneſs is increaſed, it magnifies them more and more, and moſt of all when it becomes an entire ſphere. But it is no wonder that he concludes wrong from a wrong principle; namely that the letters appear leſs in the greater ſegment than in the ſmaller, becauſe their image is beyond them in the greater ſegment and before them in the leſſer. The only juſt conſequence that can be drawn from theſe different diſtances of the image, is, that to an old man's eye the letters will appear diſtincter, by rays diverging ſomewhat leſs from the remoter image, and more confuſed by rays diverging ſomewhat more from the nearer, than if he viewed them with his naked eye. The effect of the leſſer ſegment is therefore contrary to the deſign of ſpectacles; which is not to magnify the letters, but to make them appear diſtinct, by cauſing the rays to fall upon the eye leſs diverging, or parallel, or even ſomewhat converging, according to the different age or conſtitution of the eye; and therefore it cannot be performed, but by a very nice and determinate degree of convexity. His miſtakes.

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did not ent ſpe- les.

88. Hence it is plain that our author tryed no experiments with a greater and a leſſer ſegment, to compare their effects together. For then he muſt have found out his miſtake, and muſt rather have preferred the larger ſegment for magnifying more, which is all he contends for. However let us ſuppoſe him to have followed his own doctrine, and to have tryed a leſſer ſegment only. It could not be a thin ſegment of a large ſphere, like one of our ſpectacle-glaſſes. For this could not ſenſibly magnify the letters underneath it, as he ſays it did. The moſt convex ſpectacles now made, when laid upon a book, have not this effect; becauſe they are too thin. It follows then, if he tryed any ſegment at all, that it muſt have been a ſegment of a ſmall ſphere, ſufficiently thick to magnify the letters underneath it; and therefore it muſt have been thicker than our deepeſt ſpectacles for the oldeſt mens eyes; and conſequently when applyed to their eyes, it muſt have made the letters appear confuſed by too great a quantity of the refractions. And this confuſion our author could not correct by theory and reaſon, becauſe he knew not the cauſe of it; and it is plain he made not many experiments. The diſcovery of this cauſe, together with the manner of viſion by pictures upon the retina, was firſt made by *Kepm. 25. ler* [a], above 300 years after our author's time, and alſo after ſpectacles were in common uſe. It was impoſſible then for any man before *Kepler*, even to explain the effect of ſpectacles, (that is, how they correct the confuſion in the picture upon the retina,) and much more to invent them by theory and reaſon; and of conſequence they muſt have been the reſult of ſome lucky accidents among a multitude of tryals and attempts, begun perhaps upon this hint of our author's; which is all the honour that can juſtly be paid him. What he knew of teleſcopes ſhall be conſidered in its proper place.

borrow- from *lhazen.*

89. As to his theory and applications abovementioned, they are all taken from *Alhazen* whom he frequently mentions upon other occaſions. *Alhazen* is reckoned to have lived about the year of our Lord 1100. Among his experiments made to confirm his theorems, he expreſsly mentions, that if an object be applyed cloſe to the baſe of a larger ſegment of a ſphere of glaſs, it will appear magnified [b]; which, as I obſerved, was fitter for *Bacon*'s purpoſe than his own leſſer ſegment. He alſo treats about the appearance of an object through a globe; and ſays he is the firſt that found out the refraction of rays into the eye.

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90. The proof that I promiſed of the 87th remark is this. Let *O* be the place of the eye, *E* the center of the ſpherical ſurface *AC*, and *OCED* its axis, *OAPB* any ray ſuppoſed to flow from the eye, and to fall upon any point *P* of the object *PQ*, ſuppoſed always to be a part of the baſe of the ſegment *RACS*; and *πκ* a line parallel and equal to *PQ*, and ſubtending the viſual angle *AOC*. The object *PQ* appears by refraction to be of the ſame magnitude as if it was removed to the place *πκ* and viewed there by the naked eye [b]; and therefore the apparent magnitude is to the true, as *OQ* to *Oκ* [*], Suppoſe now the arch *AC* to be very ſmall and the incident and refracted ray *OAB* to keep fixt, while the ſegment *RACS* increaſes from nothing till it becomes a whole ſphere; and the ratio of *OQ* to *Oκ* will at firſt be a ratio of equality, and then will increaſe perpetually [c]. The exact quantity of this ratio may be computed from this theorem deduced from Art. 208, $O\kappa = OQ - OE \times \frac{CQ}{CF}$, wherein *F* is the focus of rays that go parallel within the ſphere.

b Art. 104. * Art. 99, 100.

c Art. 124.

91. The famous M. *de la Hire* has endeavoured to raiſe the antiquity of lenſes or lenticular burning-glaſſes to a very great height [d], imagining he has found them in the *Clouds* of *Ariſtophanes*, Act II. Sc. 1. towards the end. *Strepſiades* an old ſtupid fellow tells *Socrates* he had found out an excellent contrivance againſt paying his debts. Strep. *Have you never ſeen at the apothecaries that fine tranſparent ſtone with which they kindle fire?* Socrat. *Do you mean the glaſs?* Strep. *Yes.* Socrat. *Bring it; what then?* Strep. *When the attorney hath written an action againſt me, I will take this glaſs, and ſtanding at a diſtance, in this manner againſt the ſun, I will melt down the letters of my action.* Socrat. *Cunningly done by the Graces.* The Scholiaſt upon this place ſays it was a round [e] thick glaſs made on purpoſe for this uſe; that they rubbed it with oyl and heated it, and then they fitted to it, or brought near, a match (for the greek word here is equivocal) and after this manner the fire was lighted. *De la Hire* cannot underſtand what the oyl was for, unleſs it was to poliſh the glaſs. But be that as it will, he ſays the Scholiaſt conceived it was convex, which ſhews that in his time, though later than *Ariſtophanes*, they uſed ſuch glaſſes to kindle a fire.

Burning-ſpheres known to the ancients.

d Hiſt. de l'Acad. R. des Sci. 1708.

e Τεγχοειδὲς round like a wheel.

92. He argues alſo from the words Ἀποστέρα σταὶς *ſtanding at a diſtance*, that this glaſs was lenticular rather than ſpherical; becauſe a ſphere burns an object only at a very ſmall diſtance from it. But Mr. *Waller* obſerves [f] that conſidering the whole deſign of this Play was to ridicule *Socrates*, it was proper enough to bring in an old coxcomb boaſting of an invention for doing what indeed it could not perform. To this remark Mr. *Waller* has added others upon ſome curious paſſages from the ancients, concerning their burning glaſſes by refraction; from which he concludes they had no other than whole ſpheres, and that theſe were chiefly uſed by Surgeons

f Philoſ. Exp. and Obſ. by *Hook* &c. 80. p. 348.

to

to burn the flesh of sick persons that needed cauterizing, and by the Vestal virgins to kindle the sacred fire. I believe *Vitellio* was the first person that demonstrated the reason of this effect. Optic. Lib. 10. Th. 48.

Their mistake about burning with speculums.

93. Considering that catoptricks was known and cultivated by the ancients long before dioptricks, it is surprising they could not account for burning by reflection from a concave metal. *Euclid* in his catoptricks says its center is the burning point[b], because all the rays which pass through it are returned directly back to it. But as the sun's diameter is so small, these rays are but very few; and the consequence would be that a very broad speculum would burn no better than a very narrow one, which is contrary to experience. It is evident from this and many other blunders in that book, that *Euclid* the geometer was not the author of it, and also that the ancients made very gross experiments.

b Prop. ult. & *Vitellio* Lib. 9. Prop. 37.

Philos. Exp. and Obs. by *Hook* &c. p. 351.

94. But since the effects of burning with solid spheres or glass bottles filled with water, was so well known to the ancients, how came it to pass that they did not also know their effects in magnifying objects. Had the greek and latin philosophers known this augmentation of objects, would they not have mentioned it frequently; and would not several metaphors and allusions to it have been brought into their language? Mr. *de la Hire* accounts for this oversight of theirs partly from their false notions about the manner of vision, viz. by certain whimsical emanations from the eye, that went out in quest of objects, or else by little representations in miniature, which came from them and sought out for our eyes; so that having no suspicion of pencils of rays nor of our focuses, they could see no analogy between a burning-glass and the manner of vision. This indeed is a sufficient reason for their not discovering its magnifying power by theory; but can it be supposed they never looked through these spheres? To this our author answers; that the focus of a sphere of glass is at the distance of half the radius from the nearest surface; so that if these spheres had been six inches in diameter, which is the most they can be supposed to be, the object in view must have been placed at one inch and a half from the sphere to be seen distinctly. But it is natural and almost necessary that when any one had looked through these spheres, the objects in view would have been farther off, which instead of appearing bigger would only have looked confused. A defined or distinct augmentation of distant objects, requires either very large spheres, which is impracticable, or portions of large spheres, as is now practised with great success. And besides they must have known how to have wrought and ground their glasses as we do; whereas in all probability the ancients knew only how to blow their glass and make vessels of it.

Upon ART. 94.

Much light stopt by the atmosphere.

95. M. *Bouguer* Professor of Hydrography at *Croisic*, has found by experiments that the light of the moon is frequently 2000 times weaker in the horizon, than at the altitude of 66 degrees. And that the proportion of her lights at the altitudes of 66 and 19 degrees is about 3 to 2. And the lights of the sun must bear the same proportion to each other at those heights; which he made choice of as being the meridian heights of the sun at the summer and winter solstices in the latitude of *Croisic*[c]. His manner of tryal was this. When the moon was 19 degrees high, he received her light perpendicularly upon a white paper, and also the light of 4 candles perpendicularly upon another paper hard by; then he ordered the distance of the candles to be varied, till he judged their light upon the paper to be equal to that of the moon upon the other paper; and found the distance of the candles from the paper to be 50 feet. Then he repeated the same experiment when the moon was 66 degrees high, and found her light equal to that of the same candles at the distance of 41 feet from the paper. The proportion of these lights is therefore as the square of 50 to the square of 41 *, or in round numbers about 3 to 2. The horizontal light of the moon was found in the same way, but is subject to greater variations, by passing through more vapours.

c Essai dioptrique sur la gradation de la lumiere pag. 2

* Art. 58.

Upon ART. 95.

The proportion of day-light and moon light by experiment. ibid. p. 28

96. By the like experiments this ingenious author finds the light of the full moon to be about 300000 times weaker than that of the sun, at a medium of several tryals. I found it by theory not much above 90000 times weaker; the difference may arise chiefly from the loss of light in the moon's body, which could not be allowed for in theory. M. *Bouguer* proceeded thus. He received the light of the sun when 31 degrees high, into a dark room through a concave lens, placed against a round hole, one line in diameter, made in the window-shutter; and at a certain distance between 5 and 6 feet from the hole, the diameter of the transmitted light upon a white paper was 108 lines. The direct light of the sun was therefore weakened in the ratio of the square of 108 to 1 *, that is 11664 times; and then it appeared equal to the light of a candle upon a paper held 16 inches from it. At another time when the full moon was also 31 degrees high, and the paper was placed at such a distance from the same lens, that the diameter of the transmitted light was 8 lines, he judged it equal to the light upon a paper coming from

* Art. 58.

the

the ſame candle 50 feet from it. To find the reſult of theſe two experiments, it is to be conſidered that the moon's light was weakened but 64 times by the concave lens; and therefore to weaken it 11664, (that is as much as the ſun's light was weakened by the ſame concave,) or to weaken the equal candle light 11664 times, the candle muſt be removed from the ſaid diſtance of 50 feet to 675 feet[a]. Becauſe the ſquares of 50 and 675 are as 64 to 11664. But the candle whoſe light equalled that of the ſun weakened 11664 times, was but 16 inches or $1\frac{1}{3}$ foot from the paper. Therefore the ſquares of the diſtances 675 and $1\frac{1}{3}$ give the ratio of two degrees of candle light equal reſpectively to thoſe of the ſun and moon. This ratio is as 256289 to 1. But by taking a medium among ſeveral experiments he concludes upon 300000 to 1, when the moon is at her middle diſtance from the earth. For when ſhe is in perigee and apogee the proportion of her lights is about 4 to 3. So far M. *Bouguer*.

[a] Art. 58.

The proportion of day-light to moon-light demonſtrated by theory. Fig. 19.

97. My rule may be thus demonſtrated. Let the little circle *cfdg* repreſent the moon's body half inlightened by the ſun, and the great circle *aeb*, a ſpherical ſhell concentrick to the moon and touching the earth; *ab* any diameter of that ſhell perpendicular to a great circle of the moon's body, repreſented by its diameter *cd*; *e* the place of the ſhell receiving full moon-light from the bright hemiſphere *fdg*. Now becauſe the ſurface of the moon is rough like that of the earth, we may allow that the ſun's rays, incident upon any ſmall part of it, with any obliquity, are reflected from it every way alike, as if they were emitted. And therefore if the ſegment *df* ſhone alone, the points *a*, *e* would be equally illuſtrated by it; and likewiſe if the remaining bright ſegment *dg* ſhone alone, the points *b*, *e* would be equally illuſtrated by it. Conſequently if the light at the point *a* was increaſed by the light at *b*, it would become equal to the full moon-light at *e*. And conceiving the ſame transfer to be made from every point of the hemiſpherical ſurface *hbik* to their oppoſite points in the hemiſphere *kaeh*, the former hemiſphere would be left quite dark, and the latter would be uniformly illuſtrated with full moon-light; ariſing from a quantity of the ſun's light which immediately before its incidence on the moon, would uniformly illuſtrate a circular plane equal to a great circle of her body, called her diſk. Therefore the quantities of light being the ſame upon both ſurfaces, the denſity of the ſun's incident light, is to the denſity of full moon light, as that hemiſpherical ſurface *hek* is to the ſaid diſk; that is, as any other hemiſpherical ſurface whoſe center is at the eye, to that part of it which the moon's diſk appears to poſſeſs very nearly, becauſe it ſubtends but a ſmall angle at the eye: that is, as the radius of the hemiſphere to the verſed ſine of the moon's apparent ſemi-diameter[b], or as 10000000 to $1106\frac{2}{3}$ or as 90400 to 1; taking the moon's mean horizontal diameter to be 16'. 7".

[b] Selecta ex *Archimede* Theoremata ſub finem Euclid. *Cantab.* 1722. p. 304. prop. 27.

98. Strictly ſpeaking this rule compares moon-light at the earth with day-light at the moon; the medium of which at her quadratures, is the ſame as our day-light; but is leſs at her full in the duplicate ratio of 365 to 366 or thereabout, that is, of the ſun's diſtances from the earth and full moon: and therefore full moon-light would be to our day-light, as about 1 to 90900, if no rays were loſt at the moon.

99. Secondly I ſay that full moon-light is to any other moon-light, as the whole diſk of the moon, to the part that appears inlightened, conſidered upon a plane ſurface. For now let the earth be at *b*, and let *dl* be perpendicular to *fg* and *gm* to *cd*; then it is plain that *gl* is equal to *dm*; and that *gl* is equal to a perpendicular ſection of the ſun's rays incident upon the arch *dg*, which at *b* appears equal to *dm*; the eye being unable to diſtinguiſh the unequal diſtances of its parts. In like manner conceiving the moon's ſurface to conſiſt of innumerable phyſical circles parallel to *cfdg*, as repreſented at *A*, the ſame reaſon holds for every one of theſe circles as for *cfdg*. It follows then that the bright part of the ſurface viſible at *b*, when reduced to a flat as repreſented at *B*, by the creſcent *pdqmp*, will be equal and ſimilar to a perpendicular ſection of all the rays incident on that part, repreſented at *C* by the creſcent *pgqlp*. Now the whole diſk being in proportion to this creſcent, as the quantities of light incident upon them; and the light falling upon every rough particle being equally rarified in diverging to the eye at *b*, conſidered as equidiſtant from them all; it follows that full moon-light is to this moon-light, as the whole diſk *pdqc*, to the creſcent *pdqmp*. Fig. 20.

100. Therefore by compounding this ratio with that in the former remark, day light is to moon-light, as the ſurface of an hemiſphere whoſe center is at the eye, to the part of that ſurface which appears to be poſſeſſed by the inlightened part of the moon.

Upon ART. 97.

101. This magnitude of a ſenſible point upon the retina was computed by the rule in art. 374 with the numbers in remark 22.

Upon Chapter 4. Concerning viſion with glaſſes.

Upon ART. 118.

Single and double microſcopes compared.

102. M. *Huygens* obſerves that a convex lens has ſome advantage over a little globule that magnifies as much; becauſe the diſtance between the object and the lens is three times greater than between the object and the neareſt part of the globule [a]; and ſo the lens allows more room for a ſide light to fall upon the object; by which its colours may be obſerved; otherwiſe, if tranſparent, it muſt be viewed by light tranſmitted through it. This indeed is ſome advantage in glaſſes that magnify but little, in others none at all; that diſtance from the lens being ſo very ſmall. But opake objects may be better obſerved by double microſcopes, in which they lye farther from you, and alſo farther from the object-glaſs; and this indeed is one of their chief advantages above ſingle microſcopes; which generally exceed them in the power of magnifying.

a *Hugenii* dioptr. prop. 59.

Upon ART. 120, 121, 122, 123.

History of the invention of teleſcopes. Dioptric. pag. 163.

103. We are obliged to *Hugenius* for the moſt judicious hiſtory of the invention of teleſcopes and microſcopes; which I will here tranſlate, and make ſome additions to it. "The moſt valuable and principal of all the ſubjects that are handled in dioptricks, is the theory of teleſcopes. The worth and excellency of this great and noble invention, can never be ſufficiently declared and commended. For to ſay nothing now of its other uſes, how great is this one, that it has opened a way to various ſubjects of contemplation upon the heavens, which could not be inveſtigated by any other means. Hence many wonderful things in nature, and in a word the whole ſyſtem of the univerſe, is laid open to our view: ſo that in what region of it this earth of ours and we its inhabitants are placed, is now no longer doubtful but indiſputably certain. And in my opinion the wit and induſtry of man has not produced any thing ſo noble, and ſo worthy of his faculties, as this ſort of knowledge: inſomuch that if any particular perſon had been ſo diligent and ſagacious, as to invent this inſtrument from the principles of nature and geometry; for my part I ſhould have thought his abilities were more than human. But the caſe is ſo far from this, that the moſt learned men have not yet been able ſufficiently to explain the reaſons of the effects of this caſual invention.

The inventors,

104. There are ſome that give the praiſe of the firſt invention of this inſtrument, though certainly caſual as I obſerved, to *James Metius* a Dutchman and citizen of *Alcmaria*. But to my certain knowledge, teleſcopes were made before this, at *Middleburgh* in *Zeland* about the year 1609, by a certain ſpectacle-maker in that city; either by *John Lipperſheim*, whom *Sirturus* mentions, or elſe by *Zacharias* whom *Borellus* makes the firſt inventor of them, in his book *de Vero Teleſcopii inventore*. The teleſcopes then made by theſe two artiſts, were but a foot and a half long. Nevertheleſs it is certain that *John Baptiſta Porta* had delivered ſome ſketches of this art, in his books upon Dioptricks and *Natural Magick*, printed 15 years before any teleſcopes were made by either of theſe Dutchmen. In theſe books he ſpeaks of certain inſtruments of his that ſhewed very diſtant objects as if they were near at hand; and of combining concave and convex lenſes [b]. But that he made no great progreſs in this art, is hence evident, that in all that time it did not become famous; and that he did not diſcover any of thoſe things in the heavens, that were obſerved afterwards. And this ſhews that his invention was not owing to his ſkill, but to ſome accidental experiments. For though he had ſome degree of knowledge in mathematicks, yet he did not underſtand thoſe fundamental principles and abſtruſe theorems in dioptricks, that are neceſſary to invent a teleſcope by reaſon; and much leſs did thoſe illiterate mechanicks that I mentioned before. But no wonder that good luck and caſual experiments ſhould produce this effect; after ſpectacles and concave glaſſes, for defective eyes of both ſorts, had been commonly uſed for above 300 years [c]. It is rather ſurpriſing that ſo obvious a thing ſhould be ſo long unknown.

b Magiæ Naturæ lib. 17. cap. 10.

c Rem. 67.

Teleſcopick obſervations by *Galileo* and *Hugenius*.

105. But as ſoon as the fame of theſe dutch teleſcopes was ſpread abroad, *Galileo* immediately made others of the like ſort, and ſoon after much better; by which he firſt diſcovered thoſe celebrated appearances in the heavens; as the mountains and valleys in the moon, the ſpots in the ſun, and thereby his rotation about himſelf; the ſatellits of Jupiter; the phaſes of Venus like thoſe of the moon, and their various apparent magnitudes; the great difference between the apparent diameters of the planets and of fixt ſtars; and of theſe a much greater multitude than had ever been known. He alſo obſerved the phænomena of Saturn as well as he could with ſuch ſhort teleſcopes; but could not diſcover his true ſhape, nor any one after him for many years. For though they had lengthened their tubes very much, yet they improved their power and efficacy but very little. As to my ſelf, I undertook this buſineſs with a better proſpect of ſucceſs; and after I had acquired a thorough knowledge of the laws of refraction, and had made my own object-glaſſes and teleſcopes above twenty feet long, with theſe I diſcovered the true

forms

forms of ſaturn, never ſeen before; and the cauſe of them to be a ring ſurrounding his body; the like of which is not to be found in any other planet. I alſo diſcovered a ſatellit revolving about him in 16 days; all which I publiſhed 26 years ago in a treatiſe by it ſelf [a]. By the encouragement of theſe diſcoveries Aſtronomers and Artificers went on to make longer teleſcopes; the beſt of which are made by *Campani* at *Rome*. By theſe, about ten years after, *Caſſini* had the good fortune to diſcover two more ſatellits beſides mine. He alſo obſerved ſome ſpots in Jupiter and Mars, by which he determined the times of their rotations about themſelves.

ystema urnium. 1659.

leſcopes proved Huge-

106. Theſe are the preſent improvements of this noble inſtrument; and this is a ſummary of thoſe things in the heavens, which it has revealed to the inhabitants of the earth. And who is ſo ſtupid as not to perceive how great and excellent they are? and who does not underſtand, that has any philoſophy, what a light they afford to the contemplation of nature? Certainly we may congratulate the happineſs of the preſent age, for ſo great acquiſitions of knowledge. Thoſe famous and excellent men, *Copernicus*, *Regiomantanus*, *Brabeus*, ſo lately excluded from it, what an immenſe treaſure would they have given for it? And thoſe ancient philoſophers, *Pythagoras*, *Democritus*, *Anaxagoras*, *Philolaus*, *Plato*, *Hipparchus*; would they not have travelled over all the countries in the world for the ſake of knowing ſuch ſecrets of nature, and of enjoying ſuch ſights as theſe? and perhaps we may ſhortly expect a diſcovery of many more and quite new ones, by practiſing a late invention of mine for uſing a long teleſcopick glaſs, without that exceſſive trouble in managing thoſe long and weighty tubes, and indeed without any tubes at all; it being as eaſy now to obſerve with an object-glaſs of 100 or 200 feet focal diſtance, as it was before with a tube of ten feet [b]. Eſpecially ſince many perſons have now undertaken to cultivate the art of grinding and poliſhing ſuch large glaſſes; which I alſo after a long intermiſſion have reſumed again with good ſucceſs.

Art. 892.

eir magfying wet not demonated.

107. But now for the cauſes and properties of this artificial eye, which are not yet ſo happily explained. The chief thing ſtill wanting and not yet demonſtrated is this. Having the form and poſition of the lenſes, to find the manner and meaſure by which they magnify objects. For *Kepler* has not done it, though highly commendable for his new diſcoveries in his dioptricks. Nor had *Des Cartes* better ſucceſs; for in truth he is quite miſtaken in his method of demonſtrating the effects of teleſcopes. This indeed is hardly credible of ſo great a man and ſo converſant in theſe things; yet it was proper to be mentioned, to ſave people the trouble of endeavouring to underſtand what can never be made ſenſe of. And though many others have ſince been labouring at the ſame problem, which is the chief of all, yet none of them have been able to ſolve it.

108. So far *Hugenius*. *Borellus* in his book *de Teleſcopii inventore* [c] has printed the affidavits of ſeveral inhabitants of the city of *Middleburgh*, made before the Conſuls in the year 1655; wherein the ſon of *Zacharias Janſen* atteſts, that his father, a ſpectacle-maker of that city, made teleſcopes in the year 1590, as he had often been informed; and preſented one of them to Prince *Maurice* and another to the Arch-Duke *Albert*; and that *Metius* and *Drebel* in the year 1620 came to *Middleburgh* and bought of theſe teleſcopes; which at firſt were but 16 inches long. Another atteſts upon his own knowledge that his neighbour *Hans Lipperhey*, a ſpectacle-maker in the ſame city, made teleſcopes before the year 1605; two others ſay before the year 1609 or 1610. Likewiſe *Antonius de Rheita* in his *Oculus Enoch & Eliæ* publiſhed at *Antwerp* in 1645, fixes *Lipperhey*'s invention in the year 1609, and that he caſually hit upon it, by happening to look through a concave and a convex ſpectacle-glaſs, both at once, while he held them in each hand; that he himſelf upon *Kepler*'s thought, made the firſt inverting teleſcope with a convex eye-glaſs; which takes in much more at a view than the other; that he alſo invented and perfected the Binocular teleſcope [d], which ſhews things much more bright and vivid and ſeemingly larger too than a ſingle teleſcope. But *Fontana* in his *Novæ obſervationes cæleſtium & terreſtrium rerum*, printed at *Naples* in 1646, ſays that he invented the inverting aſtronomical teleſcope in the year 1608, three years before *Kepler*'s dioptricks were printed. But his friend *Zupus* only teſtifies that this author ſhewed him one of them in 1614.

Additions to the hiſtory of teleſcopes.

c pag. 29.

d Art. 974.

109. *Galileo* has given us the following account of his part of the invention of teleſcopes in his *Nuncius Sidereus* publiſhed by him in March 1610. "Near ten months ago it was reported that a certain Dutchman had made a perſpective, through which very diſtant objects appeared diſtinct as if they were near; ſeveral experiments were reported of this wonderful effect, which ſome believed and others denyed. But having had it confirmed to me a few days after, by a letter from the noble *James Badovere* at *Paris*, I applyed my ſelf to conſider the reaſon of it, and by what means I might contrive a like inſtrument, which I attained to ſoon after by the doctrine of refractions. And firſt I prepared a leaden tube in whoſe extremities I fitted two ſpectacle-glaſſes, both of them plane on one ſide, and on the other ſide, one of them ſpherically convex, and the other concave. Then applying

Galileo a ſecond inventor of teleſcopes. pag. 2.

my eye to the concave, I ſaw objects appear pretty large and pretty near me; they appeared three times nearer, and nine times larger [in ſurface] than to the naked eye. And ſoon after I made another which repreſented objects above 60 times larger; and at laſt having ſpared no labour nor expence, I made an inſtrument ſo excellent as to ſhew things almoſt a thouſand times larger and above thirty times nearer than to the naked eye. Whoever would be ſatisfied of the truth of the obſervations I am going to relate, muſt have an inſtrument that ſhall make things appear 20 times nearer at leaſt. Which he may be ſure of by this experiment. Let him make two ſquares or two circles, one 400 times larger than the other, as it will be if its diameter be made 20 times longer; then having fixt them both upon a very diſtant wall, let him view them both at once, that is the leſſer with the perſpective and the larger with his other eye, which may be done conveniently enough with both eyes open at the ſame time; and then the two papers will appear equal in bigneſs if the inſtrument magnifies as was expected.

110. He adds in his *Saggiatore*, that he was at *Venice* when he heard of the effects of Prince *Maurice*'s inſtrument, but nothing of its conſtruction; that the firſt night after his return to *Padua* he ſolved the problem, and made his inſtrument the next day, and ſoon after preſented it to the *Doge* of *Venice*; who to do him honour for his grand invention, gave him the Ducal Letters, which ſettled him for life in his Lectureſhip at *Padua*, and doubled his Salary; which then became treble of what any of his predeceſſors had enjoyed before. So far *Galileo*; whoſe beſt teleſcope ſo famous for its grand diſcoveries, may be reckoned about a yard long, as appears by art. 364.

Friet *Bacon*'s pretence to this invention examined.

111. Hitherto we have made it appear that teleſcopes were certainly in uſe ſoon after the beginning of the ſeventeenth century. Let us now conſider what pretences there are for a greater antiquity of this invention. Mr. *William Molyneux* has told us that Frier *Bacon* who dyed in 1292, *did underſtand all ſorts of optical glaſſes perfectly well; and knew likewiſe the way of combining them, ſo as to compoſe ſome ſuch inſtrument as our teleſcope* [a]; and his ſon, the Honourable *Samuel Molyneux*, has declared his opinion rather more expreſsly, *that the invention of teleſcopes, in its firſt original, was certainly put in practice by an Engliſhman, Frier* Bacon; *although its firſt application to aſtronomical purpoſes may be juſtly attributed to* Galileo [b]. To examine this opinion of theſe learned and judicious perſons, which has ſince been followed by ſome others, it is neceſſary to tranſcribe a whole chapter of our author's upon viſion by refractions, which immediately follows after his diſcourſe upon viſion by reflections [c].

a Rem. 81.

b Art. 781. Chap. 2.

c Opus M jus *Lond.* 733. pag 307.

112. *De viſione fracta majora ſunt. Nam de facili patet per canones ſupradictos, quod maxima poſſunt apparere minima, & e contra; & longe diſtantia videbuntur propinquiſſime, & e converſo. Nam poſſumus ſic figurare perſpicua, & taliter ea ordinare reſpectu noſtri viſus & rerum, quod frangentur radii & flectentur quorſumcunque voluerimus; ut ſub quocunque angulo voluerimus, videbimus rem prope vel longe: & ſic ex incredibili diſtantia legeremus literas minutiſſimas; & pulveres ac arenas numeraremus propter magnitudinem anguli ſub quo videremus; & maxima corpora de prope vix videremus propter parvitatem anguli ſub quo videremus. Nam diſtantia non facit ad hujuſmodi viſiones niſi per accidens, ſed quantitas anguli. Et ſic poſſet puer apparere gigas, & unus homo videri mons, & in quacunque quantitate; ſecundum quod poſſemus hominem videre ſub angulo tanto ſicut montem & prope ut volumus. Et ſic parvus exercitus videretur maximus, & longe poſitus appareret prope, & e contra. Sic etiam faceremus ſolem & lunam & ſtellas deſcendere ſecundum apparentiam hic inferius; & ſimiliter ſuper capita inimicorum apparere; & multa conſimilia; ut animus mortalis ignorans veritatem non poſſet ſuſtinere.*

What problems in viſion he thought practicable by refractions.

113. In Engliſh thus. Greater things than theſe may be performed by refracted viſion. For it is eaſy to underſtand by the canons abovementioned, that the greateſt things may appear exceeding ſmall, and on the contrary; alſo that the moſt remote objects may appear juſt at hand, and on the contrary. For we can give ſuch figures to tranſparent bodies, and diſpoſe them in ſuch order with reſpect to the eye and the objects, that the rays ſhall be refracted and bent towards any place we pleaſe; ſo that we ſhall ſee the object near at hand or at a diſtance, under any angle we pleaſe. And thus from an incredible diſtance we may read the ſmalleſt letters, and may number the ſmalleſt particles of duſt and ſand, by reaſon of the greatneſs of the angle under which we may ſee them; and on the contrary we may not be able to ſee the greateſt bodies juſt by us, by reaſon of the ſmalneſs of the angles under which they may appear. For diſtance does not affect this kind of viſion, excepting by accident, but the quantity of the angle. And thus a boy may appear to be a giant, and a man as big as a mountain, foraſmuch as we may ſee the man under as great an angle as the mountain and as near as we pleaſe. And thus a ſmall army may appear a very great one, and though very far off yet very near us, and on the contrary. Thus alſo the ſun, moon and ſtars may be made to deſcend hither in appearance, and to appear over the heads of our enemies;

and

and many things of the like ſort, which would aſtoniſh unſkilful perſons.

and what by reflections.

114 In the foregoing chapter upon viſion by reflections, he has alſo told us, that by ſeveral ſpeculums properly diſpoſed, a ſingle ſoldier may be multiplyed ſo as to appear like an army; and one army like many armies, for terrifying infidels and enemies; that ſpeculums may be raiſed up on high againſt oppoſite armies and cities, ſo as to diſcover all the enemies hidden tranſactions; and that this may be done from any diſtance whatever. Becauſe by the Book upon ſpeculums, one and the ſame thing may be ſeen by reflection from as many ſpeculums as we pleaſe, if they be diſpoſed in proper ſituations; and therefore ſome may be placed nearer and ſome farther off; ſo that the object may be ſeen from any diſtance whatever, *ut videremus rem quantum a longe vellemus.*

They are much alike.

115. I have added this extract to the other paſſage, to ſhew that his thoughts about the manner of executing theſe deſigns by reflections and by refractions are much alike; as is evident from the paſſage laſt mentioned compared with this, *figurare perſpicua & taliter ea ordinare reſpectu noſtri viſus & rerum, quod frangentur & flectentur radii quorſumcunque voluerimus*, to diſpoſe the glaſſes in ſuch order with reſpect to the eye and objects, *that the rays ſhall be refracted and bent towards any place we pleaſe.*

His ſolution of them impracticable.

116. It ſeems then as if he did not think of performing theſe problems by a ſingle portable inſtrument like a teleſcope; but by fixing up ſeveral glaſſes in proper places at large intervals from one another; which would certainly prove ineffectual. Becauſe no ſurfaces can be figured and poliſhed ſo perfectly, as to reflect and refract light to very great diſtances, without great aberrations, or ſtraglings of the rays, from the places or points intended; and theſe aberrations would be increaſed by the interpoſition of more glaſſes; to ſay nothing of the loſs of light at every ſurface, and of colours generated by refractions from ſuch great diſtances; ſo that the object muſt appear at laſt ſo faint, ſo deformed and confuſed as to excite no determinate idea.

He relates a fact impracticable by ſpeculums.

117. What he mentions of *Julius Cæſar* [a], that he raiſed up ſpeculums to a great height upon the coaſt of *France*, to diſcover the diſpoſition of the cities and camps in *England*, when he was going to invade it, is therefore impracticable; and probably a fiction, if there be not a miſtake in the interpretation of the word *ſpecula* for glaſſes inſtead of a *watch-tower*. The ſame is to be underſtood of the ſtory mentioned by *Porta* [b], that *Ptolomy* by ſpeculums could diſcern ſhips at the diſtance of ſix hundred miles; which could not poſſibly be done by our beſt teleſcopes. Mr. *Waller* imagines he had his intelligence by *ſpeculis* or watch-towers, placed at ſeveral intermediate diſtances; where ſeveral ſigns might be given and received ſucceſſively from the firſt to the laſt [c]. Dr. *Hook* has given us ſome ingenious thoughts upon diſcourſing by ſigns at a great diſtance in the ſame book [d].

a ibid. p. 357.

b Magia Natur. lib. 17. cap. 11.

c Philoſ. Exp. and Obſ. by *Hook* &c. p. 36a.

d Ibid. p. 142.

118. But to return to our author, conſidering the falſe notions he had from the ancients about diſtinct and confuſed viſion; the falſe principle he maintains, that the apparent magnitude of an object is as the angle ſubtended at the eye by its image, and reciprocally as the diſtance of the image too; and laſtly the falſe concluſions he has drawn, and muſt always draw, from theſe principles; as I have ſhewn in his attempt upon making ſpectacles [e]; it was certainly next to impoſſible for him to have invented by theory a much nicer and more complex inſtrument, I mean a teleſcope of any ſort. Indeed Mr. *Molyneux* ſays it is manifeſt he knew what a concave and a convex glaſs was [f]. But this does not appear from the paſſage there cited, nor from any other that I have yet met with. Mr. *Molyneux* was miſled by the inaccuracy of the author's language. *Si vero corpora non ſint plana*, per *quæ viſus videt, ſed ſphærica, tunc eſt magna diverſitas; nam vel concavitas corporis eſt verſus oculum vel convexitas* &c. inſtead of this he ſhould have ſaid *ſi vero corporum ſuperficies non ſint planæ*, per *quas viſus videt, ſed ſphæricæ* &c. For in this whole chapter he treats of nothing elſe but the appearance of an object through a ſingle concave or convex ſurface of a large body or medium, the object being placed within it and not beyond it; nor do I find that he has ever conſidered the appearance of an object placed at a diſtance beyond a refracting body of any ſhape. Indeed he mentions the refractions of the ſun's rays through a ſphere, to ſhew how they burn things [g]; but not a word how an object appears through it.

He was not qualified to invent a teleſcope by theory.

e Rem. 87.

f Rem. 82.

g Opus Majus pag. 67. & 397.

119. Beſides, I have ſhewn above, that he never had handled a convex ſpectacle-glaſs when he wrote a paſſage in this ſame treatiſe [h]; and without a variety of lenſes he could try but few experiments, and could never have hit upon a proper combination of them for a teleſcope. Indeed Dr. *Jebb* the editor of the late edition of the *Opus Majus*, in his dedication to Dr. *Mead*, produces a paſſage from a manuſcript [i] to ſhew that this author actually applyed teleſcopes to aſtronomical purpoſes. *Sed longe magis quam hæc oporteret homines haberi, qui bene, immo optime, ſcirent perſpectivam & inſtrumenta ejus, —— quia inſtrumenta aſtronomiæ non vadunt niſi per viſionem ſecundum leges iſtius ſcientiæ.* To this it may be anſwered, that the ancients had ſome occaſion for perſpective in plain inſtruments, before the invention of teleſcopick ones. But as this paſſage ſtands alone, it is not eaſy to know the intent of it; however had there been any more to the

Nor by experiments for want of lenſes.

h Rem. 88.

i MS. Cot. Tib. C. V. fol. 6.

the like purpoſe, no doubt this gentleman, ſo much verſed in this author's works, would have found them out and obliged us with them.

His imagination carried him too far.

120. In ſhort this author ſpeaks only hypothetically, ſaying that glaſſes may be figured and objects may be magnified ſo and ſo; but never aſſerts one ſingle trial or obſervation upon the ſun or moon, (or any thing elſe,) though he mentions them both. On the other hand he conceives ſome effects of teleſcopes that cannot poſſibly be performed by them.

How he came by theſe notions.

121. If it be aſked how he came by theſe notions; I anſwer from the common doctrine of refractions in his Canons, and from common appearances by refraction and reflection; eſpecially from concave ſpeculums, whoſe effects were well known to him, both by the accounts of them in ancient authors, and by his own experience. And this I take to be a ſufficient ground for a man of good ſenſe and fancy to produce all that he has ſaid. I conclude then that the time of the invention of teleſcopes was not earlier than the beginning of the ſeventeenth century.

Hiſtory of the invention of microſcopes. *Hugenii* Dioptr. p. 221.

122. I will add the hiſtory of the invention of microſcopes written by *Hugenius*. "It is probable ſays he, that the uſe of ſimple microſcopes, for magnifying objects with a ſingle globule or a little lens, was known ſoon after the invention of teleſcopes; but the contrivance of compound microſcopes, being leſs obvious, ſeems to have been about ten years later. It does not appear that theſe microſcopes were made in the year 1618; becauſe *Syrturus*, who publiſhed a book that year about the origine and conſtruction of teleſcopes, would hardly have been ſilent upon ſo remarkable an invention, if it had been then known. *Fontana* indeed lays claim to it from the year 1618, in his book of Obſervations publiſhed in 1646; but the teſtimony of *Syrſalis* there printed, goes no higher than the year 1625. But that my countryman *Drebelius* made theſe compound microſcopes at *London* in the year 1621, I have often been informed by ſeveral eye-witneſſes, and that he was then reckoned the firſt inventor of them. For nothing hinders but both theſe perſons may have hit upon the ſame thing, by trying various compoſitions of glaſſes, though quite ignorant of geometry aud of the cauſes of theſe effects.

Another demonſtration of teleſcopes, Fig 21, 22.

[a] Art. 120.

123. So far *Hugenius*, who demonſtrates the effects of teleſcopes in this manner. The glaſſes L, E being placed as uſual[a], in their axis EL produced forwards, take LN equal to Lq, the focal diſtance of the object-glaſs LM. Then any ray as PNM which paſſes through N, and falls upon the object-glaſs LM, will be refracted through it into a line MA parallel to the axis LE; and falling upon the eye-glaſs AE, it will be refracted to the principal focus O; or elſe will diverge from it, if this glaſs be concave. Therefore in both teleſcopes, the object PQ appears under the angle AOE, and to the naked eye at N, under the angle PNQ; and ſo the apparent magnitude is to the true, as the angle AOE to PNQ or LNM, that is, ſince LM equals AE, as the focal diſtance LN, to the focal diſtance EO*. * Art. 60.

124. If two more equal eye-glaſſes BF, CG be ſubjoined to the aſtronomical teleſcope, as in art. 121, and O be the common focus of the glaſſes AE, BF; the ray AOB will again be refracted into a line BC parallel to the axis; and conſequently will be refracted through the laſt eye-glaſs to its principal focus D; where the eye being placed, will ſee the object upright, and magnified juſt as much as before; becauſe GD being equal to EO, the angle CDG is equal to BOF or AOE. Fig. 23.

Colours appear in teleſcopes.

[b] Phil. Tranſ. N°. 1.

125. This excellent compoſition of glaſſes was invented by ſomebody at *Rome*, probably by *Campani*[b]. It is uſual and uſeful to put a plate with a round aperture in it, at the place of the firſt image at q or of the ſecond at x, to circumſcribe the viſible proſpect, and to cut off the colours from the edges of it; though they are not more apparent in this teleſcope with three eye-glaſſes than with one; but rather leſs. For though there be more refractions, yet the colours generated by the refractions at the two firſt glaſſes, made both the ſame way, are ſomewhat corrected and contracted by th refractions at the two laſt; being both made the contrary way to the former[c]. For the edge of a lens has the like effect as the edge of a priſm.

[c] Art. 171.

A more general demonſtration of teleſcopes. Fig. 24, 25, 26, 27.

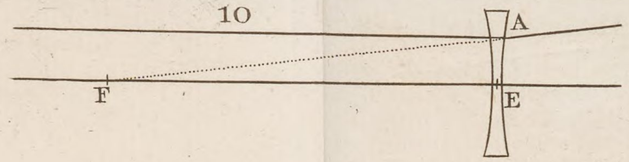
[d] Art. 128.

126. Hitherto we have ſuppoſed the interval LE between the two convex glaſſes to be equal to the ſum of their focal diſtances. Now let this interval be bigger or leſs, as is requiſite for defective eyes[d], and let EF be the focal diſtance of the eye-glaſs, and Lq that of the object-glaſs; I ſay the apparent magnitúde will be to the true as LF to FE, that is as the interval of the glaſſes diminiſhed by the focal diſtance of the eye-glaſs, to the focal diſtance of the eye-glaſs. For the axes of all the pencils which paſs through L, as PLA, will be refracted by the eye-glaſs to a focus G, where the eye being placed will ſee the whole object PQ, though the aperture of the pupil and of the object-glaſs be never ſo ſmall; and the object PQ will appear under the angle AGE. But L being a focus of incident rays upon the eye-glaſs, we have $LF : LE :: LE : LG$*, and disjointly $LF : FE :: (LE : EG ::$ the angle EGA, to the angle ELA* or PLQ*) as the apparent magnitude, to the true. * Art. 239. * Art. 60. * Art. 43.

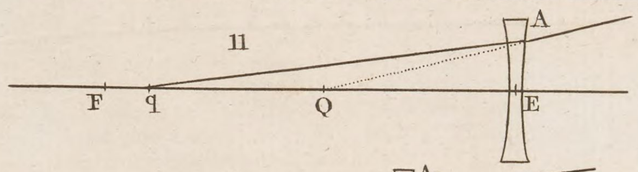
127. Hence according as the interval of the glaſſes is greater or leſs than the ſum of their focal diſtances, the apparent magnitude is to the true, in a greater or leſs proportion than that of the focal diſtances.

128.

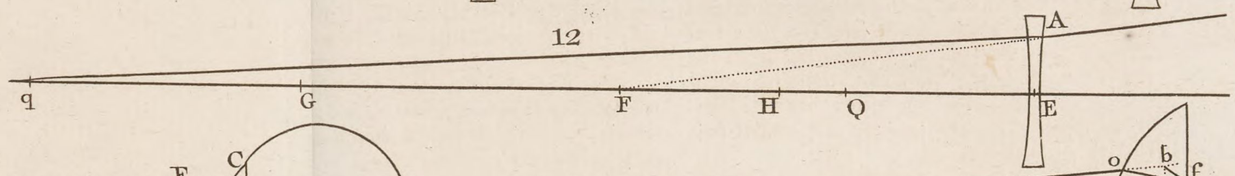
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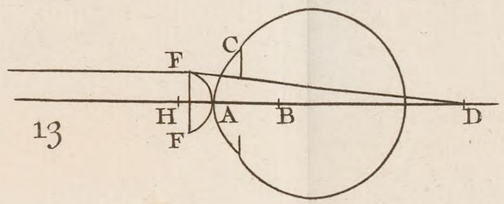
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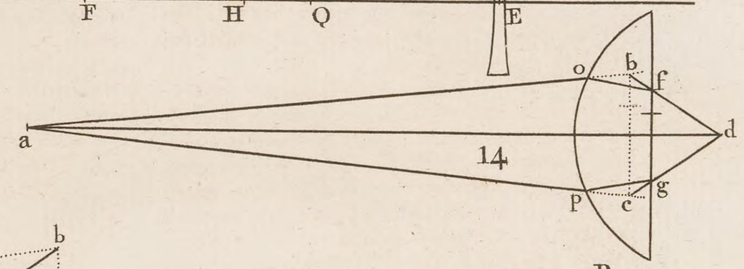
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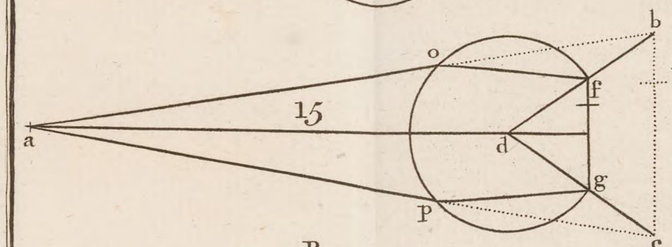
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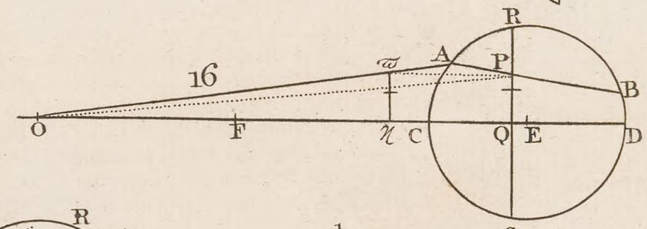
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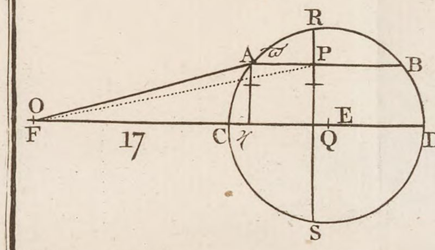
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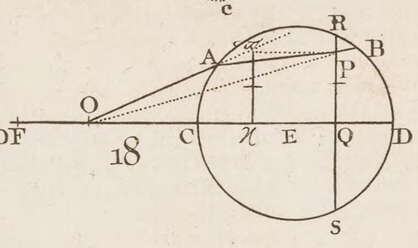
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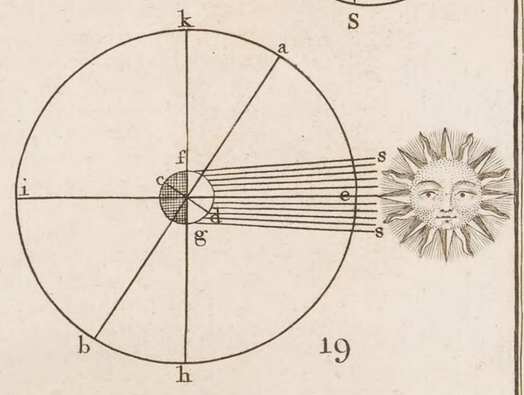
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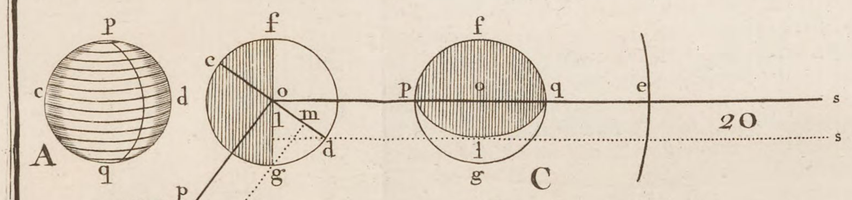
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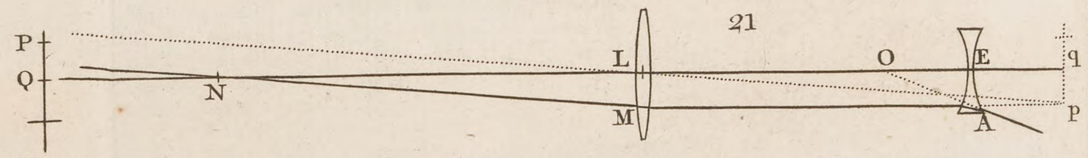
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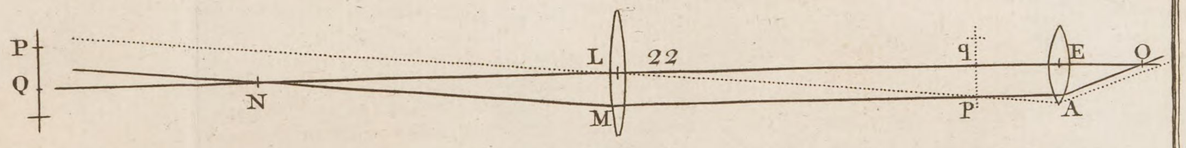
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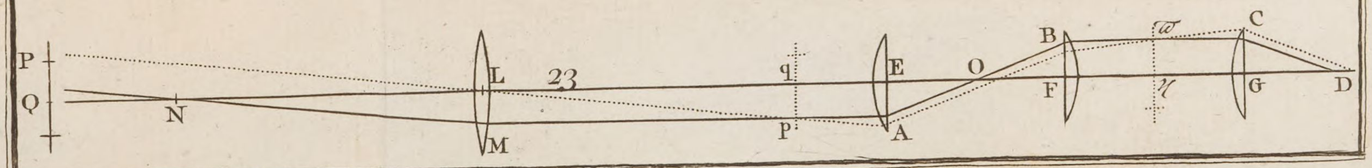
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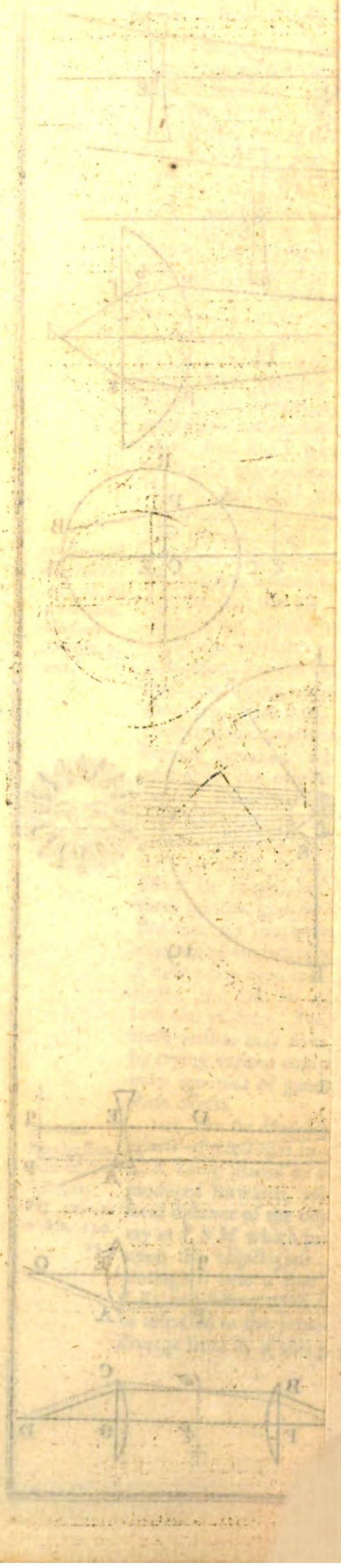


22



23





To fit a telescope for observing eclipses and spots in the sun.

128. When the telescope is lengthened, the place of the image or focus q is farther from the eye-glass than its focal distance; and therefore this glass will make the emergent rays converge and form a second image or picture $\varpi\varkappa$ of the object PQ, upon any white surface, placed at the distance $q\varkappa$, which is a third proportional after qF and qE *. And the apparent magnitude of this picture, seen by the naked eye from a distance equal to $\varkappa G$, is the same as if it could be seen distinctly from G through the telescope; the angles $\varpi G\varkappa$, and AGE being equal; and consequently is somewhat greater than when the length of the telescope is adjusted for distinct vision. The same demonstrations are applicable to *Galileo*'s telescope.

* Art. 239.

Brightness of the sun's image.

129. This is the way used by some astronomers, in observing eclipses of the sun and spots upon his body; which may also be observed in looking through the telescope, by applying a plane smoaked glass to the eye, or by smoaking the eye-glass it self. In the other way the image will appear brighter if the end of the telescope be put through a hole in the window-shutter of a dark room. And then if the round image upon the paper be equal to the aperture of the object-glass, it will appear as bright as if the paper was illustrated by the direct light of the sun; supposing no rays were intercepted by reflections from the surfaces of the glasses. And therefore as much as the image is made larger than this, by shortening the telescope a little, it will be so much the fainter, and not quite so distinct. But in this matter experience is the best guide.

Telescopes made with three glasses.

130. In the next book I have shewn the construction of telescopes with three glasses, but those made with two or four are the best, for shewing more of the object at one view, and less coloured about its circumference, as *Hugenius* has observed [a]. An object may be seen upright through two lenses, but not distinct nor much magnified and but little of it at one view; and besides, the distance between the glasses must be much greater than ordinary.

[a] Dioptric. p. 54.

Upon ART. 125.

Theory of refracting telescopes. Lem. 4.

131. After the discovery of the true law of refraction, according to the given ratio of the sines [b], *Des Cartes* and other mathematicians soon found that all the rays of a large pencil could not possibly be collected to a distinct point by any object-glass composed of spherical surfaces [c]; and that the aberrations of the rays from that point were increased with the breadth of the glass. And this they took to be the cause of the apparent indistinctness of an object seen in a telescope, when the focal distance of the eye-glass is too much shortened. For as a shorter eye-glass increases the area of the picture upon the retina [d], it is necessary to inlarge the area of the object-glass for the reception of more light [e]; and this inlargement increases the aberrations of the rays in its focus, and consequently in the picture upon the retina.

[b] p. 81.

[c] Art. 120.

[d] Art. 129.

Hyperbolick and elliptical lenses why introduced. Fig. 28.

132. These aberrations, caused by the spherical surfaces of glasses, were then thought the only impediment to the perfection of telescopes; and this engaged the mathematicians in determining what figure a glass must have to refract all the rays of a pencil to a given point; and among others they found, that glasses figured according to the surfaces described by conick sections, turned about their axes, would have that effect [f]. For example if F and G be the focuses of two opposite hyperbola's, whose axis CD is to FG in the ratio of the sines of incidence and refraction, suppose of 2 to 3; and if one of these hyperbola's ACB be turned about its axis GCD, and a portion of the solid so generated be cut off by a plane AB perpendicular to the axis; then all the rays which fall perpendicular upon this plane will be refracted by the convexity ACB to the outward focus F.

[f] *Cartesii* Dioptr. Cap. 8. *Newtoni* Princip. Phil. Lib. 1. pr. 97. *Huygens* de la Lumiere p. 101. *Dechales* Tom. 3. pag. 681.

They do not answer the purpose.

133. This discovery presently engaged all the mathematicians and mechanicks to contrive engines for grinding and polishing glasses according to the shapes of these conick surfaces; and amongst the rest Sir *Isaac Newton* himself, in the beginning of the year 1666. But having a curiosity at the same time to try the celebrated phænomena of the colours generated by refraction of the sun's rays through a triangular prism, by which he happily discovered their causes, he presently left off his glass works [g]. "For I saw, says he, that the perfection of telescopes was hitherto limited, not so much for want of glasses truly figured according to the prescription of optick authors, (which all men have hitherto imagined,) as because light it self is a heterogeneous mixture of differently refrangible rays [h]. So that were a glass so exactly figured, as to collect any one sort of rays into one point, it could not collect those also into the same point, which having the same incidence upon the same medium, are disposed to suffer a different degree of refraction. This, says he, made me take reflections into consideration; and finding them regular, so that the angle of reflection of all sorts of rays was equal to their angle of incidence, I understood that by their mediation optick instruments might be brought to any degree of perfection imaginable; provided a reflecting substance could be found which would polish as finely as glass, and reflect as much light as glass transmits, and the art of communicating to it a parabolick figure could also be obtained.

[g] Phil. Transf. N°. 80.

[h] Art. 171, &c.

Newton's reflecting telescope when invented.

134. Amidst these thoughts, he tells us, he was forced from *Cambridge* by the intervening Plague in 1666, and proceeded no farther till above two years after. When having thought of a

a tender way of poliſhing, proper for metal, by degrees he perfected an inſtrument 6 inches long, which magnified between 30 and 40 times, and which he firſt deſcribed in the Phil. Tranſ. N°. 80, and afterwards in his Opticks, page 91. 8°. And this is indiſputably the greateſt improvement that teleſcopes have ever received ſince their firſt invention.

Mr. *Gregorie's* was invented before it.

135. Nevertheleſs it muſt be acknowledged that Mr. *James Gregorie* of *Aberdeen* was the firſt inventor of a reflecting teleſcope. But his conſtruction is quite different from Sir *Iſaac Newton*'s, and not near ſo advantageous, as Sir *Iſaac* himſelf has ſhewn in the Phil. Tranſ. N°. 83. Mr *Gregorie* deſcribes this teleſcope at the end of his *Optica promota* publiſhed in 1663; and was led into the invention of it, not by the conſideration of the different refrangibility of rays, which was not then known, but by an inconvenience he foreſaw would follow from an hyperbolick object-glaſs. For he obſerves that if it be ſufficiently broad, to receive light enough into a teleſcope that ſhall magnify very much, it muſt of conſequence be very thick; and then the cleareſt glaſs would hinder too much of the light from being tranſmitted. He might alſo have added another inconvenience, that though it will collect a pencil of rays coming parallel to its axis into a ſingle point, yet it cannot collect the rays of an oblique pencil ſo accurately as a glaſs conſiſting of ſpherical ſurfaces will do; as has been found by experience [a]; and therefore ſpherical lenſes upon this and ſome other accounts are fitter for optical uſes than thoſe of any other figure [b].

[a] *Dechales* Tom. 3. p. 686.
[b] *Newtoni* Princip. lib. 1. prop. 98. Schol.

Mr. *Hadley* brought them both into uſe.
[c] See Art. 782.

136. Theſe reflecting teleſcopes were firſt brought to perfection in practice about the year 1719, by the great ingenuity and induſtry of Mr. *John Hadley* [c]; Sir *Iſaac Newton*'s firſt, and Mr. *Gregorie*'s ſoon after; which in ſmall lengths has an excellent effect and is exceedingly commodious. The following deſcription of it differs from the author's chiefly in this; that he directs his larger reflecting concave to be made of a parabolick figure, and his leſſer of an elliptical one, inſtead of the ſpherical ſurfaces now uſed; which are the only figures that can be poliſhed without inſuperable difficulties.

Mr. *Gregorie's* reflecting teleſcope deſcribed. Fig. 29.

137. It is propoſed to make a reflecting teleſcope with two concave metals and a convex eye-glaſs and to ſhew its effects. Let the given focal diſtances of the leſſer and the larger concave and of the convex eye-glaſs, be equal reſpectively to the lines $t, \mathcal{T}, q$; and in a given line $ctqCl$, deſigned for their common axis, take in one and the ſame direction, $ct = t$, $tq = \mathcal{T}$, $qC = \frac{t \times t}{\mathcal{T}}$ and $ql = q$; and place the eye-glaſs at l, the leſſer concave at c, and the larger at C; ſo that their concavities may reſpect each other; and let the incident rays, as $\mathcal{Q}A$, $\mathcal{Q}B$, be reflected from the larger to the leſſer concave, and from thence to the larger again, where let them paſs through a moderate hole made in the middle of it at C, and then be refracted through the eye-glaſs kl to the eye at o; I ſay a remote object will appear diſtinct and upright and magnified in the ratio of $\mathcal{T} \times \mathcal{T}$ to $t \times q$; that is, of the ſquare of the focal diſtance of the larger concave, to the rectangle under the focal diſtances of the leſſer concave and of the eye-glaſs.

138. For a pencil of rays $\mathcal{Q}A, \mathcal{Q}B$ coming parallel to the common axis, will be reflected from the larger concave ACB to its principal focus $\mathcal{T}$; where croſſing one another and falling upon the leſſer concave acb, they will be reflected from it to the point q. For ſince the focal diſtance $\mathcal{T}C = \mathcal{T} = tq$ by conſtruction; by taking away the common part $\mathcal{T}q$, we have $t\mathcal{T} = qC = \frac{t \times t}{\mathcal{T}}$ by conſtruction; that is, we have $t\mathcal{T}, tc, tq$ continual proportionals, as they ſhould be [d]; and ſince pl is the focal diſtance of the eye-glaſs kl, the rays that flow from q will emerge from it in parallel lines, and therefore will produce a diſtinct appearance of the remote point $\mathcal{Q}$ from which they came.

[d] Art. 20

139. Let $S\mathcal{T}$ be the image of the object $P\mathcal{Q}$ formed by reflection from the large concave; and it will be terminated by the line PES, drawn through E, the center of this concave, parallel to the rays PA, PA that flow from P *. Again the rays that flow from this image $S\mathcal{T}$, will be reflected from the leſſer concave and form a ſecond image pq; which will be terminated by the line Sep drawn through the center e of this concave [e]; and the rays that diverge from p will emerge from the eye-glaſs kl in the lines ko parallel to the line pl, drawn through the center of the eye-glaſs [f]. Therefore the object $P\mathcal{Q}$ will appear upright, becauſe the rays ko lye on the ſame ſide of the common axis $\mathcal{Q}lo$ as the point P from which they came. Fig. 30.

* Art. 21
[e] Art. 21
[f] Art. 46

140. In the ſecond image pq take a line qs equal to the firſt image $\mathcal{T}S$; and if the image pq was equal to qs, the object would appear through the eye-glaſs under an angle equal to qls *; which is to the angle $PE\mathcal{Q}$ or $SE\mathcal{T}$, under which it appears to the naked eye at E, as $\mathcal{T}E$ or $\mathcal{T}C$ to ql *; and ſo the object would be magnified in the ſame ratio as in Sir *Iſaac Newton*'s teleſcope. But ſince the triangles $epq, eS\mathcal{T}$ are ſimilar; and ſince we had tq to te (as te to $t\mathcal{T}$ *, and disjointly as eq to $e\mathcal{T}$, that is,) as pq to $S\mathcal{T}$ or qs; it appears that pq is bigger than qs, and alſo the viſual angle kol or plq bigger than qls, in the ſaid ratio of tq to te. And ſo the object being farther magnified in this ratio of tq to te or, by conſtruction, of $\mathcal{T}C$ to tc, is magnified in the whole in the compound ratio of $\mathcal{T}C$ to tc, and of $\mathcal{T}C$ to ql, that

* Art. 10 111.
* Art. 60
* Rem. 138.

that is in the ratio of TC ſquare, to the rectangle under tc and ql.

141. The magnifying power may alſo be demonſtrated in Mr. *Huygens*'s manner[a]; that is, by conſidering a ray of an oblique pencil to go parallel to the axis between the leſſer concave and the eye-glaſs, and by determining the ratio of the angles in which it will cut the axis.

a Rem. 123.

142. For viewing near objects the little concave muſt be removed a little from the large one by the contrivance mentioned in Art. 924. Becauſe while a remote object approaches, its image TS will alſo approach towards t; and while tT is diminiſhed, its reciprocal tq will be increaſed[b].

To adapt it to a near object.

b Rem. 138. And to ſhort-ſighted eyes.

143. Therefore to fit this teleſcope for a ſhort-ſighted perſon, ſince the eye-glaſs is uſually fixt, the little ſpeculum muſt be moved ſomewhat nearer to the large one. For then the interval tT will alſo be diminiſhed and its reciprocal tq will be increaſed; and ſo the rays will fall upon the eye-glaſs diverging from a nearer point than its focal diſtance; and conſequently will emerge from it diverging upon the eye.

144. By a farther contraction of the interval between the concaves, the image pq may be projected through the hole in the large concave, to any given place behind it; and by removing the eye-glaſs to the ſame diſtance from the image as before, the viſion would become diſtinct again; and the object would be more magnified than before, as much as the ratio of tq to te or tc is made bigger than the ratio of TC to tc; as appears by the demonſtration*. But by enlarging the image pq, it becomes more obſcure and imperfect, as ſhall be ſhewn hereafter, and conſequently the appearance of the object leſs bright and diſtinct. Beſides, as the image becomes larger, the leſs of it, and of the object, can be ſeen at one view through a given eye-glaſs.

A more general rule for the magnifying power.

* Rem. 140.

145. All things being fixt in their places, the diameter of an object taken in at one view, is proportionable to the breadth of the eye glaſs, if the hole in the large concave does not limit it. For the angle of reflection pce, at the middle point of the leſſer concave, being equal to the angle of incidence ecS; it appears, that while pq and kl are increaſed or diminiſhed in any ratio, the image ST and the object PQ will alſo be increaſed or diminiſhed in the ſame ratio.

The viſible area is as the breadth of the eye-glaſs.

146. Now if an eye-glaſs of a given focal diſtance and convexity, be made very broad, it will become too thick; and ſo the rays will fall too obliquely upon one or both its ſurfaces near the margin of it; and this obliquity will cauſe too many of them to be reflected, and the reſt that are tranſmitted, to be too much refracted, in compariſon to thoſe pencils that paſs through the middle of the ſaid lens[c]. Therefore to increaſe the viſible area of the object, it is neceſſary to project the image pq two or three inches beyond the hole in the large concave, and to intercept the rays that are tending towards it, with a thinner and broader convex glaſs fg put cloſe to the backſide of this concave; which glaſs will cauſe the rays to converge quicker than before, and to form an image vx nearer to it, and ſmaller, than pq; both being terminated by a line pvg drawn through the center of this glaſs[d]. And then the rays of each pencil diverging from this new image vx, muſt be received by another convex eye-glaſs hi, that ſhall make them emerge towards the eye in parallel lines. A meniſcus glaſs, whoſe convex ſide is placed towards the converging pencils fvh, is fitteſt for this purpoſe; becauſe the rays will paſs through its edges leſs obliquely, than through a glaſs of any other ſhape.

Is inlarged by two eye-glaſſes.

c Art. 73.

Fig. 31, 32.

d Art. 55.

147. Having the places and focal diſtances of theſe eye-glaſſes, I could eaſily give a rule for finding the magnifying power of the teleſcope[e]; but the meaſures of ſuch ſmall diſtances being liable to errors, it is better to find it by experiment: either in *Galileo*'s way of viewing two unequal circles, one with the naked eye and the other through the teleſcope; or by comparing this teleſcope with a dioptrick one, whoſe magnifying power is known or eaſier to be found[f]. One of theſe teleſcopes 16 inches long, is reckoned to magnify as much as a dioptrick one of 15 or 16 feet.

This teleſcope compared with a common one.

e See Rem. 156.

f Art. 128.

148. To prevent collateral rays, that paſs by the ſides of the ſmaller concave, through the hole in the larger, and thoſe alſo which are reflected from the imperfect margins of them both, from entering into the eye; it is neceſſary to place a thin plate with a proper hole in it to circumſcribe the image at x, and alſo another very ſmall hole at o, where all the pencils croſs one another immediately before they enter the eye. The breadth of this latter hole muſt be no bigger than that of the principal pencil at o, and the places of them both muſt be exactly adjuſted; otherwiſe the teleſcope can have no good effect.

The eye-ſtops.

149. If the focal diſtance of the leſſer concave be a given line t, and it be required to place it ſo, that the rays ſhall be reflected from the given focus T to a given point q; biſect Tq in m, and to mT erect a perpendicular Tn equal to the line t; and joining mn, towards T ſet off mt equal to mn; and t will be the point where the focus of the leſſer concave muſt be placed. For let a ſemicircle, deſcribed with the center m and ſemidiameter mn or mt, cut the axis again in z; and we have qz equal to Tt, and conſequently Tz equal to tq; we have alſo Tn a middle proportional between the ſegments tT, Tz of the diameter tz*, that is, the focal diſtance t or tc is a middle proportional between tT and tq, and therefore the

Some problems ſolved.

Fig. 33.

* Eucl. VI. 13.

rays that flow from T, will be reflected from the
* Art. 107. leſſer concave to the given point q *.

150. Hence alſo, if it be required to find the focal diſtance of the leſſer concave, whoſe focus being placed at a given point t, ſhall cauſe the rays to be reflected from a given point T to a given point q; biſect Tq in m, and with the center m and ſemidiameter mt deſcribe a circle, cutting an indefinite perpendicular erected from T, in the point n; and you have Tn equal to the focal diſtance required.

151. The larger concave, and the convex eye-glaſs, and the interval Tq between the two images of a remote object being given; if it be required to find the focal diſtance and place of the leſſer concave, that ſhall cauſe the teleſcope to magnify the object in any propoſed ratio; ſince this given ratio is compounded of the given ratio of TC to qC and of tq to tc *, this latter ratio is alſo given; for which putting n to 1, take tT to Tq as 1 to $nn-1$; and you have tT; then take tc to tT as n to 1; and you have the poſition and magnitude of tc. For ſince the unknown lines tT, tc, tq are continual proportionals in the given ratio of 1 to n, we have tT to tq as 1 to nn *, and disjointly, tT to Tq as 1 to $nn-1$.

* Rem. 144.

* Eucl. VI. 20. cor. 2.

The little concave ſpeculum may be changed for a convex one. Fig. 34.

152. Teleſcopes of this kind are ſometimes made with a little convex ſpeculum inſtead of the concave one. If their focal diſtances be equal, and the vertex of the convex de, be placed at e, where the center of the concave was, the teleſcope will magnify in the ſame ratio as before; but will ſhew the object inverted; unleſs it be ſet upright by three convex eye-glaſſes, as in a dioptrick teleſcope. For a pencil of rays converging from the large concave towards its focus T, being intercepted by the little convex de, will be reflected by it to the ſame point q as before by the little concave bc. For the point t being the principal focus of both theſe little ſpeculums, we have tT, te (or tc) and tq continual proportionals as before [a]. Through any point S of the firſt image ST and through the center e of the little concave, draw Sep terminating the image pq formed by this concave [b]; in like manner through c the center of the little convex de, and through the ſame point S, draw cSr terminating the image qr formed by this convex. Theſe images qp, qr lye on contrary ſides of the axis, and therefore the object appears in contrary poſitions. But theſe images are equal, and of conſequence the object appears equally magnified. For we have $tq : te :: te : tT :: tq \mp te : te \mp tT$, that is $:: eq : eT :: cq : cT$. And the triangles peq, TeS being ſimilar, and alſo qcr, Tcs, we have $pq : ST$ ($:: eq : eT :: cq : cT ::$) $qr : ST$; therefore pq is equal to qr.

[a] Art. 207.

[b] Art. 215.

Upon ART. 127.

153. Hence we may compute the magnifying power of a double microſcope in this manner. When the object appears diſtinct, meaſure the diſtances LQ and LE, and alſo Eq the focal diſtance of the eye-glaſs. Then by ſubſtracting Eq from EL we have Lq, and alſo the quotient of Lq divided by LQ. Again by dividing the meaſure of the leaſt diſtance from which we commonly view minute objects, that is 6 or 8 inches, by the meaſure of the focal diſtance Eq, we have another quotient; which being multiplyed by the former quotient, gives the number of times, by which the diameter of the object is magnified: as I ſaid in the preſent article. For the triangles pLq, PLQ being ſimilar, the object PQ is contained in its image pq, as often as LQ is contained in Lq. But the rule being more general than this demonſtration, which ſuppoſes the rays of each pencil to emerge parallel from the eye-glaſs, or the image pq to fall upon its principal focus, I will add another demonſtration of it.

To compute the magnifying power of a double microſcope. Fig. 285.

154. Let the image pq fall at any diſtance from the eye-glaſs AE, and let EF be its focal diſtance; and with the center E and ſemidiameter EF, deſcribe an arch FG cutting the axis PLA of any oblique pencil, in G. Draw GE, and AO parallel to it; and the ray PLA will be refracted into the line AO *. Draw EK parallel to AO or GE, and ſuppoſing the naked eye placed at any point Ω in the axis LQR produced, join $P\Omega$. Now ſince the angle PLQ or FLG * is very ſmall, the arch FG may be taken for a ſtright line perpendicular to the axis LE; and therefore the figures $LPQR$, $LGFE$ are ſimilar [c]. Conſequently we have $QR : QL :: FE : FL$. Whence $QR = \frac{QL \times FE}{FL}$. But the apparent magnitude of the object ſeen at O, is to its apparent magnitude ſeen at Ω, as the angle AOE or PRQ, to the angle $P\Omega Q$; that is, as $Q\Omega$ to QR * or $\frac{QL \times FE}{FL}$; that is, as $Q\Omega \times FL$ to $QL \times FE$, or as $\frac{Q\Omega}{FE} \times \frac{FL}{LQ}$ to 1.

Another demonſtration of double microſcopes. Fig. 35.

* Art. 52.

* Art. 43.

[c] Art. 204.

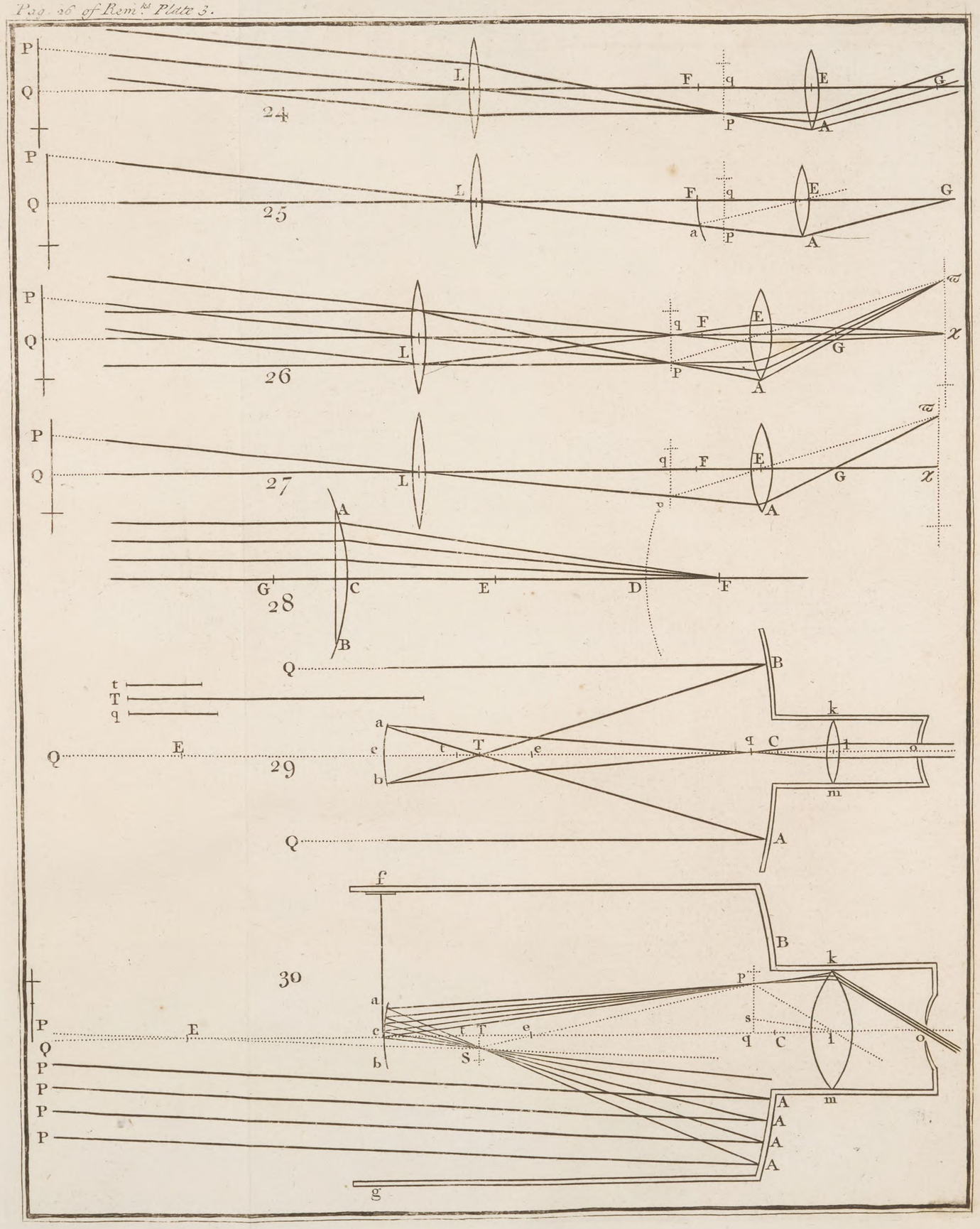
* Art. 60.

155. Hence it appears that the apparent magnitude of the object may be increaſed, either by bringing it nearer to the object-glaſs, and conſequently by inlarging its image, or by viewing the ſame image through a ſmaller eye glaſs. But in this proceeding there are two limitations. The firſt is, that the aperture of the object-glaſs muſt be increaſed to admit more light [d], which will increaſe the imperfections of the image [e]; and the ſecond is, that the viſible area of the object will be diminiſhed, either by increaſing the image or by viewing it through a ſmaller and narrower

Viſible area conſidered.

[d] Art. 129.

[e] Art. 81.


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er eye-glaſs. The firſt limitation is conſidered in the next book, and the ſecond may be removed as follows.

A microſcope with two eye-glaſſes. Fig. 36. * Art. 55.

156. When it is required to ſee much of an object at one view, it is uſual to interpoſe a pretty broad convex glaſs AE, between the object-glaſs L and the image pq formed by it. For this glaſs AE will contract the image pq into a ſhorter one $\varpi\varkappa$, terminated by the line pE *; and then the rays that diverge from this image $\varpi\varkappa$, may all be received upon a narrower eye-glaſs ae, and be refracted, to the eye at o, either into parallel or in diverging lines, as ao. When all theſe glaſſes are fixt at proper intervals to be found by experience, diſtinct viſion may be procured by a gradual alteration of the diſtance $L\mathcal{Q}$. Then let all the diſtances $L\mathcal{Q}$, LE, Ee be meaſured, and alſo the focal diſtances EF and ef of the two eye-glaſſes, by Art. 63; and by ſaying as LF to LE ſo LE to Ll, we have the line Ll; and by taking Lf from it, we have fl; and the apparent magnitude of the object ſeen through the microſcope, will be to its apparent magnitude ſeen by the naked eye from the diſtance $\mathcal{Q}\Omega$, as $\frac{\mathcal{Q}\Omega}{\mathcal{Q}L} \times \frac{FL}{FE} \times \frac{fl}{fe}$ to 1.

157. For with the centers E, e and ſemidiameters EF, ef deſcribe the arches FG, fg; and let the axis $PLGA$ of an oblique pencil, cut FG in G; and joining GE, the ray LA will firſt be refracted into the line Al parallel to GE *; and conſequently ſince the triangles LGE, LAl are equiangular, we have LF to LE (or LG to LA *) as LE to Ll. Let the ray Al cut the arch fg in g, and the eye-glaſs ea in a, and it will be refracted into a line ao parallel to ge *: and ſo the eye being placed at o will ſee the object $P\mathcal{Q}$ under the angle aoe. But this angle aoe or feg is to the angle flg as fl to fe *; and again this angle flg or FEG is to FLG as FL to FE *; and laſtly this angle FLG or $PL\mathcal{Q}$ is to $P\Omega\mathcal{Q}$ as $\mathcal{Q}\Omega$ to $\mathcal{Q}L$ *. And by compounding theſe ratios, the angle aoe is to $P\Omega\mathcal{Q}$ as $\mathcal{Q}\Omega \times FL \times fl$ to $\mathcal{Q}L \times FE \times fe$, or as $\frac{\mathcal{Q}\Omega}{\mathcal{Q}L} \times \frac{FL}{FE} \times \frac{fl}{fe}$ to 1. Fig. 37. * Art. 51. * Art. 204. * Art. 51. * Art. 60. * Art. 60. * Art. 60.

158. This middle glaſs is only uſeful for ſeeing more of the object at one view. For the more glaſſes are uſed, the more light is loſt by reflections at their ſurfaces; and a ſingle eye-glaſs will magnify more, and more diſtinctly, than two of them.

Upon Chapter 5. Concerning our ideas acquired by ſight.

Upon ART. 132.

159. I muſt here make my acknowledgements, once for all, to my highly honoured friend the learned and judicious Dr. *Jurin*, for obliging me with ſome of his curious remarks and diſſertations, written at my requeſt upon the ſubjects of Art. 132, 135, 137, 160; which I have ſpecified in the margin, as follows.

Dr. *Jurin*'s ſolution of Mr. *Molyneux*'s problem.

160. It is a thing well known that when ſeveral ideas have been habitually joined and connected together, if one or two of theſe be accidentally preſented to the mind, the reſt of the claſs which have been uſually ſo connected, are immediately excited in us. In hot weather the ſight of ſun-ſhine is inſtantly accompanied with the ideas of heat and uneaſineſs, and the view of a grove of trees does as immediately produce thoſe of ſhade and refreſhment. In winter the ſight of the ſun, or of a fire in the room we come into, is as readily followed by the ideas of warmth and pleaſure.

161. But this connexion or aſſociation of ideas is no where more conſtant, and at the ſame time no where leſs attended to by the generality of mankind, than in the different ideas of ſight and touch. When a ſolid body is preſented to our view, the idea firſt and immediately excited in us, is no other than that of a various complication of light and ſhade, which inſtantly raiſes in our mind the other ſubſequent ideas belonging to the touch, of ſolidity, convexity, or angularity, which have been uſed to accompany ſuch a ſenſation of ſight, or to ſpeak in the language of the acute and judicious author of the *Theory of Viſion*, the viſible idea excites in us thoſe tangible ideas which have been uſed to go along with it. And this is ſo inſtantly and imperceptibly done, that we can hardly avoid eſteeming that to be a bare ſenſation of our ſight, which is indeed the act of our memory and of our judgement, the greater part of mankind thinking they ſee a globe to be convex and a cube to be angular, when really they only judge them to be ſo. The truth of this doctrine has been ſo well demonſtrated by Mr. *Locke* and the abovementioned ingenious writer, and is ſo clearly confirmed in this very chapter, particularly by the curious obſervations of Mr. *Cheſſelden*, as to put the matter out of all doubt. But I muſt confeſs, I do not ſee that the famous queſtion of Mr. *Molyneux*, as it is by him ſtated, concerning the globe and cube ſet before a man born blind and now made to ſee, is of any uſe towards eſtabliſhing this doctrine: nor could I ever bring myſelf to think that he had rightly ſolved his own problem. I am not afraid to ſay ſo, though Mr. *Locke* himſelf has been pleaſed to declare, that

he agrees with that thinking gentleman. For notwithſtanding this piece of complaiſance, Mr. *Locke*'s determination appears to me to be expreſſed in ſuch a manner, and with ſuch limitations not to be found in the problem propoſed by Mr. *Molyneux*, as intirely convince me that the judgment of this great and clear reaſoner was widely different from that of his friend. In order to make this appear more plainly, I ſhall here ſet down Mr. *Molyneux*'s problem, with his determination of it, and that of Mr. *Locke*, in their own words.

Eſſay on Human Underſtanding. Book 2. Chap. 9. Sect. 8.

162. "*Suppoſe a man born blind, and now adult, and taught by his touch to diſtinguiſh between a cube and a ſphere of the ſame metal, and nighly of the ſame bigneſs, ſo as to tell when he felt one and the other, which is the cube, which the ſphere. Suppoſe then the cube and ſphere placed on a table, and the blind man to be made to ſee.* Quære, *whether by his ſight, before he touched them, he could now diſtinguiſh and tell, which is the globe, which the cube.* To which the acute and judicious Propoſer anſwers: *Not. For though he has obtained the experience of, how a globe, how a cube affects his touch; yet he has not yet obtained the experience, that what affects his touch ſo or ſo, muſt affect his ſight ſo or ſo; or that a protuberant angle in the cube that preſſed his hand unequally, ſhall appear to his eye as it does in the cube.* I agree with this thinking gentleman, whom I am proud to call my friend, in his anſwer to this his problem; and am of opinion, that the blind man, at firſt ſight, would not be able with certainty to ſay which was the globe, which the cube, whilſt he only ſaw them; though he could unerringly name them by his touch, and certainly diſtinguiſh them by the difference of their figures felt. So far Mr. *Locke.*

163. Here it is to be obſerved that by Mr. *Molyneux*, the blind man now made to ſee, is debarred of no other help towards diſtinguiſhing which is the globe and which the cube, except only that of touching thoſe bodies. He is left at full liberty to make uſe of his ſight as he pleaſes, to look at them again and again, and to view their ſeveral ſides by walking round them; as likewiſe to uſe his memory and his reaſon, if they can any way aſſiſt him. Theſe laſt are ſo far from being excluded by Mr. *Molyneux*, that the uſe of them is manifeſtly implied by the words of the problem and his determination of it. For the blind man is ſuppoſed to be *now adult*, and to have *obtained the experience of how a globe and how a cube affects his touch*; though the author is of opinion that this experience will be of no uſe to him on the preſent occaſion.

164. But does Mr. *Locke* allow the blind man all theſe advantages? No. As ſoon as he is made to ſee, he requires him to pronounce, *with certainty at firſt ſight*, which is the globe, which the cube; without giving him leave to take a ſecond view, much leſs to recollect himſelf and to reaſon upon what he ſees. The queſtion therefore determined in the negative by Mr. *Locke* is widely different from that propoſed by Mr. *Molyneux*; and we have no reaſon to think that ſo accurate a writer would have made ſuch a material alteration in the conditions of the problem, if, without doing ſo, he could have ſubſcribed to his friend's determination.

165. Setting aſide therefore the authority of Mr. *Locke*, or rather taking it in to my own aſſiſtance, I proceed to prove againſt Mr. *Molyneux*, that the blind man, now brought to ſight, ſhall be able to diſtinguiſh and tell which is the globe, and which the cube, before he touches them.

166. But here, in order to prevent miſtakes, I muſt obſerve, that the queſtion is not, whether this man, upon ſeeing the two bodies, ſhall of himſelf know them to be a globe and cube: this I take to be utterly impoſſible for him to diſcover, without touching them. All that Mr. *Molyneux* queries, is, whether he can diſtinguiſh and tell which is the globe, which the cube; which queſtion manifeſtly implies the two following conditions.

Firſt. That the blind man ſhall by ſight perceive the globe as one thing, diſtinct from the cube and all other bodies; and ſhall likewiſe perceive the cube as one thing diſtinct from the globe and all other bodies.

Secondly. That he ſhall be told, the two bodies he ſees are one a globe and the other a cube; without which information it is to no purpoſe to aſk him which is the globe and which the cube.

This therefore being done, I will ſuppoſe him to take a careful and repeated view of the two bodies in open ſky-light, and by walking round the table on which they are placed, to ſee and obſerve them in all their different ſituations; after which he will be able to reaſon in this manner.

167. The two bodies before me, I am told, are one a globe and the other a cube.

They are therefore of a different figure one from the other.

The ſenſations I receive from them, are likewiſe different.

I learn therefore from the little experience I have already had of this new ſenſe of ſeeing, that it is differently affected by diverſity of bodies. And my reaſon tells me it ought to be ſo. For, if two bodies are alike in figure, I muſt needs be affected in like manner by them both: and on the contrary, when I am differently affected by two or more bodies, I muſt conclude that this diverſity of ſenſations is occaſioned by the diverſity of figure of thoſe bodies.

168. Farther, I find my ſenſe to be differently

ly affected by one of these bodies, when I view its several sides, and observe its several parts.

These parts therefore are in themselves different from one another, for the reason just now mentioned, and consequently the body composed of them is not all over alike.

The other body, in what manner, or on what side soever I view it, always gives me the same sensation, and is therefore all over alike.

Now I remember, when a globe and cube were given me to handle, before I received my sense of seeing, that by reasoning then in the same manner upon what occurred to my touch, as I now do upon what is presented to my sight, I discovered not only that a globe and cube were bodies of a different make; but that a globe was a body alike all over, and that a cube was not so, but composed of parts greatly differing from one another.

This body therefore, which my sight informs me is alike all over, is undoubtedly the globe, and the other is the cube,

169. Thus, I think the blind man will unerringly distinguish between the two bodies, and that by the use of this single principle, that his senses were not given to deceive him; but that the different sensations, which several bodies raise in him, are caused by the difference of those bodies; without which our senses would not only be fallacious, but utterly useless.

170. I cannot omit taking notice that, upon talking about this problem not long since, with my highly esteemed friend the author of this treatise, I had the satisfaction to be informed by him, that the greatest blind philosopher that ever was, whom I need not call Dr. *Sanderson*, is of the same opinion with me, and reasons upon the problem much in the same manner. And I must observe that this learned author himself, by using these words, *by sight alone*, which seem to exclude the use of reason, as much as those of Mr. *Locke*, *at first sight*, appears not to be intirely of the same sentiment with Mr. *Molyneux*.

Upon ART. 135.

Jurin's remark on the association ideas.

171. Though it be beside the present purpose of the author, yet it may not be wholly improper to observe here, that when the ideas of sight excite in us those tangible ideas, that have been used to go along with them, this is not owing to any peculiar virtue or property in the ideas of sight to exhibit other ideas, but only to the general faculty of our memory; which upon any idea being presented to the mind, does readily suggest to us such other ideas, whether of the same or of any different sense, as have usually accompanied it.

172. That this is true of the ideas of hearing is evident from the use of language; the various sounds of which do readily excite in us such ideas, whether visible, or tangible, or of any other sense, with which those sounds have usually been connected. And any person who is ever so little attentive to what passes in his own mind, will find the same to hold in every one of our senses.

173. As I go home in the dark, I find by my touch several different objects in my way. I feel but a part of them, and immediately the visible ideas of the whole are excited. I know the one to be a post, the second a man, and the third a house.

174. I hear a rattling noise behind me, and presently the visible idea of a coach and horses arises in my mind, and with it the corresponding tangible ideas likewise, and I get out of the way to avoid the danger.

175. I pass by two several places, where I meet with very different scents, which raise in me the visible ideas, here of woodbinds growing in a court yard, and there of a dunghil.

176. I come home and go into a room, where I feel for a table with several sorts of fruit upon it. I taste one, and immediately know it to be a bergamot pear; another I find to be a golden pippin, the visible idea of each being instantly suggested by their taste.

177. If these instances are not thought sufficient, or not sufficiently clear, the imagination of every thinking person will readily supply him with innumerable others, to prove that the ideas received by any one of our senses do readily excite such other ideas, either of the same sense or of any other, as have habitually been associated with them. So that, if on this account we are to suppose, with a late ingenious writer, that the ideas of sight constitute a visual language, because they as readily suggest to us the corresponding ideas of touch, as the terms of a language excite the ideas answering to them, I see not but we may for the same reason allow of a tangible, an audible, an olfactory, and a gustatory language as well as a visual: though it must be owned that, as sight is without comparison the most comprehensive of all our senses, the visual language will be abundantly more copious than any of the rest.

Upon ART. 137.

Dr. *Jurin's* dissertation upon squinting.

178. A person is said to squint, when the axes of both his eyes are not directed to the same object. This defect, I think, is by the generality of physicians supposed to proceed from the want of a due correspondence in the muscles of the eyes, which not acting in a proper concert with one another, as in persons free from this blemish, are not able to point both eyes at one object. But the famous Mr. *de la Hire* a who has since been followed by a very great and learned Professor, whose sentiments are more generally embra-

a Mem. de Math. & Phys. Ann. 1694. Accid. de la vue. art. 10.

embraced by thoſe of the faculty throughout *Europe*, than perhaps thoſe of any phyſician ſince the time of *Galen*, is of a contrary opinion; and imagines this deformity not to ariſe from any depraved habit, or want of conſent in the muſcles of the eyes, but from a defect in the eye it ſelf, which he explains after this manner.

The opinion of *de la Hire* and others.

179. He ſuppoſes that, in the generality of mankind, that part of the retina, which is ſeated in and about the axis of the eye, is of a more delicate ſenſe and perception, than what the reſt of that coat is endowed with; and therefore that we direct both axes to the ſame object, not only for the ſake of direct viſion, whereby the image of the object may be more diſtinctly and perfectly painted upon the retina; but likewiſe and indeed chiefly, in order to receive the picture upon that part of the retina, which can beſt and moſt accurately perceive it. But in perſons who ſquint, he conceives the moſt ſenſible part of the retina of one eye, not to be placed in the axis, but at ſome diſtance from it on the one ſide, or on the other; and that therefore in the eye ſo unuſually framed, not the axis, but this more ſenſible part of the retina is turned toward the object, on which the axis of the other eye is fixed; and conſequently both axes are not directed to the ſame point.

Confuted by experiments.

180. If this opinion be true, that the eye is thus diſtorted for the ſake of ſeeing more diſtinctly with it; then, if the other eye be ſhut, and the diſtorted eye alone be uſed to look at an object, it muſt ſtill be as much diſtorted as before, for the ſame reaſon. The contrary of which is true in fact, as may eaſily be experienced.

181. Deſire ſuch a perſon to cloſe his other eye, and to look at you with that which is uſually diſtorted. He will immediately turn the axis of it directly towards you. Bid him open the undiſtorted eye, and look at you with both eyes. You will find the axis of this laſt now pointed at you, and the other eye turned away from you and drawn cloſe to the noſe, or perhaps to the upper eye-lid, as I have ſometimes obſerved it. Make as many tryals as you pleaſe, you will always find the ſame event.

182. Likewiſe, if a ſquinting perſon uſes a glaſs to read with, or to look at any minute object, with the eye uſually diſtorted, you may eaſily obſerve the axis of that eye to be pointed directly at the object.

Squinting perſons ſee objects diſtinctly with one eye only.

183. From theſe tryals it plainly appears, that the eye is thus diſtorted not for the ſake of ſeeing better with it, but rather to avoid ſeeing at all with that eye, as much as poſſible. For its pupil being generally drawn cloſe to the noſe, cannot receive the image of the object, towards which the other eye is directed, but in a very oblique and indiſtinct manner; and conſequently can be no more affected with it, than the eyes of other perſons are by ſuch objects as lye at a conſiderable diſtance laterally from that which we look at. So that in reality a ſquinting perſon ſees the object before him diſtinctly with one eye only; namely that whoſe axis is pointed directly at the object. Conſequently we may conclude, that this defect is not cauſed by any ſuch præternatural make of the eye, as thoſe two learned gentlemen have ſuppoſed.

Other opinions confuted.

184. Nor is it occaſioned by any defect in the muſcles of the diſtorted eye. For when the other is ſhut, this eye is moved by the action of its muſcles, in all poſſible directions, as freely as that of any other perſon,

185. Neither is it owing to the want of correſpondence in the muſcles of both eyes, ſuch as to hinder them from being both moved the ſame way at the ſame time. For when both eyes are open, and the undiſtorted eye is moved either upwards or downwards, or to the right or left, the other always accompanies it, and is turned the ſame way at the ſame inſtant of time.

The diſpoſition of the axes of common eyes.

186. But to underſtand particularly wherein this defect conſiſts, it will be neceſſary to conſider the diſpoſition and ſituation of the eyes, in perſons who are not affected with ſuch a blemiſh.

For ſeeing remote objects.

187. When we look directly forwards at a diſtant object, the pupil of each eye lies in the middle of the aperture formed by the eye lids, ſo that the diſtance between the two pupils conſiſts of the breadth of the noſe and half the breadth of the aperture of each eye: and howſoever obliquely we turn our eyes, this very ſame diſtance between the two pupils is always preſerved.

For ſeeing near objects.

188. In looking at near objects the diſtance between the pupils is ſomething leſs; but ſtill this ſame leſſer diſtance is always preſerved in all oblique directions of the eyes, as well as when we look ſtraight forwards. By which means the axes of both eyes are directed towards the ſame object, in both theſe caſes of looking at diſtant aud near objects.

The diſpoſition of the axes of ſquinting eyes.

189. But in thoſe who ſquint, when the pupil of the undiſtorted eye is ſeated in the middle of the aperture, as in looking directly forwards, the pupil of the other eye is drawn cloſe to the noſe, and conſequently the diſtance between the two pupils is conſiderably leſs than in other perſons; and this leſſer diſtance between the two pupils continues the ſame in all oblique directions of the eyes; ſo that the two axes are never pointed at the ſame object, though the muſcles do ſo far act in concert with each other, as to move both eyes the ſame way at the ſame inſtant of time,

One probable cauſe of ſquinting.

190. This vitious habit may eaſily be contracted by a child, if he is often laid into his cradle in ſuch a poſition, as to be able to ſee either the

light

light, or any other remarkable object, with one eye only.

The common methods of cure why ineffectual.

191. And when by this means he is brought to squint, and is afterwards confirmed and settled in the practice of doing so, I apprehend, it will be in vain to attempt a cure by his wearing tubes, or shels with small holes in them to look through. Do what you will of this kind, he will continue to see through them distinctly with one eye only, and will still distort the other.

The true method of cure.

192. The true method of cure I take to be this. When the child is arrived at such an age as to be capable of observing directions, place him directly before you, and let him close the undistorted eye and look at you with the other. When you find the axis of this eye fixt directly upon you, bid him endeavour to keep it in that situation, and open his other eye. You will now immediately see the distorted eye turn away from you towards his nose, and the axis of the other will be pointed at you. But with patience and repeated tryals he will by degrees be able to keep his distorted eye fixt upon you, at least for some little time, after the other is opened. And when you have brought him to continue the axes of both eyes fixt upon you, as you stand directly before him, it will be time to change his posture, and to set him first a little to one side of you, and then to the other, and so to practise the same thing: and when in all these situations he can perfectly and readily turn the axes of both eyes towards you, the cure is effected. An adult person may practise all this by a glass, without any director; though not so easily as with one. But the older he is, the more patience will be necessary.

An instance of it.

193. About twenty years ago I attempted a cure after this manner, upon a young gentleman about nine years of age, with promising hopes of success; but was interrupted by his falling ill of the small pox, of which he dyed.

Another opinion confuted.

194. I must not forget to take notice that Mr. *de la Hire*, in art 62, supposes squinting to proceed sometimes from another cause, namely the oblique situation of the crystalline humor in one of the eyes: but this is easily refuted by the experiments abovementioned, in the same manner as his former opinion. So far Dr. *Jurin*.

A remarkable instance of double vision.

195. After this 137th article was printed, I received an account of another remarkable instance of double vision, from my highly honoured friend *Martin Folkes* Esquire. Who tells me, he was informed by Dr. *Hepburn* of *Lynn*, that the late Reverend Mr. *Forster* of *Clenchwharton* in that neighbourhood, having been blind for some years with a *gutta serena*, was restored to sight by salivation; and that upon his first beginning to see, all objects appeared to him double; but afterwards, the two appearances approaching by degrees, he came at last to see single; and as distinctly as he did before he was blind.

Opinions upon the cause of double vision.

196 Another instance or two of this kind may be seen in Dr. *Briggs*'s *Nova Visionis Theoria* pag. 25; wherein he proposes a theory, or rather an hypothesis, to account for the single and double appearances of an object, by means of equal degrees of tension of the fibres, of both the optick nerves, continued from the brain to corresponding parts of the two retinas; in which parts the two pictures of an object usually fall. So that the isochronous vibrations of these corresponding fibres agitated by the rays, may stir up a single sensation in the mind; in like manner as unisons in musick are hardly distinguished from one sound. But when the two pictures of an object fall upon parts of the retina, where the tensions of the fibres are different; their discording vibrations may stir up two distinct sensations in the mind; as in musical concords and discords. But for a fuller account of this hypothesis I chuse to refer the reader to the author himself; because there are several hints and observations in that little treatise, and in his *Anatomy of the Eye*, which may be useful and entertaining to the curious. Sir *Isaac Newton*'s sentiments upon this subject of single and double vision, may be seen in the 15th Query at the end of his Opticks. *Kepler* has observed[a], that the supposed union of the nerves, any where between the two retinas and the brain, is repugnant to the double appearance of an object. Because, if the fibres of the nerves were there united, he reckons we should always see an object single, but never double. See also *Dechales*'s Cursus Math. Tom. 3. pag. 410.

[a] Dioptr. prop. 62.

Upon ART. 138.

The design.

197. In the present article, to avoid being tedious and troublesome to readers unprejudiced by received opinions, I gave but a summary view of the result of my thoughts and experiments upon apparent distance. But as this subject has never been rightly settled, and as it is higly necessary thereunto, that the principles of it should be clearly explained, and firmly established; for farther satisfaction it may not be amiss, to be somewhat more explicit. I observed that, by abundance of experiments made with glasses of all sorts, I found that an object always appeared to approach while its apparent magnitude increased, either by moving the glass, the eye or the object, backwards or forwards; and that it always appeared to recede while its apparent magnitude decreased; just as in vision with the naked eye. A few of the easiest and plainest experiments will be abundantly sufficient to establish this truth.

I. Exper. with a concave lens at rest.

198. Let any concave lens be fixt between your eye and any remote objects; and while your eye is drawn back from the lens, (the other being

being ſhut,) the apparent magnitudes of the objects will continually decreaſe, and their apparent diſtances from you will continually increaſe; and the contrary will appear while your eye approaches to the lens.

II. Exper. with a concave lens in motion.

199. Or if your eye be fixt, and the concave lens be gradually removed from it; the apparent magnitudes of the remote objects will continually decreaſe, and their apparent diſtances from you will continually increaſe; even though the lens were carried half way towards the objects: and the contrary will appear while the lens is brought back to your eye.

III. Exper. with a convex ſpeculum.

200. Or if a convex looking-glaſs be uſed inſtead of the concave lens, the apparent magnitudes and diſtances of remote objects, lying laterally behind you, or of your own face, will alſo vary reciprocally; as they did in both the foregoing experiments.

IV. Exper. with a convex lens at reſt.

201. Now let any convex lens be fixt between your eye and any remote objects; and when your eye is firſt put cloſe to the lens, and then drawn back from it, the apparent magnitudes of the objects will continually increaſe, and their apparent diſtances from you will continually decreaſe; ſo long as the objects appear upright. And even after they are inverted, if the eye be drawn farther back, their apparent magnitudes will continually decreaſe, and their apparent diſtances from you will continually increaſe.

V. Exper. with a convex lens in motion.

202. Or if your eye be fixt, and the convex lens be applied cloſe to it, and then be gradually removed from it; the apparent magnitudes and diſtances of remote objects will correſpond reciprocally, and in the ſame order, as they did before; even though the lens were carried half way towards the objects.

VI. Exper. with a concave ſpeculum.

203. Or if a concave looking-glaſs of a large ſphere, be uſed inſtead of this convex lens; the apparent magnitudes and diſtances of remote objects, lying laterally behind you, or of your own face, will always vary reciprocally with reſpect to each other. The reaſon of theſe variations of apparent magnitude, may be ſeen in Art. 106 and 110; and the correſponding variations of apparent diſtance are the facts, for which we are to aſſign a reaſon, that ſhall hold good in all caſes.

VII. Exper. with a plane ſpeculum.

204. But firſt it is obſervable that while the apparent magnitude varies quicker or ſlower, the apparent diſtance does alſo vary quicker or ſlower; as will be evident by moving the eye or the glaſs, quicker or ſlower, or by uſing glaſſes of leſſer or larger ſpheres. And even the ſlow variations of apparent magnitude and diſtance, are alſo obſervable in a plane looking-glaſs; eſpecially by attending to the minute parts of objects, as they ſeem to depart from you; otherwiſe the imagination ſupplies a magnitude, which the ſenſe does not perceive. But if the plane of the looking-glaſs be not perfect, it will frequently diſtort objects at a great diſtance, and gently increaſe their apparent magnitudes, and thereby deceive you.

Hence apparent diſtance reciproc as appare magnitu

205. It is alſo obſervable in all theſe experiments, when the eye and the glaſs are cloſe together; that the apparent magnitudes and diſtances of all objects, are the ſame as to the naked eye; and when the eye and the glaſs are ſeparated, that the apparent diſtance varies reciprocally in the ſame proportion as the apparent magnitude varies; that is, when one becomes double or treble, the other becomes half or one third reſpectively; as near as the ſenſe can diſtinguiſh: as any one will find by comparing the appearances of the ſame objects, ſeen at one view through the lens, and by the ſides of it with the naked eye.

Uſes of t general principle

206. It is true that the ſenſe alone cannot accurately determine ſuch ratios of apparent diſtances, or even of magnitudes, as ſhould be expreſſed by larger numbers; and therefore a general rule derived from the ſimpleſt ratios and experiments, is the more uſeful and neceſſary to conduct our inquiries in more complicated caſes; and to examine how near the appearances of things, and the cauſes aſſigned for them, do agree in Quantity with each other. For as this is the ſureſt and the beſt means of diſtinguiſhing true cauſes from falſe ones, ſo the general neglect of it has been the chief occaſion of all the errors in philoſophy.

The certainty of foundatio

207. The apparent diſtances of things ſeen clearly by the naked eye, are unalterable by the power of imagination; and therefore being determinate in themſelves, they have determinate ratios to each other, and determinate cauſes: and thoſe that look into glaſſes will be ſenſible of the ſame thing. And that all people agree in their judgments of the meaſure of apparent diſtance in glaſſes, will appear by this experiment. I well remember, when ſeveral perſons were trying to read a gazette at a great diſtance, through one of Mr. *Gregory*'s reflecting teleſcopes, that I aſked them one by one, how near they thought it appeared through the teleſcope; and whether as near as my face appeared to their naked eye, when I placed it before them, by the ſide of the viſual rays coming from the gazette; and removed it backwards and forwards, as they directed, till they judged the two objects appeared equidiſtant, by the ſide of each other. And by marking the ſeveral ſtations to which they reduced me, I found their differences were very little, even in this groſs way of tryal; though the objects were of different kinds, and the ſpectators of different ages; ſome of them being children. Therefore ſince determinate judgments have adequate cauſes, the judgments may be meaſured with the ſame certainty as you can meaſure the cauſes.

he reaſon f it.

208. This conſtant and regular connection between the quantities of apparent magnitude and apparent diſtance, being proved to be matter of fact, let us now conſider how it comes to be ſo. In deſcribing the experiments above I have ſuppoſed the objects to be pretty remote; not becauſe they will not ſucceed when the objects are near, but becauſe they are ſomewhat ſimpler and plainer; and alſo becauſe the eye can comprehend at one view a larger ſyſtem of remote objects in all manner of oblique and direct ſituations to the viſual rays. Which ſhews that their diſtances are ſuggeſted to the mind by the ſame cauſe, acting in the ſame manner in all theſe caſes. Now ſince any one will acknowledge, at firſt ſight of theſe objects through a concave lens, that they all appear ſmaller, exactly in the ſame manner as if he ſaw them with his naked eye from a greater diſtance; what can be plainer than that this ſmaller appearance ſuggeſts to him the uſual idea of that greater diſtance, which has been conſtantly annext to it by experience from his infancy [a]? And the like may be ſaid of the greater and nearer appearances of objects ſeen through a convex lens or in a concave looking-glaſs. Intending to prove this general ſolution of theſe Phænomena in a more particular manner in a remark upon Art. 148, at preſent I will proceed to conſider the received principles and opinions of optick writers upon this ſubject.

[a] Art. 135. and Rem. 171.

Dr. *Barrow's* difficult caſe of apparent diſtance ſolved. [b] Rem. 201.

209. The famous difficulty which ſo much diſtreſſed the learned Dr. *Barrow*, was to account for the apparent diſtance of an object ſeen as in our 4th, 5th and 6th experiments [b]; that is when the viſual rays fall upon the eye converging towards an image behind it. This difficulty aroſe naturally from a principle univerſally received in opticks, viz. that an object ſeen by reflections and refractions, appears always in the place of its image from which the rays diverge upon the eye. Which principle, though it happens to agree with experience in two or three common caſes, is in general quite diſagreeable to it, and to reaſon too; though not ſo manifeſtly as in that uncommon caſe which the Doctor thus deſcribes.

His own deſcription of it.

210. *Hæc ſunt, quæ circa partem opticæ præcipue mathematicam dicenda mihi ſuggeſſit meditatio. Circa reliquas, (quæ φυσικώτερα ſunt, adeoque ſæpiuſcule pro certis principiis plauſibiles conjecturas venditare neceſſum habent) nihil fere quicquam admodum veriſimile ſuccurrit, a pervulgatis (ab iis, inquam, quæ* Keplerus, Scheinerus, Carteſius, *& poſt illos alii tradiderunt) alienum aut diverſum. Atqui tacere malo, quam toties oblatam cramben reponere. Proinde receptui cano; nec ita tamen ut prorsùs diſcedam anteaquam improbam quandam difficultatem (pro ſinceritate quam & vobis & veritati debeo minime diſſimulandam) in medium protulero, quæ doctrinæ noſtræ, hactenus inculcatæ, ſe objicit adverſam, ab ea ſaltem nullam admittit ſolutionem. Illa, breviter, talis eſt:* lenti vel ſpeculo cavo EBF *exponatur punctum viſibile* A, *ita diſtans ut radii ex* A *manantes, ex inflexione verſus axem* AB *cogantur. Sitque radiationis limes (ſeu puncti* A *imago, qualem ſupra paſſim ſtatuimus) punctum* Z. *Inter hoc autem & inflectentis verticem* B *uſpiam poſitus concipiatur oculus. Quæri jam poteſt ubi loci debeat punctum* A *apparere? Retrorſum ad punctum* Z *videri non fert natura (cum omnis impreſſio ſenſum afficiens proveniat a partibus* A*) ac experientia reclamat. Noſtris autem e placitis conſequi videtur, ipſum ad partes anticas apparens, ab intervallo longiſſime diſſito, (quod & maximum ſenſibile quodvis Intervallum quodammodo exſuperet) apparere. Cum enim quo radiis minus divergentibus attingitur objectum, eo (ſecluſis utique prænotionibus & præjudiciis) longius abeſſe ſentiatur; & quod parallelos ad oculum radios projicit, remotiſſime poſitum æſtimetur. Exigere ratio videtur, ut quod convergentibus radiis apprehenditur, adhuc magis, ſi fieri poſſet, quoad apparentiam elongetur. Quin & circa caſum hunc generatim inquiri poſſit, quidnam omnino ſit, quod apparentem puncti* A *locum determinet, faciatque quod conſtanti ratione nunc propius, nunc remotius apparcat? Cui itidem dubio, nihil quicquam ex hactenus dictorum* analogia, *reſponderi poſſe videtur, niſi debere punctum* A *perpetuo longiſſime ſemotum videri. Verum experientia ſecus atteſtatur, illud pro diverſa oculi inter puncta* B, Z, *poſitione varie diſtans, nunquam fere (ſi unquam) longinquius ipſo* A *libere ſpectato, ſubinde vero multo propinquius apparere; quinimo, quo oculum appellentes radii magis convergunt eo ſpeciem objecti propius accedere. Nempe, ſi puncto* B *admoveatur* oculus, *ſuo (ad lentem) fere nativo in loco conſpicitur punctum* A *(vel æque diſtans, ad* ſpeculum;) *ad* O *reductus oculus ejuſce ſpeciem appropinquantem cernit; ad* P *adhuc vicinius ipſum exiſtimat; ac ita ſenſim, donec alicubi tandem, velut ad* Q, *conſtituto oculo objectum ſumme propinquum apparens, in meram confuſionem incipiat evaneſcere. Quæ ſane cuncta rationibus atque decretis noſtris repugnare videtur, aut cum iis ſaltem parum amice conſpirant. Neque noſtram tantum ſententiam pulſat hoc experimentum; at ex æquo cæteras quas norim omnes, veterem imprimis ac vulgatam, noſtræ præ reliquis affinem, ita convellere videtur, ut ejus vi coactus doctiſſimus* A. Tacquetus *iſti principio (cui pene ſoli totam inædificaverat* Catoptricam *ſuam) ceu infido ac inconſtanti renunciarit, adeoque ſuam ipſe doctrinam labefactarit; id tamen, opinor, minime facturus, ſi rem totam inſpexiſſet penitius, atque difficultatis fundum attigiſſet. Apud me vero non ita pollet hæc, nec eouſque præpollebit ulla difficultas, ut ab iis, quæ manifeſte rationi conſentanea video, diſcedam; præſertim quum, ut hic accidit, ejuſmodi difficultas in ſingularis cujuſpiam caſus diſparitate*

Fig. 18, 19.

 fande-

fundetur. Nimirum in præſente caſu peculiare quædam, naturæ ſubtilitati involutum, deliteſcit, ægre fortaſſis, niſi perfectius explorato videndi modo, detegendum. Circa quod nil, fateor, hactenus excogitare potui, quod adblandiretur animo meo, nedum plane ſatisfaceret. Vobis itaque nodum hunc, utinam feliciore conatu, reſolvendum committo.

Tranſlated.

211. I beg leave to borrow the following tranſlation from an ingenious writer. "I have here delivered what my thoughts have ſuggeſted to me, concerning that part of opticks which is more properly mathematical. As for the other parts of that ſcience (which being rather phyſical, do conſequently abound with plauſible conjectures inſtead of certain principles) there has in them ſcarce any thing occurred to my obſervation, different from what has been already ſaid by *Kepler*, *Scheinerus*, *Deſcartes*, and others. And methinks, I had better ſay nothing at all, than repeat that which has been ſo often ſaid by others. I think it therefore high time to take my leave of this ſubject: but before I quit it for good and all, the fair and ingenuous dealing that I owe both to you and to truth, obligeth me to acquaint you with a certain untoward difficulty, which ſeems directly oppoſite to the doctrine I have been hitherto inculcating, at leaſt, admits of no ſolution from it. In ſhort it is this. Before the double convex glaſs or concave ſpeculum *EBF*, let the point *A* be placed, at ſuch a diſtance that the rays proceeding from *A*, after refraction or reflection, be brought to unite ſomewhere in the axis *AB*. And ſuppoſe the point of union (i. e. the image of the point *A*, as has been already ſet forth) to be *Z*; between which and *B*, the vertex of the glaſs or ſpeculum, conceive the eye to be any where placed. The queſtion now is, where the point *A* ought to appear? Experience ſhews that it doth not appear behind at the point *Z*, and it were contrary to nature that it ſhould; ſince all the impreſſion which affects the ſenſe comes from towards *A*. But from our tenets it ſhould ſeem to follow, that it would appear before the eye at a vaſt diſtance off, ſo great as ſhould in ſome ſort ſurpaſs all ſenſible diſtance. For ſince, if we exclude all anticipations and prejudices, every object appears by ſo much the farther off, by how much the rays it ſends to the eye are leſs diverging; and that object is thought to be moſt remote, from which parallel rays proceed unto the eye; reaſon would make one think, that object ſhould appear, at yet a greater diſtance, which is ſeen by converging rays. Moreover it may in general be aſked concerning this caſe, what it is that determines the apparent place of the point *A*, and maketh it to appear after a conſtant manner, ſometimes nearer, at other times farther off? To which doubt, I ſee nothing that can be anſwered, agreeable to the principles we have laid down, except only that the point *A* ought always to appear extremely remote. But on the contrary, we are aſſured by experience that the point *A* appears variouſly diſtant, according to the different ſituations of the eye between the points *B* and *Z*. And that it doth almoſt never (if at all) ſeem farther off, than it would if it were beheld by the naked eye, but on the contrary, it doth ſometimes appear much nearer. Nay, it is even certain, that by how much the rays falling on the eye do more converge, by ſo much the nearer does the object ſeem to approach. For the eye being placed cloſe to the point *B*, the object *A* appears nearly in its own natural place, if the point *B* is taken in the glaſs; or at the ſame diſtance, if in the ſpeculum. The eye being brought back to *O*, the object ſeems to draw near; and being come to *P*, it beholds it ſtill nearer. And ſo on by little and little, till at length the eye being placed ſomewhere, ſuppoſe at *Q*, the object appearing extremely near, begins to vaniſh into mere confuſion. All which doth ſeem repugnant to our principles, at leaſt, not rightly to agree with them. Nor is our tenet alone ſtruck at by this experiment, but likewiſe all others that ever came to my knowledge are, every whit as much, endangered by it. The ancient one eſpecially (which is moſt commonly received, and comes neareſt to mine) ſeems to be ſo effectually overthrown thereby, that the moſt learned *Tacquet* has been forced to reject that principle, as falſe and uncertain, on which alone he had built almoſt his whole catoptricks, and conſequently by taking away the foundation, hath himſelf pulled down the ſuperſtructure he had raiſed upon it. Which, nevertheleſs, I do not believe he would have done, had he but conſidered the whole matter more throughly, and examined the difficulty to the bottom. But as for me, neither this, nor any other difficulty ſhall have ſo great an influence on me, as to make me renounce that which I know to be manifeſtly agreeable to reaſon: eſpecially when, as it here falls out, the difficulty is founded in the peculiar nature of a certain odd and particular caſe. For in the preſent caſe ſomething peculiar lies hid, which being involved in the ſubtilty of nature, will, perhaps, hardly be diſcovered till ſuch time, as the manner of viſion is more perfectly made known. Concerning which, I muſt own, I have hitherto been able to find out nothing that has the leaſt ſhew of probability, not to mention certainty. I ſhall, therefore, leave this knot to be untied by you, wiſhing you may have better ſucceſs in it than I have had.'

Fig. 38, 39.

His principle and the old one compared.

212. Dr. *Barrow*'s manner of propoſing and deſcribing this difficult caſe, ſo oppoſite to his own theory, which he had cultivated throughout his Lectures, is indeed a noble declaration of his regard

regard to truth, and highly worthy of a great mind. The old principle that he mentions, which *Euclid*, *Alhazen*, *Tacquet*, and almoſt all optick writers had hitherto followed, is this. Any given viſible point of an object, appears at the interſection of the reflected or the refracted viſual ray produced, and of a line drawn through the viſible point perpendicular to the reflecting or refracting ſurface, whether plane or ſpherical. This interſection always coincides with our image of the viſible point in a plane looking-glaſs, and alſo in reflections and refractions at plane and ſpherical ſurfaces, provided the angle of incidence be very ſmall as it generally is. But whatever be the magnitude of the angle of incidence Dr. *Barrow*'s principle is, that the viſible point appears by reflections or refractions at a certain point, from which thoſe rays of a ſlender pencil diverge, in falling on the eye, that will enter the pupil. Which point in ſtrictneſs of geometry, is always ſome point of a cauſtick formed by reflections or refractions of all the rays that flow from the viſible point[a]: and therefore differs widely from the place of our image, when the eye lyes wide of the axis of the cauſtick, that is when the angle of incidence is large, but nearly coincides with it when the eye is near the axis, as in the preſent experiments.

[a] Art. 154.

And both proved to be inſufficient by experiments with lenſes. Fig. 40, 41.

213. In the mathematical part of his Lectures this learned Author has made many uſeful improvements; but in the phyſical part he ſeems to have failed in his principal deſign; which was to determine the apparent place or diſtance of an object univerſally and more accurately than by the anciently received principle. Now the reaſon of this failure and conſequently of the falſity of both theſe principles, appears to me very plain by the following experiments. Holding any concave lens very cloſe to your eye, repeat the Doctor's experiment with his lens or ſpeculum, and you will preſently find that the concave eye-glaſs will not ſenſibly alter the apparent diſtance of the object. This will be evident by ſlipping the eye-glaſs ſideways off and on your eye alternately, while fixt at any given diſtance from the convex object-glaſs; and by comparing the appearances. Now if the focal diſtance Bb of this concave eye-glaſs B, be leſs than the focal diſtance of the convex object-glaſs A; the rays which flow from a point Q of a remote object, and after refractions through the object-glaſs A, go converging towards a point q, being intercepted and refracted through the concave eye-glaſs B, will fall upon the eye diverging from a point k; provided the interval of the glaſſes A, B be ſmall enough for the principal focus b of the concave eye-glaſs B, to fall between it and the image q*. It is plain then that the object appears at one and the ſame diſtance, whether ſeen by rays diverging from the laſt image k, or converging towards the firſt image q when the eye-glaſs is taken away; and conſequently that the apparent diſtance of the object has no dependance upon the places of its images. For theſe may be ſtill varied at pleaſure, by applying different glaſſes, either concave or convex, to the eye in the ſame place B; and yet the apparent diſtance will always be the ſame as if they were taken quite away. And by drawing your eye and eye-glaſs back towards the image q; while the principal focus b approaches to q and paſſes over it, the ſecond image k will run out to an infinite diſtance, and then will return back the contrary way, from an infinite diſtance behind the eye[c]; and yet all this while the apparent diſtance will vary exactly in the ſame proportion as in Dr. *Barrow*'s manner of tryal, when the rays fell converging, upon his naked eye, towards a fixt image q. Which plainly ſhews that his principle of divergency of rays, has nothing at all to do with apparent diſtance, to a ſingle eye at leaſt.

* Art. 48.

[c] Art. 48.

And with a looking-glaſs.

214. And the like experiments, tryed upon objects ſeen in a common looking-glaſs, will ſhew that concave or convex glaſſes, held cloſe to the eye, do not alter the apparent diſtances of the objects; though the laſt image, or place from which the rays diverge upon the eye, or towards which they converge, after paſſing through the different eye-glaſſes, is varied thereby at pleaſure. By which it is alſo evident to me, that no author, that I have ſeen, has given the true reaſon of the apparent diſtance of an object ſeen in a common looking-glaſs. See Art. 146. and Rem. upon it.

The origine of this falſe principle.

215. What miſled them was this. Finding by experience that the apparent place of an object ſeen in a plane looking-glaſs, is always as far behind the glaſs, as the real place of the object is before it; finding alſo, by the known law of reflections, that the rays muſt diverge upon the eye, as if they had come from that place[d]; from this coincidence of facts, (in this and in a like caſe, of ſeeing objects that lye under water[e], which are the moſt obvious of all others,) they concluded too haſtily, that the divergency of the viſual rays from the place of the image of an object, was the cauſe of its appearing in that place; and of conſequence that the ſame cauſe muſt hold good in viſion by reflections and refractions at ſpherical ſurfaces. But this principle being falſe even in that ſimpleſt caſe of a looking-glaſs, as we ſhewed above, it is no wonder that ſo many difficulties have ariſen from it in more complex caſes; as in thoſe obſerved by Dr. *Barrow*, *Gregory*, *Tacquet*, *Molyneux* and all the beſt writers.

[d] Art. 23.

[e] Art. 146.

Not true in viſion with the naked eye.

216. But the fundamental error is ſtill unobſerved. Thoſe that uſe ſpectacles and concave glaſſes to help the defects of their eyes, perceive objects through them very diſtinctly; and

ſo do others that have no defects, provided the convexities and concavities of the glaſſes be not too great. Now when theſe glaſſes are put cloſe to their eyes, all objects appear to them nearly of the ſame magnitudes and at the ſame diſtances, as they do to the naked eye a. But now the rays of the pencils do not diverge upon the eye from the place of the object, but as if they had come ſtraight through the concave from a place much nearer, or ſtraight through the convex from a place much remoter, than that of the object, or even converged towards a place lying behind the eye. And yet the object appears in its uſual place. And conſequently the divergency of the rays upon the naked eye from this real place, cannot be the cauſe of its appearing there. Nor indeed are we certain that it does appear there, but only thereabouts when the place is near to us. But when we look at very remote objects, they evidently do not appear, even to the naked eye, in the real places and poſitions which we know they poſſeſs; but ſometimes nearer, ſometimes remoter; ſome towards the one hand and ſome towards the other. See inſtances of this in Art. 138. pag. 51 line 11; and in Art. 158 to 163. and in Art. 169, 170. and Rem. upon it. Thus by arguing analytically from obſervations and experiments, it is evident to me, that the divergency of rays from an object in any place, is not the cauſe that ſuggeſts its apparent place, even to the naked eye.

a Art. 117.

herwiſe nfuted in t. *Berke*y's new heory. ſect. 13.

217. Upon this occaſion I cannot omit taking notice, that this received principle was firſt rejected (upon proof of its inſufficiency by arguments *à priori*) by the moſt ingenious and learned Author of the late *Eſſay towards a New Theory of Viſion*; concerning the manner wherein we perceive by ſight, the diſtance, magnitude and ſituation of objects: a Work highly entertaining and uſeful to all that are duly qualified to conſider it. And herein, had this worthy gentleman, whom I greatly eſteem, thought proper to have joined ſome experiments with glaſſes, or even geometrical inferences from glaſſes, with the other helps which his moſt eminent qualifications afforded him for this ſubject; I cannot but think, that he might have been furniſhed with ſeveral reaſons for rejecting another principle or two, which he ſubſtituted inſtead of that old one; and which, in purſuance of my deſign of ſettling this ſcience upon a firm foundation, I cannot avoid conſidering in the following remarks.

218. Having done with the divergency of rays, which is not perceived by ſenſe; let us now conſider the degrees of apparent confuſion, that frequently reſult from different divergencies. For the confuſion being certainly perceived, may be reaſonably thought to have an influence upon the mind in its judgments of diſtance: and indeed one can hardly help thinking ſo, upon peruſing the following argument of the ingenious writer abovementioned.

His new principle; that apparent confuſedneſs ſuggeſts diſtance. ibid. ſect. 21.

219. "An object placed at a certain diſtance from the eye, to which the breadth of the pupil bears a conſiderable proportion, being made to approach, is ſeen more confuſedly: and the nearer it is brought, the more confuſed appearance it makes. And this being found conſtantly to be ſo, there ariſeth in the mind an habitual connexion between the ſeveral degrees of confuſion and diſtance; the greater confuſion ſtill implying the leſſer diſtance, and the leſſer confuſion, the greater diſtance of the object.

And is the occaſion of thoſe judgments attributed to diverging rays. ibid. ſect. 22.

220. "This confuſed appearance of the object doth therefore ſeem to be the medium, whereby the mind judgeth of diſtance in thoſe caſes, wherein the moſt approved writers of opticks will have it judge by the different divergency, with which the rays, flowing from the radiating point, fall on the pupil No man, I believe, will pretend to ſee or feel thoſe imaginary angles, that the rays are ſuppoſed to form according to their various inclinations on his eye. But he cannot chooſe ſeeing whether the object appears more or leſs confuſed. It is therefore a manifeſt conſequence from what has been demonſtrated a, that inſtead of the greater, or leſſer divergency of the rays, the mind makes uſe of the greater or leſſer confuſedneſs of the appearance, thereby to determine the apparent place of an object'.

a Ibid. ſect. 13. See Rem. 217.

And is ſhewn to agree with Dr. *Barrow*'s difficult caſe. ibid. ſect. 31. Fig. 38, 39.

221. "Let us now ſee how Dr. *Barrow*'s experiment agrees with our tenets. The eye the nearer it is placed to the point *B* in the foregoing figures, the more diſtinct is the appearance of the object; but as it recedes to *O*, the appearance grows more confuſed; and at *P* it ſees the object yet more confuſed; and ſo on till the eye being brought back to *Z* ſees the object in the greateſt confuſion of all. Wherefore by Rem. 219 the object ſhould ſeem to approach the eye gradually, as it recedes from the point *B*, that is at *O* it ſhould (in conſequence of the principle I have laid down in the aforeſaid Remark) ſeem nearer than it did at *B*, and at *P* nearer than at *O*, and at *Q* nearer than at *P*; and ſo on, till it quite vaniſhes at *Z*. Which is the very matter of fact, as any one that pleaſes may eaſily ſatisfy himſelf by experiment.'

Ibid. ſect. 36.

222. Dr. *Berkeley* farther obſerves; that though the confuſion of near objects, ſeen by the naked eye, reſults from too great a divergency of the rays, and in Dr. *Barrow*'s experiment, from a convergency of them; yet that equal degrees of confuſion, either way produced, will have the ſame effect upon the mind. For, ſays he, "the eye, or (to ſpeak truly) the mind perceiving only the confuſion it ſelf, without ever conſidering the cauſe from which it proceeds, doth conſtantly annex the ſame degree of diſtance

ſtance to the ſame degree of confuſion. Whether that confuſion be occaſioned by converging, or by diverging rays, it matters not. Whence it follows, that the eye viewing the object Z through the glaſs QS (which by refraction cauſeth the rays ZQ, ZS, &c. to converge) ſhould judge it to be at ſuch a nearneſs, at which if it were placed, it would radiate on the eye with rays diverging to that degree, as would produce the ſame confuſion, which is now produced by converging rays, i. e. would cover a portion of the retina equal to DC in figure 42. But then this muſt be underſtood (to uſe Dr. *Barrow*'s phraſe) *ſecluſis prænotionibus & præjudiciis*, in caſe we abſtract from all other circumſtances of viſion, ſuch as the figure, ſize, faintneſs, &c. of the viſible objects; all which do ordinarily concur to form our idea of diſtance; the mind having by frequent experience obſerved their ſeveral ſorts or degrees, to be connected with various diſtances.' So far Dr. *Berkeley*.

223. Now ſuppoſing his principle to be true; namely that apparent diſtance (or the idea of diſtance) is ſuggeſted by apparent confuſion, I agree with him perfectly in this his concluſion, viz. "that the eye viewing the object through the glaſs, ſhould judge it to be at ſuch a nearneſs, at which if it were placed, it would radiate upon the eye with rays diverging to that degree, as would produce the ſame confuſion, which is now produced by converging rays [a]." This is a juſt conſequence from the principle; but it ſhews, that an object ſeen but the leaſt confuſedly in glaſſes, ought always to appear within a foot or two of the eye at the fartheſt. Becauſe moſt people perceive by the naked eye, little or no confuſion in objects, placed at thoſe diſtances, or even much ſmaller. But in Dr. *Barrow*'s experiment, objects appear confuſed at all degrees of apparent diſtance, not exceeding their apparent diſtance to the naked eye. For the object being placed any where beyond the focal diſtance of the glaſs, will appear confuſed though the eye ſhould touch the glaſs; becauſe the rays fall converging upon the eye; and if the object be removed from the glaſs gradually, or which comes to the ſame, if the eye and the glaſs cloſe together, be gradually drawn from the object, the confuſion will increaſe [b] with the apparent diſtance; which is always the ſame very nearly as it would have been to the naked eye thus drawn back [c]. And by uſing glaſſes of different convexities put cloſe to the eye, the apparent confuſion may be altered at pleaſure, without altering the real and apparent diſtance. It follows then, from the diſagreement of theſe appearances with the concluſion abovementioned, that the principle it ſelf is inſufficient; and conſequently that the agreement between greater nearneſs and confuſedneſs, when the eye is drawn back from the glaſs, is only accidental with reſpect to confuſedneſs; and neceſſarily owing to the apparent magnitude increaſing with the confuſedneſs [d].

224. Again, let a concave eye-glaſs, of a much ſhorter focal diſtance than the convex lens uſed in the laſt experiment, be held cloſe to your eye, and alſo to this lens, and then the object will appear confuſed through both, by reaſon of too great a divergency of the rays in falling upon the eye. But while theſe glaſſes are gradually ſeparated, the confuſion will firſt decreaſe to nothing, and then increaſe again, while the apparent diſtance of the object is decreaſing continually. The reaſon of theſe variations of confuſion, will appear from the motion of the ſecond image k in Rem. 213; and ſhall be farther explained in the next remark but one.

225. And by the way, this experiment ſhews that pretty long perſpectives may be uſed without tubes; it being ſufficient to hold the head of your cane with the eye-glaſs in one hand, and any other part of the cane with the object-glaſs in the other hand; and then to ſlide this glaſs to a proper diſtance, ſoon found by tryal, for ſhewing the objects moſt diſtinctly. Thus you may uſe the glaſſes of a perſpective as long as your arm, as I frequently do when I carry them about with me. But to procure a brighter appearance of objects, it is better to have a broader object-glaſs than thoſe that are commonly included in tubes.

226. But to return. Let the order of the glaſſes be now inverted; that is, hold a convex lens cloſe to your eye, whoſe focal diſtance is ſomewhat greater than that of the concave lens or the convex ſpeculum, uſed in the three firſt experiments [e]; and by repeating them over again with this convex eye-glaſs, you will find the ſame variations of apparent diſtance and magnitude as without the eye-glaſs [f]. But now the apparent confuſion and diſtance will always increaſe both together, or decreaſe both together. Which is contrary to the principle of confuſion. The reaſon of theſe variations of confuſion is this. Let the concave object-glaſs be at A, the remote object at Q, its image formed by this concave at q, from which the rays diverge in falling upon the convex eye-glaſs B; and let the ſecond image formed by this glaſs be at k, from which the rays diverge, or to which they converge in falling upon the eye at B. Now ſince the focal diſtance Bb of the convex eye-glaſs B, is ſuppoſed to be longer than Aq; it follows, when the glaſſes are near to each other, that the image q will be nearer to the eye glaſs B, than its principal focus b; and then the rays will diverge upon the eye as from the image k, and the viſion will be diſtinct, if k be not too near the eye. But while the glaſſes are drawn aſunder, and the interval qb decreaſes to nothing

[a] but it diſagrees with Dr. Barrow's experiment in the quantity of apparent diſtance.

[b] Art. 48.

[c] Art. 117.

[d] Rem. 208. And with the ſame expetiment tryed with a concave eye glaſs: Fig. 40.

[A digreſſion upon the uſe of perſpectives without tubes.]

[e] And with other expetiments tryed with a convex eye-glaſs. Rem. 198.

[f] Art. 117.

Fig. 44, 45.

Fig. 45.

thing and then becomes negative, the diſtance *B k* will increaſe to infinity and then become negative and decreaſe on the other ſide of the eye[a]; and conſequently as the rays fall more and more converging upon the eye, the confuſion will increaſe; while the apparent diſtance decreaſes[b], as it did without the eye-glaſs[c].

a Art. 48.

b Rem. 198.

c Art. 117.

Objection anſwered.

227. As to excluding prænotions and prejudices; that is, as the author expreſſes it, in caſe we abſtract from all other circumſtances of viſion, ſuch as the figure, ſize, faintneſs &c. of the viſible objects[d]; I anſwer, that when a given fixt object is viewed ſucceſſively through different eye-glaſſes put cloſe to the eye, fixt at any given diſtance from the fixt object-glaſs; the figure, ſize and faintneſs of the viſible object are not ſenſibly altered by the different eye-glaſſes; and yet the confuſedneſs, though very ſenſibly altered, or reduced to diſtinctneſs, produces no ſenſible alteration of the apparent diſtance of the object in any of my tryals.

d Rem. 222.

Therefore apparent confuſion does not ſuggeſt diſtance.

228. It appears to me by theſe experiments and many more that I have tryed, that confuſedneſs of appearance does not ſuggeſt nearneſs or any diſtance at all. Perhaps the reaſon may be, that moſt children and young people do not perceive any confuſedneſs in near objects; becauſe they commonly pore upon their letters in learning to read and write; their eyes, like other parts of their bodies, being then more pliant than afterwards. And therefore though a confuſed appearance of near objects, may gradually come upon them as they grow older; yet they rather avoid it than attend to it as a ſign of nearneſs; becauſe they did not want it, having conſtantly been uſed to other ſigns of nearneſs ſufficient for the purpoſe.

Not does the ſtraining of the eye.

229. But we are told that a ſtraining of the eye, to prevent confuſedneſs, is a ſenſation that ſupplies the place of it in ſuggeſting nearneſs[e]. I anſwer that this ſenſation being ſomewhat painful might hinder young people from poring. But they do pore, and therefore ſeem to have no ſenſation of ſtraining; nor any need of it, to ſuggeſt nearneſs, though it afterwards comes upon them. For as they grow older they ſtrain their eyes to avoid confuſedneſs, or to reſtore the uſual diſtinctneſs and largeneſs, together with a perception of the minuter parts of things; and are not theſe uſual perceptions ſufficient to ſuggeſt the uſual idea of nearneſs? When this ſtraining is no longer effectual, they are forced to uſe ſpectacles, by which the old aſſociated ideas, of diſtinctneſs, largeneſs and nearneſs, are again revived in their minds. Small objects ſeen through ſpectacles, by thoſe that want their help, appear much diſtincter than to the naked eye; and therefore ſhould appear much remoter, according to the principle of confuſedneſs; but I am told that they appear rather nearer, as they ſhould do by theory; becauſe they appear rather larger, by reaſon of a ſmall diſtance of the ſpectacles from the eyes.

e Ibid. ſect. 27.

Nor does faintneſs or appearance, perceived through pin-holes.

230. Laſtly for farther ſatisfaction, that apparent diſtance does not depend upon apparent confuſedneſs, nor upon the ſtraining of the eye, nor upon degrees of brightneſs and faintneſs of appearance, let the following experiments be tryed. To prevent confuſedneſs and the ſtraining of the eye, make a fine pin-hole in a card, or a paper, and having applyed it cloſe to your eye, repeat any of the experiments abovementioned; and you will find that the objects will always appear diſtinct, even when the rays converge towards the eye; and yet the degrees of apparent diſtance will continue the ſame as before; and will be ſomewhat more determinate, becauſe the apparent magnitudes are now more diſtinctly defined by diſtincter out-lines[f]. It is beſt to look at objects without doors, becauſe they radiate ſtronger than thoſe within doors, and appear more evident through a ſmall hole. Now by ſlipping the hole to one ſide of the pupil, and by reſtoring it alternately, while the eye, the object and glaſſes are fixt; you cannot be certain of a ſenſible alteration of their apparent diſtances; though their diſtinctneſs and brightneſs be greatly altered. And the ſame holds good in uſing the pin-hole without any glaſſes. Becauſe, when it is cloſe to the eye, it does not ſenſibly alter the apparent magnitudes of objects placed at any diſtances whatever; and yet it cauſes very near ones to appear diſtinct.

f Art. 117.

Or through ſmaller apertures of teleſcopes.

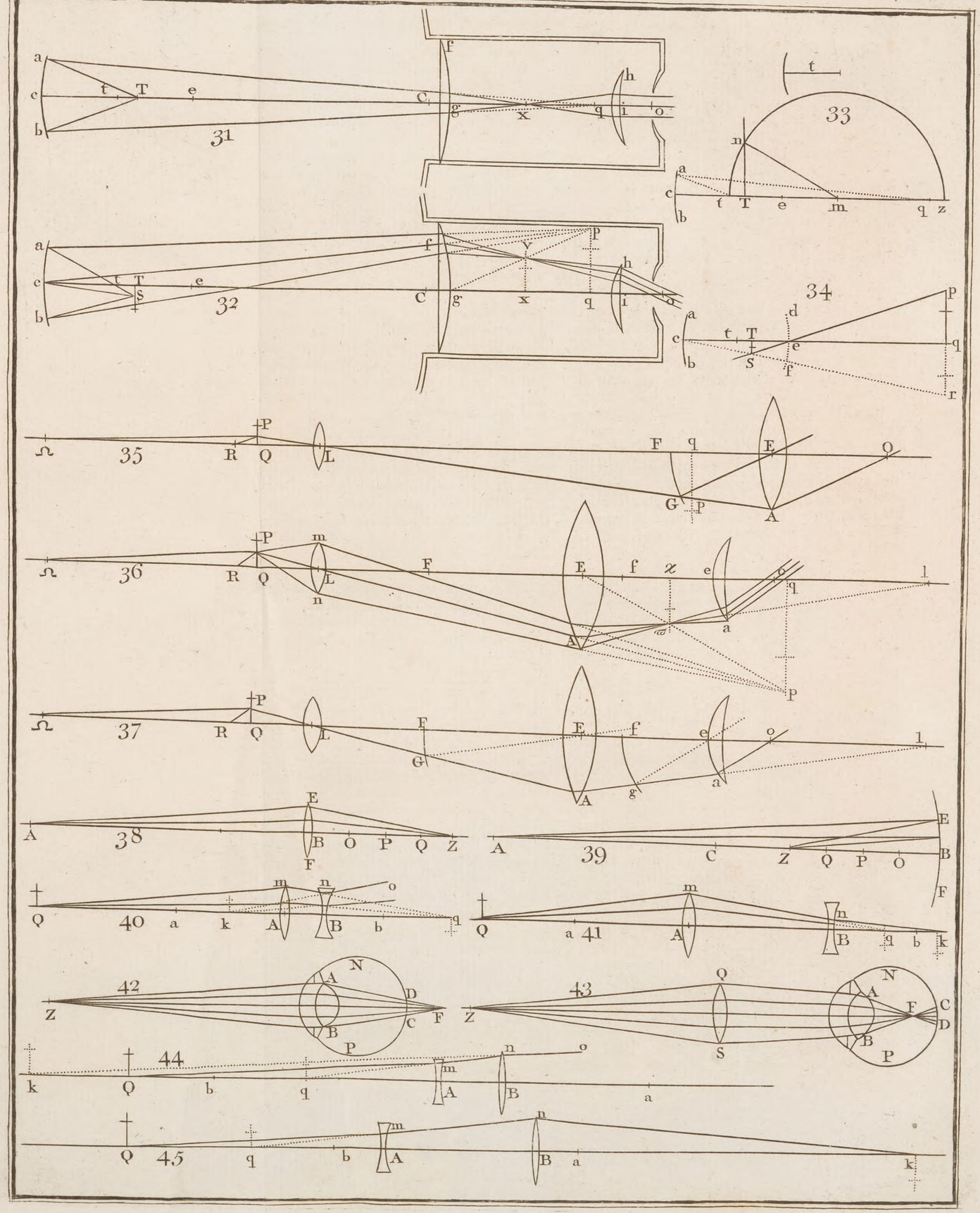
231 In like manner if the aperture of the object-glaſs of a teleſcope be varied, by covering it ſucceſſively with circles of paper, having holes in their centers of very different ſizes; it is well known that the apparent brightneſs of any given object, will be varied in proportion to thoſe apertures. But its apparent diſtance will not be varied thereby, ſo long as the ſame eye-glaſs is retained. But if it be changed for other eye-glaſſes of ſhorter focal diſtances, the apparent diſtance of the object will now be diminiſhed; notwithſtanding that its faintneſs is increaſed[g]. This may be accurately tryed in the manner deſcribed in Rem. 207. Therefore the increaſe of the apparent magnitude is the cauſe of the greater nearneſs notwithſtanding the faintneſs; becauſe nothing elſe is ſenſibly altered.

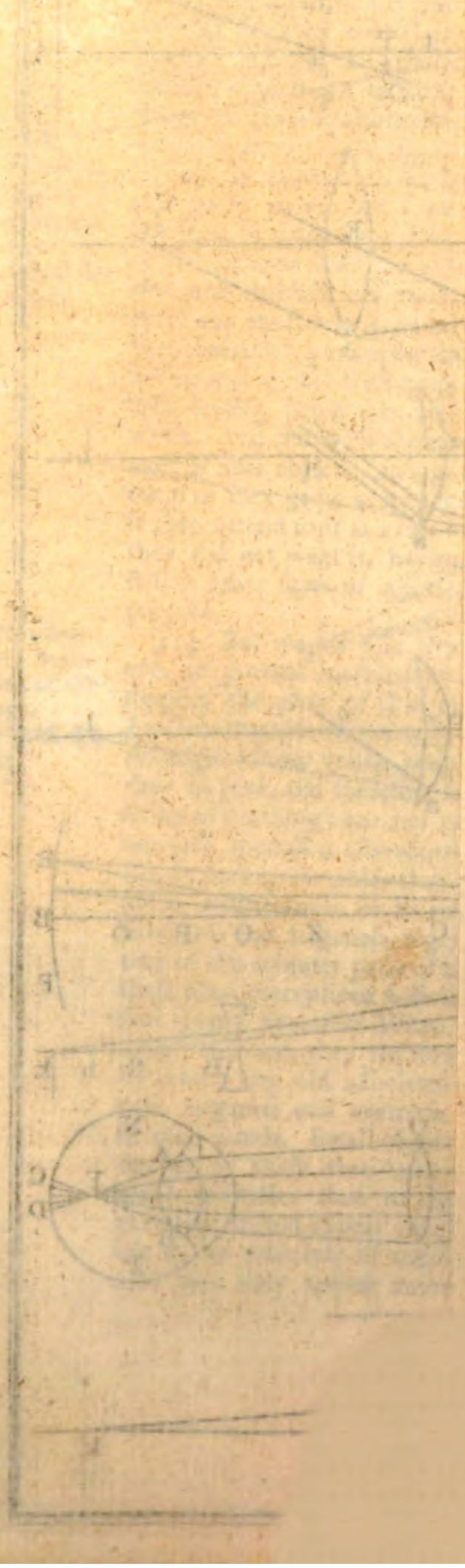
g Art. 348.

General concluſion.

232. I conclude then by induction of particulars in all theſe experiments (to ſay nothing of many more) that, whatever be the variations of the divergency and convergency of rays, or of diſtinctneſs and confuſedneſs, or of brightneſs and faintneſs, the apparent diſtance of a given ſyſtem of known objects ſeen with glaſſes, in the field or in any ſpacious place where the fancy has room enough to work in[h], is ſuggeſted to us either principally or ſolely by its apparent magni-

h See Rem. on Art. 151.


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magnitude. And ſince our judgments of appearances in glaſſes, are indiſputably derived from our experience in viſion with the naked eye; it follows that here alſo the ſame general concluſion takes place. And therefore this concluſion, thus diſcovered by analyſis, may be aſſumed as a principle in ſyntheſis, in order to explain the phænomena of diſtance in viſion of all ſorts; according to the beſt method of philoſophiſing deſcribed by our great Philoſopher in the following words.

233. "As in mathematicks, ſo in natural philoſophy, the inveſtigation of difficult things by the method of analyſis, ought ever to precede the method of compoſition. This analyſis conſiſts in making experiments and obſervations, and in drawing general concluſions from them by induction, and admitting of no objections againſt the concluſions, but ſuch as are taken from experiments, or other certain truths. For hypotheſes are not to be regarded in experimental philoſophy. And although the arguing from experiments and obſervations by induction be no demonſtration of general concluſions; yet it is the beſt way of arguing which the nature of things admits of, and may be looked upon as ſo much the ſtronger, by how much the induction is more general. And if no exception occurs from phænomena, the concluſion may be pronounced generally. But if at any time afterwards any exception ſhall occur from experiments, it may then begin to be pronounced with ſuch exceptions as occur. By this way of analyſis we may proceed from compounds to ingredients, and from motions to the forces producing them; and in general, from effects to their cauſes, and from particular cauſes to more general ones, till the argument end in the moſt general. This is the method of analyſis: and the ſyntheſis conſiſts in aſſuming the cauſes diſcovered and eſtabliſhed, as principles, and by them explaining the phænomena proceeding from them, and proving the explanations.' So far Sir *Iſaac Newton*.

234. In the 139th article and thoſe that follow, I proceeded ſynthetically upon the principle of apparent magnitude; thereby to determine the apparent diſtances of objects ſeen by reflections and refractions in all caſes. And in the following remarks I intend to take notice of ſuch exceptions to the general determination, as have hitherto occurred to me; and alſo to ſhew the reaſon why they muſt be exceptions. But at preſent I will proceed a little farther with the extent of this principle.

235. To ſhew that objects do not always appear to the naked eye in the real places from which the rays diverge, but frequently in other places, ſuggeſted to the imagination by their apparent magnitudes, I inſtanced in perſpective and painting [a]: in which theſe maxims are univerſally followed. Firſt, to diminiſh the dimenſions of the figures of given objects in proportion as the objects themſelves are remoter from the eye; ſo that the magnitudes of the figures may always diſcover the diſtances of the objects: And ſecondly, to make the contours that bound the figures more faint or more ſenſible according as the objects are more or leſs removed from the eye; and laſtly to leave out the minuter parts of ſmall figures, eſpecially about their contours, and to ſketch out the reſt more ſlightly and indeterminately, in proportion as the objects are more remote. Becauſe while an object recedes from the eye, the apparent magnitude of the whole and of its ſeveral parts are continually decreaſing; and the contour that bounds it, being but a phyſical line, ſoon appears faint and confounded with the colours of contiguous objects, and then diſappears, and next to that, the minuter exteriour parts; till at laſt nothing remains but the appearance of the groſſer parts of a confuſed, indeterminate figure. And what is all this but chiefly the decreaſe of apparent magnitude of the whole and of its parts?

Deſcription of a perſpective draught. Fig. 46.

236. But to be ſomewhat more particular, let the figure *ABCD* contain a perſpective draught of the inſide of a long gallery, ſeen from one end of it, when the axis of the eye is directed in a line parallel to the length of the room. And let this axis cut the plane of the draught in *o*; and by the known rules of perſpective [b], the repreſentations of all lines, which in the room are parallel to the axis of viſion, or in this inſtance to the length of the room, muſt be drawn converging towards the ſaid point *o*; thence called the center of the draught. Such are the lines *Aa*, *Bb*, *Cc*, *Dd*, which repreſent the four interſections of the two ſides of the room with the floor and cieling. It is manifeſt then, that the repreſentations of all equal lines, which in the room are perpendicular to theſe parallels, and are intercepted between them, or between any other parallels to them, are continually diminiſhed as they fall nearer and nearer to the center *o*. Which is the ſame thing as to ſay, that all the linear dimenſions of the repreſentations of any row of equal objects, ranged in the room upon any line parallel to the axis of viſion, either on the floor below it, or in the cieling above it, or on either ſide of it, are continually diminiſhed as they fall nearer to the center *o*, and conſequently as they belong to remoter objects. See Art. 156. FIG. 266.

[b] See Art. 156.

The effects of it upon the eye.

237. Theſe are the proportions of the parts or figures of the draught; and in looking upon it, we find, that the diminiſhed parts, placed in all poſitions round about the center *o*, have a like effect upon the mind in ſuggeſting diſtance from the eye. So that this effect is not owing to any particular poſition of the ſmaller parts, placed

[a] Art. 138.

ced above or below or on either ſide of the center *o*, but to their magnitudes only. Which is ſtill more evident in the perſpectives of ſmall rooms, churches and all ſhort proſpects; in which all the larger parts and figures, though drawn almoſt with equal ſtrength, do yet ſuggeſt unequal diſtances by their unequal magnitudes. And it is acknowledged by all the Artiſts, that I have conſulted, that the bare out-lines of a draught juſtly executed, will ſufficiently diſcover the diſtances of the original objects, eſpecially when viewed with one eye placed in the point of view. Becauſe the bare ſhapes of figures are frequently ſufficient to diſcover what objects are repreſented by them. And herein the imagination is helped by looking through a ſhort tube, or through ones hand coiled round like a tube, to hide the appearance of the collateral parts of the paper, not put into perſpective.

The lights and ſhadows of a draught.

238. As to the diſpoſition of the lights in a draught, they are frequently made as ſtrong in and about its center as any where elſe; and ſometimes ſtronger. Becauſe a ſide light, thrown equally upon the original objects, is reckoned the beſt; and becauſe a ſtrong light is the more neceſſary for diſtinguiſhing the ſmaller figures near the center of the draughts. The ſhadows indeed of remote objects are generally omitted, being originally too weak to affect the eye.

Aereal perſpective what.

239. Theſe are the principal maxims and effects of Linear perſpective; which in painting is improved by Aereal perſpective, or the art of imitating the apparent degradation of the colours of natural bodies, in proportion as the bodies are gradually removed from the eye. This degradation is partly cauſed by the tranſmiſſion of the rays of ſome colours through the air and vapours, in greater plenty than the rays of other colours; but chiefly by their mixture with the rays of an azure colour, reflected from very long quantities of intermediate air and vapours.

Its effects give place to thoſe of linear perſpectives.

240. The aerial perſpective of a remote country is entirely preſerved when we view it through a concave lens. And here the linear perſpective, from which the rays diverge upon the eye, is accurately proportioned in the image of the country, formed by the lens upon a plane paſſing through its principal focus at right angles to its axis. Now when the eye is applyed cloſe to the lens, this imaginary perſpective, or indeed the ſimilar perſpective upon the retina, excites the ſame idea of the country as is uſually excited by an equal[a] perſpective of it formed upon the retina of the naked eye. But when the lens is fixt and the eye is drawn back from it, and conſequently from the imaginary perſpective at its focus, the ſimilar perſpective upon the retina is now diminiſhed[b], and excites an idea of a greater diſtance of the country, though the aereal perſpective of colours continues the ſame as before. But in looking through plane glaſſes very lightly tinged with the ſmoke of a candle or with any faint and tranſparent colours, I do not remember that either the apparent magnitudes or the apparent diſtances of remote objects were ſenſibly altered; though the natural degradation of their own colours was quite diſturbed by the tincture of the colour of the glaſs. By putting theſe things together, it ſeems evident to me, that the linear perſpective of a painted landſkip has a more powerful effect in ſuggeſting diſtances, than the aereal perſpective of colours.

Objection anſwered.

241. To this it may poſſibly be objected, that certain large and remote objects, as mountains or cities, ſeen very clearly through an air more pure than uſual, appear ſenſibly nearer than they generally do when the air is groſſer[c]. I have been aſſured of two or three inſtances of this kind by very good judges in theſe matters; particularly by Dr. *Berkeley* himſelf; who has told me, that when he travelled in *Italy* and *Sicily*, where the air is generally much clearer than here in *England*, he was frequently ſurpriſed with appearances of this ſort: viz. that a diſtant city, or the like object, that appeared very lively and ſtrong, appeared alſo much nearer to him, than he knew it to be, by more certain ways of eſtimating its diſtance. But in my opinion theſe phænomena are not at all diſagreeable to the uſual relation between apparent magnitude and diſtance. For our idea of ſmaller or greater magnitude is not a ſimple idea of a ſmaller or greater uniform ſurface; but includes alſo an idea of a ſmaller or greater number and variety of diſtinct parts of the known object, not imagined, but actually perceived by the eye; together with various degrees of their evidency. Now ſince the minuter parts of a known object, which are uſually obſcured by a groſſer air, become more evidently revealed in a purer; the object ought to appear ſomewhat nearer through the purer than through the common groſſer air: and almoſt at the ſame nearneſs, at which it would appear, if the ſpectator approached ſo far towards it, till he could ſee the ſame number of minute parts as clearly in a groſſer air, as he ſaw them in the purer from a greater diſtance: I ſay almoſt at the ſame nearneſs; becauſe the greater apparent magnitude of the whole object, ſeen from the nearer ſtation, if it be known and familiar to us, will alſo contribute to leſſen its apparent diſtance: not to mention the view of a ſmaller extent of the country interpoſed.

Idea of magnitude not ſimple but compound.

The anſwer confirmed by teleſcopes.

242. Now though this explanation does not totally exclude all concurring ſuggeſtions of nearneſs by the aereal perſpective abovementioned, (as I do not intend it ſhould,) yet I think it is farther confirmed by its cloſe agreement with the principal, if not the only, cauſe that ſuggeſts

[a] Art. 117.

[b] Art. 106.

[c] See *Hook* Exper. an Obſer. pu by *Derha* 8°. p. 14

ſuggeſts the nearneſs of objects ſeen through teleſcopes, (eſpecially ſuch as are adapted for
a Art. 356. obſervations by day-light [a];) which cauſe does not conſiſt ſo entirely in the greater apparent magnitude of the whole object, (which frequently cannot be wholly ſeen at one view, through a fixt teleſcope,) as it conſiſts alſo in a clear and diſtinct perception of a much greater number and variety of minuter parts, than can poſſibly be perceived by the naked eye.

Why objects appear ſmaller through teleſcopes than we are apt to imagine.

243. Hence (by the bye) it comes to paſs that perſons unexperienced in looking through teleſcopes, are often diſappointed in their expectations of their performance. For example, when the face of a man, ſitting at 100 yards diſtance, is propoſed to be viewed through a teleſcope that magnifies 100 times in diameter; they expect to ſee a gigantick ſort of a face at leaſt as broad as the full moon: (which I am ſatisfied was Frier *Bacon*'s notion of this matter, and ſhews that he never looked through a teleſcope [b]:) and indeed it is natural enough, conſidering they are apt to think, that the object will appear in the ſame place and at the ſame diſtance, as it does to the naked eye; and conſequently like a ſurface 100 times broader than that of the real face. But upon tryal it appears about 100 times nearer; and then this nearneſs of appearance becomes the only cauſe of their admiration. For ſince the face appears in the teleſcope of the uſual ſize, and with the uſual diſtinctneſs and variety of features and minuter parts, as it would do to the naked eye about
b Rem. 111.
c Art. 143. 100 times nearer to it [c], they are not ſurprized with any uncommon circumſtance in this appearance, excepting that of the nearneſs it ſelf. Another thing is, that in looking at the face with the naked eye at 100 yards diſtance, we are apt to conceive it larger, in proportion to the diſtance, than if it was nearer. For the general air of a known perſon ſuggeſts to our memory a general notion of ſeveral features, which upon examination are not perceived by the eye: as we are plainly ſenſible in looking at ſtrangers or any unknown objects, which require a more particular perception of their minuter parts. So that this prænotion contributes alſo to fruſtrate the obſerver's expectation of the performance of the teleſcope, though very compleat and admirable in diſcovering the minuter parts of things. And this is the juſt and true ſenſe of the definition of apparent magnitude in Art. 98 and 104.

An obſervation upon the relievo in painting.

244. *Leonardo da Vinci* has made a curious obſervation not improper to be mentioned in this place [d]: that a painting, though conducted with the greateſt art, and finiſhed to the laſt perfection, both with regard to its contours, its lights, its ſhadows and its colours, can never ſhew a relievo equal to that of the natural objects; unleſs theſe be viewed at a diſtance and with a ſingle eye. Which he thus demonſtrates. If an object *C* be viewed by a ſingle eye at *A*, all Fig. 47. objects in the ſpace behind it, included as it were in a ſhadow *ECF* caſt by a candle at *A*, are inviſible to the eye at *A*; but when the other eye at *B* is opened, part of theſe objects become viſible to it; thoſe only being hid from both eyes that are included as it were in the double ſhadow *CD*, caſt by two lights at *A* and *B* and terminated in *D*; the angular ſpace *EDG* beyond *D* being always viſible to both eyes. And the hidden ſpace *CD* is ſo much the ſhorter, as the object *C* is ſmaller and nearer to the eyes. Thus he obſerves that the object *C* ſeen with both eyes becomes as it were tranſparent, according to the uſual definition of a tranſparent thing; namely, that which hides nothing beyond it. But this cannot happen when an object, whoſe breadth is bigger than that of the pupil, is viewed by a ſingle eye. The truth of this obſervation is therefore evident; becauſe a painted figure intercepts all the ſpace behind its apparent place; ſo as to preclude the eyes from the ſight of every part of the imaginary ground behind it.

Some advantages in viſion with both eyes.

245. Hence we have one help to diſtinguiſh the place of a near object more accurately with both eyes than with one; in as much as we ſee it more detached from other objects beyond it; and more of its own ſurface, eſpecially if it be roundiſh. And therefore ſuppoſing we judged of its diſtance by nothing elſe but by its apparent magnitude, our judgments muſt be ſomewhat different with one eye and with both; for this other reaſon alſo, that with both eyes we ſee objects much clearer and ſtronger as well as larger. And upon ſecond thoughts [e] I will not
e See Art. 138. p. 51. line 25.
diſpute whether the feeling of the turn of our eyes, in directing their axes ſucceſſively from a remoter object to a nearer, or on the contrary, may not alſo contribute to correct our judgment of its diſtance: certainly, as ſoon as this poſition of the eyes is obtained, our perception of the object becomes much clearer and ſtronger than before. For before that, by attending nicely we may always perceive two fainter appearances of the object approaching each other ſwiftly and tranſverſely, till they jump together into one clear image. And therefore it is no wonder that perſons upon loſing one eye, are ſubject to miſtakes in ſmall diſtances, as in pouring out liquors from one veſſel to another, or in ſnuffing a candle, or in the like actions. But by degrees they become more expert, as I am well informed by an experienced perſon; and the Hon. Mr. *Boyle* takes notice of the ſame thing [d]; and adds, that by an experiment purpoſely made, he found ſeveral times, that with his two eyes together he ſaw an object in another place, than he could do by either of them alone. Inſtances of this kind may be ſeen in Art. 137. But when the objects are pretty

d Treatiſe of painting p. 178. *Lond.* 1721.

d 7th obſ. upon vitiated ſight.

large and pretty remote in compariſon to the interval between the eyes; all theſe ſmaller helps become ineffectual, except that of the greater vivacity or ſtrength of appearance.

In what caſe greater faintneſs ſuggeſts greater diſtance and how.

246. Laſtly, when the moſt remote objects of unknown magnitudes are ſeen in the horizon one above another; as mountains above mountains, houſes above houſes, and the like; no doubt it is the greater blueneſs, or faintneſs, and the greater height together, which ſuggeſt the idea of a greater diſtance of the bluer and higher objects; becauſe we are ſenſible of the ſame thing in painting, and becauſe we have no other means of information in both theſe caſes. Nor do I deny, that ſome degrees of faintneſs, ſtill ſmaller than theſe, by obſcuring the contours and ſome of the ſmaller parts of an object, placed at a leſs diſtance than that of the horizon, may cauſe it to appear ſomewhat ſmaller, and conſequently almoſt as much remoter, as it would appear to the eye removed ſo much farther from it in a clearer air, till it ſhall appear of the ſame ſize and with the ſame number and variety of minute parts as it did before in the groſſer air a.

a Rem. 241.

The principal point eſtabliſhed.

247. But, when we take a view of a ſcene or landſkip of known objects, not ſo remote as to be ſenſibly obſcured by the air, nor ſo near at hand or ſo ſmall as to appear ſenſibly different to one eye or to both [b]; but ſituated at any moderate diſtances whatever, at which we uſually perceive them and their diſtances, moſt diſtinctly, clearly, accurately and in the utmoſt perfection; what I chiefly contend for is this; that their apparent diſtances from the eye, as well as from one another, are principally, if not ſolely, ſuggeſted to us by their apparent magnitudes: which I think has been ſufficiently proved.

b Rem. 245.

The concluſion of this diſſertation.

248. To conclude, it is the buſineſs of opticians firſt to find out and eſtabliſh the moſt general cauſe that ſuggeſts the diſtances of things, when ſeen in the utmoſt perfection; then, in order to account for the like appearances when the viſion is leſs perfect, we muſt conſider the extent and limitations of the regular operations of that general cauſe upon our minds; and alſo the impediments to its operations, ſuch as the littleneſs and the faintneſs of objects too remote, the confuſedneſs of objects too near, and the like. So that in order to the ſolution of any given phænomenon of diſtance, the effects of the general cauſe, if found inadequate to the given appearance, may be equated or corrected by the effects of the particular impediments, when proved to be ſuch in the given caſe. Juſt as the place and diſtance of a planet muſt firſt be computed from the law of its mean motion; and then be corrected by proper equations that meaſure its irregularities. This is the only juſt method of proceeding in all phyſical enquiries; and therefore till the law of the principal cauſe, anſwering to this mean motion, be well adjuſted, it will be in vain to apply the equations.

Upon ART. 142.

249. Pag. 53. line 2. *Fourthly, when a ray PO coming directly to the eye, makes an angle PO$\mathfrak{Q}$ equal to AOC or $\varpi O\varkappa$:* which problem may be ſolved as follows. In the axis of any convex lens whoſe center is E, take EG and EH ſeverally equal to twice its given focal diſtance, and let the diſtance $E\mathfrak{Q}$, of the object from the lens, be bigger than EH; and taking OG to GE as EH to $H\mathfrak{Q}$, and placing it from the lens, the eye being placed at O will ſee the object $\mathfrak{Q}$, through the lens, in the ſame place as if the lens was taken quite away. Likewiſe, if the place of the eye be given, the place of the object may be had by the ſame proportion: which may be demonſtrated from Art. 139, or from the firſt term of the ſeries in Art. 247, by putting the value of the apparent diſtance $O\Pi = OP$. Fig. 203. Fig. 48.

Upon ART. 146.

250. line 1. *Therefore in theſe two caſes the object appears in the place of its image.* This will be farther evident by producing $P\varpi$ till it cuts the refracting plane CA at right angles in D. For ſuppoſing rays as PA to flow from P, ſince $D\varpi$ is to DP, in the given ratio of the ſine of incidence to the ſine of refraction [b]; it follows that while the focus P is moved along the object $P\mathfrak{Q}$, the correſponding focus ϖ will deſcribe an image $\varpi\varkappa$, parallel and equal to $P\mathfrak{Q}$ *. Fig. 209. b Art. 113. * Art. 244.

251. In the other caſe of reflections from the plane CAD; ſince $D\varpi$ is equal to DP, all the rays that flow from P will diverge from ϖ after reflections [c]. And conſequently while the focus P is moved along the object $P\mathfrak{Q}$, the focus ϖ will deſcribe an image $\varpi\varkappa$ parallel and equal to $P\mathfrak{Q}$. Fig. 210. c Art. 202.

252. I touched upon ſome other caſes in Art. 142, in which a ſmall object is equal to its image and appears in its place, when it touches any thin lens, or any reflecting or refracting ſurface; and alſo when it is placed in the center of a concave ſpeculum; or in the center of a ſingle ſpherical ſurface of a denſe medium.

253. I think there is but one caſe more of this kind, in ſingle bodies, viz. when the diſtance $E\mathfrak{Q}$ of an object $P\mathfrak{Q}$, from the center of a convex lens or of a ſphere or of a ſingle convex ſurface of a denſe medium, is equal to the ſum of its two focal diſtances. For then the oppoſite diſtance Eq of its image, is alſo equal to that ſum [d]; and conſequently the object $P\mathfrak{Q}$ is equal to its image pq *; and ſhould appear in its place, by the general rule mentioned at the end of the preſent article; viz. that $O\varkappa$, the apparent diſtance of the object, is to Oq, the diſtance of its image, Fig. 49, 50. d Art. 236. * Art. 245.

image, as the object PQ or $\varpi\varkappa$ is to its image pq; as is evident by the similar triangles, $O\varpi\varkappa$, Opq.

254. By which it is very plain that the object cannot appear in the place of its image, (at least to a single eye,) but when they are equal to each other.

Upon ART. 148.

255. Since the phænomena of apparent motions in glasses, were deduced from the definition that I gave of apparent distance [a], by no other mediums than those of geometry; it may possibly be more satisfactory to explain them a little farther, by means of a closer comparison of them with the like phænomena to the naked eye. And hereby we shall better understand what insuperable difficulties and contradictions will arise, in attempting to explain them by the received principle of the divergency of rays from the place of the image.

256. Other things remaining as they were, join OP and through the center of the lens draw PE, cutting the visual ray OA (produced) in p; then the line pq, perpendicular to the axis, will be the image of the object PQ*; and when the eye is put close to the lens, the angles pOq, POQ will be very nearly equal to each other [b].

257. *Case* 1. Hence while the eye is receding from the lens to any point O, if the distance Oq be less than OQ, the angle pOq will decrease quicker, or in a greater proportion, than the angle POQ does [c]; and consequently the apparent magnitude of the fixt object PQ, seen in the lens, will also decrease quicker than it would do to the naked eye if the lens was taken away [d]. But supposing at the same time, that the object PQ were moved from the naked eye into the successive parallel places $\varpi\varkappa$, so quick as to subtend a decreasing visual angle $\varpi O\varkappa$ constantly equal to the angle pOq, formed by the refracted rays; its apparent magnitude to the naked eye, would then become constantly equal to its apparent magnitude in the lens: and therefore it would appear to recede from the naked eye, with the same velocity, with which it will appear to recede in the lens, while it stands fixt in the place PQ.

258. For the two pictures upon the retina being both distinct and constantly equal and similar to each other; and the sense being not at all affected by the refractions of the rays, but only by the magnitude of the picture thence resulting; the mind must needs make the same judgment upon the sensations of this decreasing picture, as it usually and constantly makes upon the like sensations of a like decreasing picture of an object moved forwards from the naked eye.

259. *Case* 2. Now let the distance Oq be greater than OQ; and while the eye is receding from the lens to any point O; the angle pOq will decrease slower, or in a lesser proportion, than the angle POQ does [c]. And consequently the apparent magnitude of the object PQ, seen in the lens will also decrease slower than it would do to the naked eye, if the lens was taken away [f]. But supposing at the same time, that the object PQ were moved towards the naked eye into the successive parallel places $\varpi\varkappa$, so quick as to subtend a decreasing visual angle $\varpi O\varkappa$ constantly equal to the angle pOq, formed by the refracted rays; its apparent magnitude to the naked eye, would then become constantly equal to its apparent magnitude in the lens: and therefore it would appear to approach towards the naked eye with the same velocity, with which it will appear to approach in the lens, while it stands fixt in the place PQ*.

260. *Case* 3. Lastly, if the fixt image pq be behind the eye, moving backwards from the lens to any point O; the angle AOE or pOq will now increase, while the angle POQ is decreasing: and of consequence the apparent magnitude of the object, seen in the lens, will increase, while its apparent magnitude to the naked eye would decrease, if the lens were taken away. But if the object be moved towards the naked eye into the successive parallel places $\varpi\varkappa$, so quick as to subtend an increasing visual angle $\varpi O\varkappa$ constantly equal to pOq or AOE, its apparent magnitude to the naked eye would become constantly equal to its apparent magnitude in the lens; and therefore it would appear to approach towards the naked eye with the same velocity, with which it will appear to approach in the lens while it stands fixt in the place PQ*: especially if the object be viewed through a pin-hole to procure distinct vision.

261. When the eye is put close to a single refracting surface AC, let the refracted visual ray DCp cut a line $P\varpi$, drawn parallel to the axis, in r; and the object PQ will appear in the place of the perpendicular rs*. And while the eye is receding from C to any point O, the apparent distance of the fixt object PQ may be shewn, as above, to vary in the same proportion, as it would vary to the naked eye, if the equal object rs were moved from the place rs, with such a velocity as always to subtend the variable visual angle pOq or AOC.

262. Hence while the eye recedes from a refracting plane surface AC, the object PQ will appear immoveably fixt in the place of its image pq; because it coincides with the place and magnitude of the perpendicular rs*.

263. For since we are constantly accustomed to certain known degrees of increase of the apparent magnitude of a fixt object, while we are approaching towards it; and to the like degrees of decrease while we are receding from it; it is necessary that the degrees of apparent mag-

[a] A fuller explanation of the apparent motions of a fixt object seen in a fixt glass. Art. 139.

Fig. 51, 52, 53.

[b] Art. 55. or 245. Art. 43.

Fig. 51.

[c] Art. 60.

[d] Art. 106. 108.

Demonstration.

Fig. 52.

[e] Art. 60.

[f] Art. 106. 108.

* Rem. 258.

Dr. *Barrow*'s difficult case demonstrated.

Fig. 53.

* Rem. 258.

Fig. 54.

* Art. 139.

Fig. 51.

* Rem. 250.

Demonstration.

nitude perceived in glaſſes, as well as by the naked eye, ſhould vary quicker than theſe, in order to excite an idea of the departure or approach of an object.

Fig. 56, 57. 264. When the eye is put cloſe to a reflecting ſurface AC, let the reflected viſual ray DCp cut the line $P\varpi$ drawn parallel to the axis, in r; and the object PQ will appear in the place * Art. 139. of the perpendicular rs*. And while the eye is receding from C to any point O, the variations of apparent diſtance may be explained as above; by conceiving the motion of the equal object rs, ſeen by the naked eye, to begin from the place rs.

265. In reflections at any ſingle ſurface it might eaſily be ſhewn that the diſtance Cs is equal to CQ; and in refractions at a ſingle ſurface, that Cs is to CQ in the ratio of the ſines that meaſure the refractions [a].

a Rem. 250.

Fig. 58. 266. Hence alſo while the eye is receding from a plane looking-glaſs, the object PQ will appear immoveably fixt in the place of its image pq, becauſe it coincides with the place and magnitude of the perpendicular rs; and for the reaſon juſt mentioned in Rem. 263.

The phænomena agree with this explanation. b Art. 139. 267. Theſe and many more concluſions of the like ſort, deduced from the definition of apparent diſtance in glaſſes [b], I find are agreeable to the phænomena, as well when the image is behind the eye as when it is before it. But the image is here introduced, not as a medium that affects the ſenſe, but the underſtanding only; in order to determine, by geometry, the variations of the viſual angle ſubtended by it. When the image is behind the eye the phænomena of diſtance are allowed, by Dr. *Barrow*, Mr. *James Gregory* * and the beſt writers, to be inexplicable, upon the received principle of the divergency of rays from it. But ſtill they contend that the object appears in the place of its image when before the eye; which indeed, by a coincidence of the effects of different cauſes, is agreeable to reaſon and to the phænomena in two or three obvious caſes abovementioned [c], but in general it is equally oppoſite to both, as I ſhewed above [d], and will now ſhew it again in a different manner.

* Optica promota prop. 29.

c Rem. 250. &c.

d Rem. 213. &c.

But are inexplicable by the divergency of rays. 268. For, ſetting aſide the ſenſations of different degrees of apparent magnitude, why does not the object appear always in the place of its image, when bigger or leſs than it ſelf, as well as when equal? And upon moving the eye backwards or forwards, why does the object appear at reſt in the latter caſe, and in motion in the former; ſometimes backwards and ſometimes forwards; with certain degrees of apparent velocity in certain caſes? Allowing that the object appears in the place of its fixt image to the eye fixt in any one given place, it ought alſo to appear there to the eye in any other given place; that is, it ought to appear fixt in that place, while the eye is in motion from place to place. I think nothing can be plainer than this concluſion, but ſtill we may conſider it in this other light. Suppoſing a real object, equal and ſimi- Fig. 51 to lar to the image pq of the object PQ, to be put 57. into the place pq; it would certainly appear at reſt to the naked eye approaching towards it or receding from it; and its picture upon the retina being conſtantly equal and ſimilar to the picture of the object PQ ſeen in the glaſs; this object PQ ought likewiſe to appear at reſt. Which being directly contrary to the general phænomena of viſion in all ſorts of glaſſes, might be ſufficient to ſhew the unreaſonableneſs of the received hypotheſis; viz. that an object appears alin the place of its image even before the eye.

Apparent lateral motions explained. 269. But beſides this, there is another plain conſequence from it, as oppoſite to experience as the former; to wit, whenever an object appears in the place of its fixt image, it ought to appear fixt, not only to the eye moving backwards or forwards, but ſideways alſo. Nevertheleſs in moſt caſes theſe apparent lateral motions are very great, though that of the eye be very ſmall; and I find both by theory and experience that they are reducible to the following rules.

Fig. 51, 56. 270. When the fixt image q lyes between the eye at O and the apparent place $\varkappa$ of the point Q; the apparent lateral motion of the point Q (like a real motion of the point $\varkappa$ carried by the angular motion of the line $Oq\varkappa$ about the fixt point q,) will tend the contrary way, from the axis of the glaſs, to the eye's lateral motion from it. Becauſe the viſual rays that fall ſucceſſively upon the eye in different places, do all diverge from the fixt point q very nearly; and becauſe we refer the appearance of the point Q to the place $\varkappa$ beyond the image q.

Fig. 55, 58. 271. Conſequently when the points q and $\varkappa$ coincide, there can be no apparent lateral motion of the point Q; which agrees with common experience, of objects ſeen by reflection from a plane looking-glaſs, if it be wrought perfectly plane, or from ſtagnating water; and alſo of objects placed under water and ſeen by refractions thro' its ſuface; provided in this latter caſe, wherein the image is imperfect [a], that the lateral motion of the eye be not very great.

a Rem. 273.

Fig. 52, 57. 272. But when the image q lyes beyond the apparent place $\varkappa$, the apparent lateral motion of the object Q, agreeable to the real motion of the point $\varkappa$, will tend the ſame way from the axis as the eye's lateral motion does; as likewiſe, Fig. 53. when the image q is infinitely remote or behind the eye. This is evident for the like reaſons as in the firſt caſe, Rem. 270.

273. I have here ſuppoſed all the rays flowing from the point Q, to belong to the fixt point q, after reflections or refractions, either accu-

accurately or very nearly; becauſe when the apertures of the glaſſes or the lateral motions of the eye are not too great, the aberrations of the outward rays from q will be found too ſmall and therefore inſufficient to account for theſe large quantities of apparent lateral motion; even upon Dr. *Barrow*'s hypotheſis, that the point Q appears in the ſucceſſive points of a portion of a cauſtick adjoining to q. See Art. 154.

Upon ART. 151.

Geometrical conſtructions of the apparent diſtance magnitude and poſition of an object ſeen in any glaſs.

Glaſs and object fixt, while the eye is moved. Fig. 59 to 67.

274. Though the manner, in which I have explained this theory, is the eaſieſt that I could think of for determining any particular quantity of apparent diſtance, and alſo for demonſtrating the reaſon of its increaſe or decreaſe in general; yet a more accurate and comprehenſive idea of it in all caſes may be eaſier had by inſpection of the figures, which I am going to deſcribe

275. *Caſe* 1. Let an object be fixt at any point Q of the axis QC of any given lens or ſpeculum fixt at C, whoſe focal diſtance is CF; and let the image of the object Q be at the point q, to be found by Art. 207 or 236. To the axis CQ erect a perpendicular CK equal to CQ, and join Kq; and while the eye at O is moved along the axis CO, let it carry a line OD always perpendicular to the axis, and terminated at D by the fixt line Kq produced; I ſay, this perpendicular OD will be continually equal to the ſucceſſive apparent diſtances of the fixt object Q from the moving eye at O: and when OD tends upwards, the object will appear upright, otherwiſe inverted.

276. The figures repreſent all the different poſitions of the line Kq to the axis of the glaſs, occaſioned by the different poſitions of the fixt object Q and its image q with reſpect to the principal focus F.

Glaſs and eye fixt, while the object is moved. Fig. 68 to 76.

277. *Caſe* 2. Let the eye be fixt at any point O of the axis OC of any given lens or ſpeculum fixt at C, whoſe focal diſtance is Cf; and let the image of the eye O be at the point o, to be found by Art. 207 or 236. To the axis CO erect a perpendicular $C\Omega$ equal to CO, and join Ωo; and while the object Q is moved along the axis QC, let it carry a line QD always perpendicular to the axis, and terminated at D by the fixt line Ωo produced; I ſay, this perpendicular QD will be continually equal to the ſucceſſive apparent diſtances of the moving object Q from the eye fixt at O: and when QD tends upwards, the object will appear upright, otherwiſe inverted.

Eye and object fixt, while the lens is moved. Fig. 77, 78, 79.

278. *Caſe* 3. Let the eye be fixt at any point O, and the object at any other point Q of the axis OCQ of any given lens C, whoſe focal diſtance is the given line f. Upon the interval OQ, between the eye and object, build a ſquare $QOK\Omega$; and let the ſide $K\Omega$, oppoſite to the interval QO, be an ordinate to the axis of a parabola $KD\Omega$, whoſe parameter belonging to its axis, is the given focal diſtance f; and whoſe legs tend towards OQ if the lens be concave, otherwiſe from it; and while the lens C is moved along its axis OQ, let it carry a line CD always perpendicular to the axis, and terminated at D by the firſt parabola $KD\Omega$; I ſay, this perpendicular OD will be continually equal to the ſucceſſive apparent diſtances of the fixt object Q from the eye fixt at O: and when OD tends upwards, that is, towards $K\Omega$, the object will appear upright, otherwiſe inverted.

Limits of inverſion determined. Fig. 80.

279. If the interval OQ between the eye and object, be bigger than four times the focal diſtance f of the convex lens; from each end of it take off Off and Qff ſeverally equal to twice the focal diſtance f, leaving the part ff in the middle; and between this part ff and OQ, take a middle proportional GH, and place it alſo in the middle; I ſay, the parabola will paſs through the points G, H; and therefore while the convex lens is paſſing over either of them, the apparent object will invert it ſelf. See Art. 151.

Eye and object fixt, while the ſpeculum is moved. Fig. 81, 82.

280. *Caſe* 4. Let the eye be fixt at any point O, and the object at any point Q of the axis QOC of any given ſpeculum C, whoſe principal focus is F. Upon the interval OQ as a baſe, make a rectangular parallelogram $QOKL$, whoſe height OK is equal to the focal diſtance CF; and let the ſide KL, oppoſite to the interval QO, be an ordinate to the axis of a parabola DKL, whoſe parameter, to its axis, is the focal diſtance CF, and whoſe concavity ſhall contain the interval OQ; and while the ſpeculum C is moved along its axis CF produced, I ſay, that a line drawn from F perpendicular to FC, and terminated at D by the fixt parabola DKL, will be continually equal to the ſucceſſive apparent diſtances of the fixt object Q from the eye fixt at O: and if the parallelogram QK be placed upwards for a concave ſpeculum and downwards for a convex one, the object will appear upright or inverted according as FD tends upwards or downwards.

281. And if the eye and object be conceived to change places, the apparent diſtance and poſition of the object, will continue the ſame as before.

Limits of inverſion determined.

282. Biſect OQ in M, and a circle deſcribed upon the center M through the point K or L, will cut the axis in the points G and H through which the parabola will paſs; and while the focus F is paſſing over G, the apparent object will invert it ſelf.

Fig. 83, 84.

283. Hence when the interval OQ between the eye and object is contracted to nothing, that is, when the eye views it ſelf in a moving ſpeculum

lum, the lines *MK, MG, MH* become ſeverally equal to the focal diſtance *CF*.

Speculum fixt, while the eye and object are moved. Fig. 85, 86.

284. *Caſe* 5. If the interval *OQ* between the eye and object be given, while both are in motion along the axis of any given ſpeculum fixt at *C*; (as when one end of a ſtick is viewed in it while the other is kept cloſe to the eye;) biſect the interval *OQ* in *M*, and the ſemidiameter *CE*, of the ſpeculum, in *F*; and let *CELK* be a rectangular parallelogram whoſe height *CK* is to *MQ* as *MQ* to *CF*; then let the given ſide *KL*, oppoſite to the ſemidiameter *CE*, be an ordinate to the axis of a parabola *DKL*, whoſe parameter to its axis, is *CF*, and whoſe concavity contains the ſemidiameter *CE*; I ſay, that while the ſtick *OQ* is moved along the axis *CE* produced, the perpendicular *MD* to the ſtick, always terminated at *D* by the fixt parabola, will be continually equal to the ſucceſſive apparent diſtances of the object *Q* from the eye at *O*: and if the parallelogram *CKLE* be placed upwards upon *CE* for a concave ſpeculum, and downwards for a convex one, the object will appear upright or inverted according as *MD* tends upwards or downwards.

Limits of inverſion determined. Fig. 87, 88.

285. Take *FG* and $FH = \pm\sqrt{CF^2 + MQ^2}$, and the parabola will cut the axis in the points *G* and *H*; and when *M* paſſes by *G* the apparent object will invert it ſelf.

286. When the length of the ſtick *OQ* is contracted to nothing, that is, when the eye views it ſelf in motion, the parabola will paſs through *C* and *E*; and *FV*, the diſtance of its vertex from *F*, will be equal to its parameter *FC* or *FE*.

How the apparent magnitude varies in all theſe caſes.

287. In all theſe Conſtructions the apparent magnitude of the object is always reciprocally as the moveable perpendicular that repreſents its apparent diſtance. The demonſtrations of all the caſes are very ſhort, but having dwelt ſo long upon this ſubject, I will leave them to be deduced from Art. 139 or 247 or 389 by thoſe that take pleaſure in Geometrick Places; and will conclude with a few general cautions and directions for a farther examination of this theory by experiments.

General directions for examining this theory by experiments.

288. In every tryal the obſerver muſt take due notice, whether the poſitions of things about him, and about the glaſs, and about the object in view, allow room enough for his imagination to work in. For inſtance, ſtanding at a diſtance from a perſon placed near the far end of a room, ſuppoſe that I view him through a concave glaſs. While I gradually withdraw the glaſs from touching my eye, he will appear ſmaller and ſmaller and to recede a little; but though he appears continually diminiſhed, yet I cannot conceive him farther removed than to the end of the room, though I ſee part of it alſo diminiſhed; becauſe I ſee the reſt of it more evidently with my naked eye by direct rays paſſing by the circumference of the glaſs. But if I view him in an open field, he will appear to recede continually farther and farther from the place in which I ſee him with my naked eye by the ſides of the glaſs; provided I remove the glaſs in a line directed exactly towards him, or even a little above him, or to either ſide of him. But if the line of direction falls below him, he will appear in the glaſs between me and his apparent place ſeen by the naked eye; in which caſe, according to the uſual order of apparent objects on the ground, I ſhall be apt to think him nearer to me than his true place is; becauſe the view of the intermediate ground by my naked eye confines the extent of my imagination in the glaſs. In fig. 89, the man or object is *PQ*, the obſerver's eye is at *O*, the upper direction of the lens at *A*, is *OAC*; and the lower direction of the ſame lens at *B* is *OBD*, terminated by the ground *CD* at *D* on this ſide of the object *PQ*. Fig. 89.

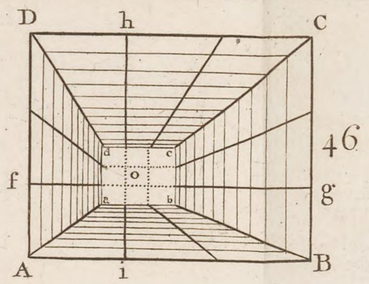
Apparent progreſſive motions ſtopt ſeveral ways.

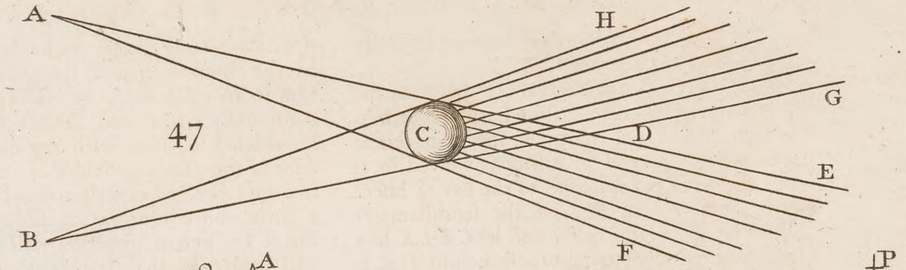
289. In like manner if we conceive the line *CD* to repreſent the ſide of a long building, or of a ſtreet, inſtead of the ground; and the concave lens to be directed to any part of the houſes nearer to the eye than the place of the object *PQ*, it will appear nearer than its true place does. Alſo if this figure be conceived to be inverted, ſo that *CD* may repreſent the cieling of a long gallery or of a cloiſter; the directing of the glaſs towards a nearer part of the cieling than that of the object upon it, will make it ſeem nearer than its true place. Which ſhews that the impediment to the imagination lyes equally in the nearneſs of the parts of thoſe ſurfaces, that are ſeen by the naked eye; and not in their lower or higher ſituation above an horizontal line drawn through the eye. Which reaſon is farther confirmed, by fixing the ſame glaſs in a hole made in a broad thin board; or by looking into the wrong end of a ſmall perſpective, which will diminiſh objects, tho' it be cloſe to the eye, and ſo it will intercept the direct view of all collateral objects from the naked eye, as well as the broad board; and then any diminiſhed object will appear equally remote, whether the glaſs in the board or the perſpective be directed exactly towards it, or below it, or above it, or towards either ſide of it.

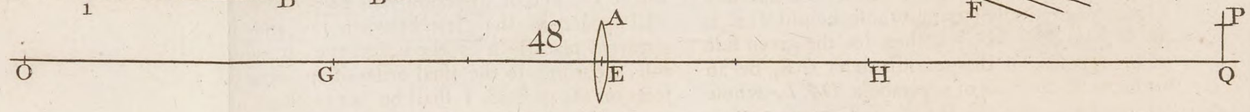
The fields are the propereſt places for theſe experiments. a Art. 113, 114.

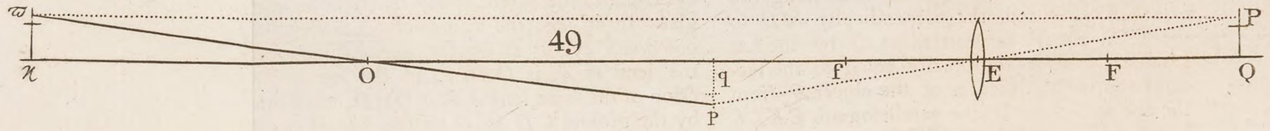
290 Another obſtacle to the power of the imagination, not much different from the former, is the continual diminution of the quantity or number of objects, or of the parts of an object, ſeen through a concave lens, in drawing it farther and farther from the eye [a]; it being very difficult to conceive one part of a known object or ſyſtem of objects to recede from the reſt, that appear to the naked eye to be fixt in their places. The ſame may be ſaid of the approach of objects

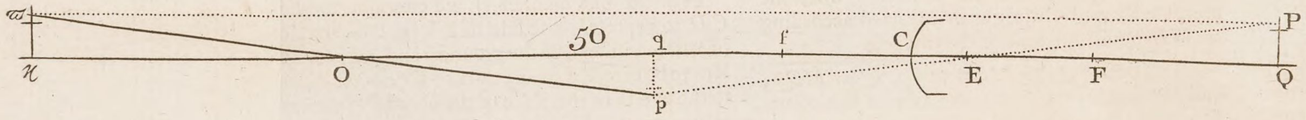
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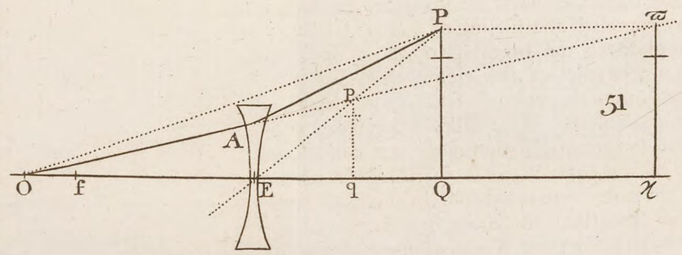

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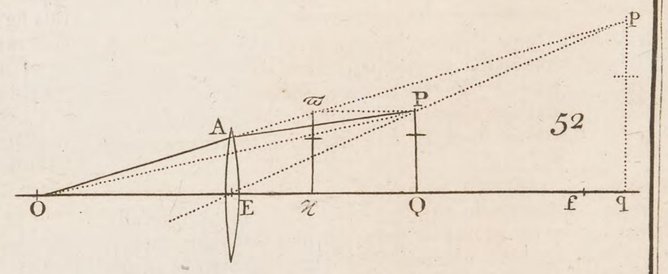

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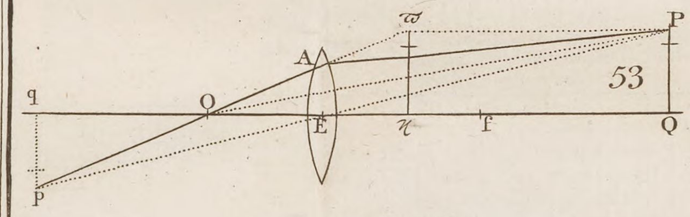

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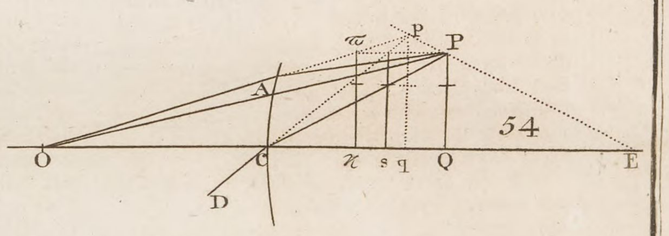

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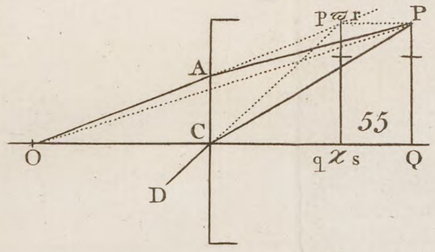

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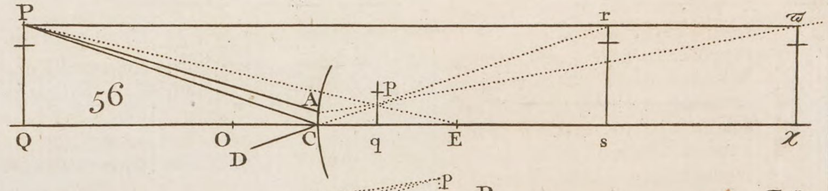

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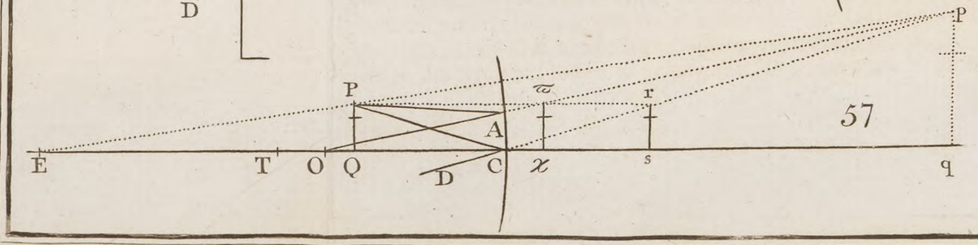

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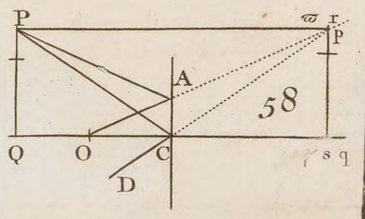

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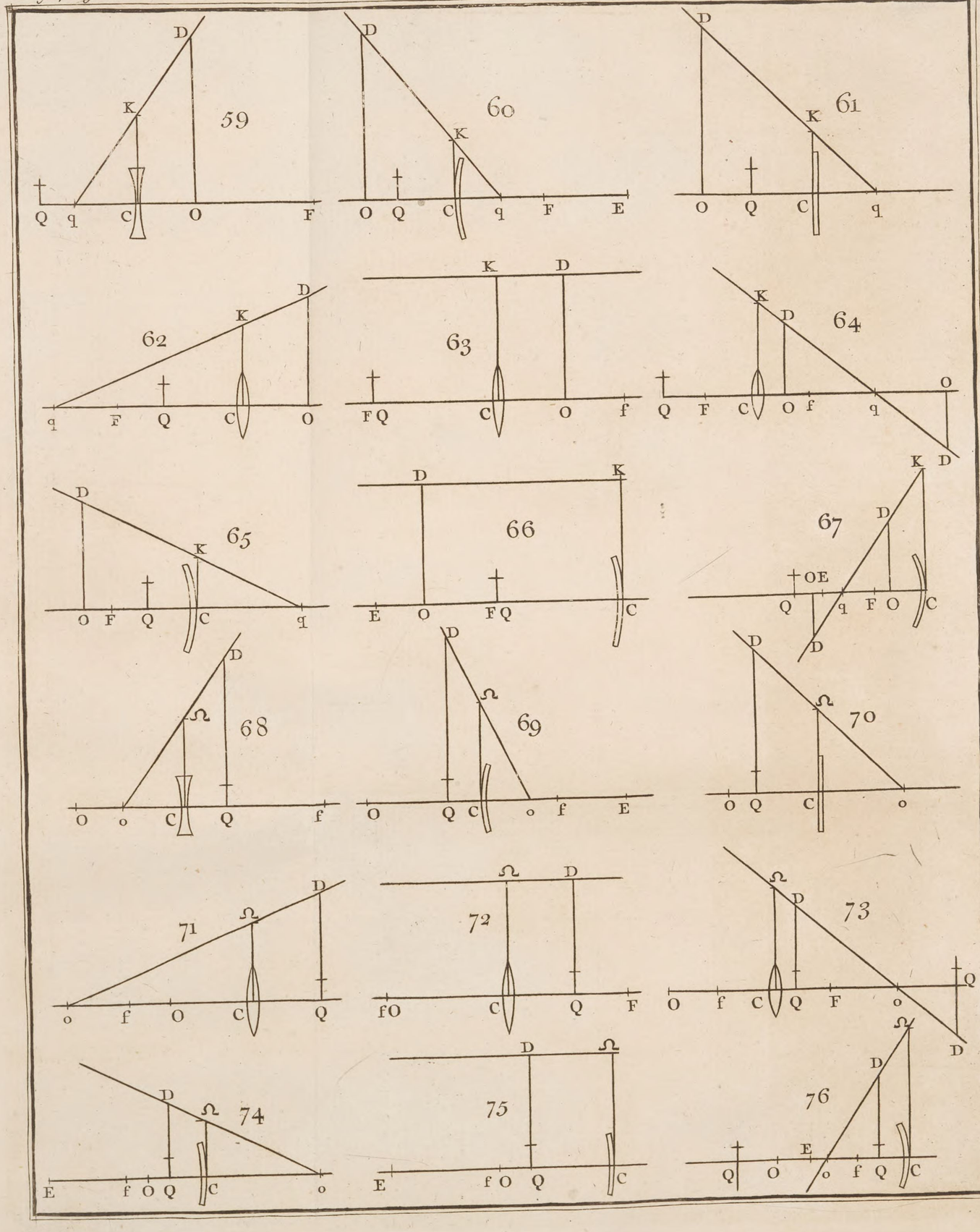

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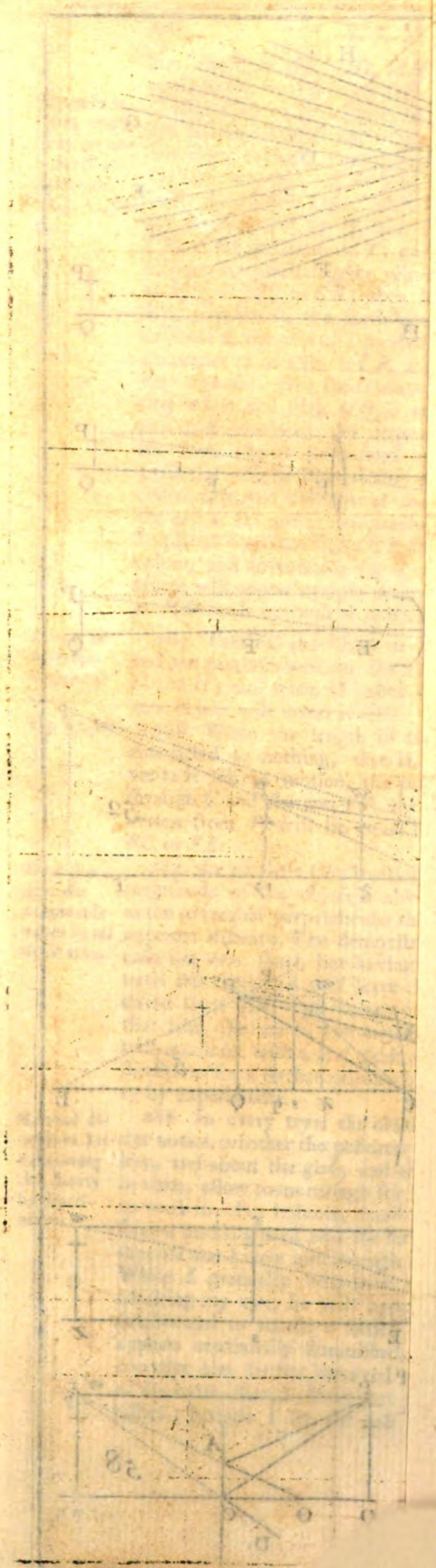

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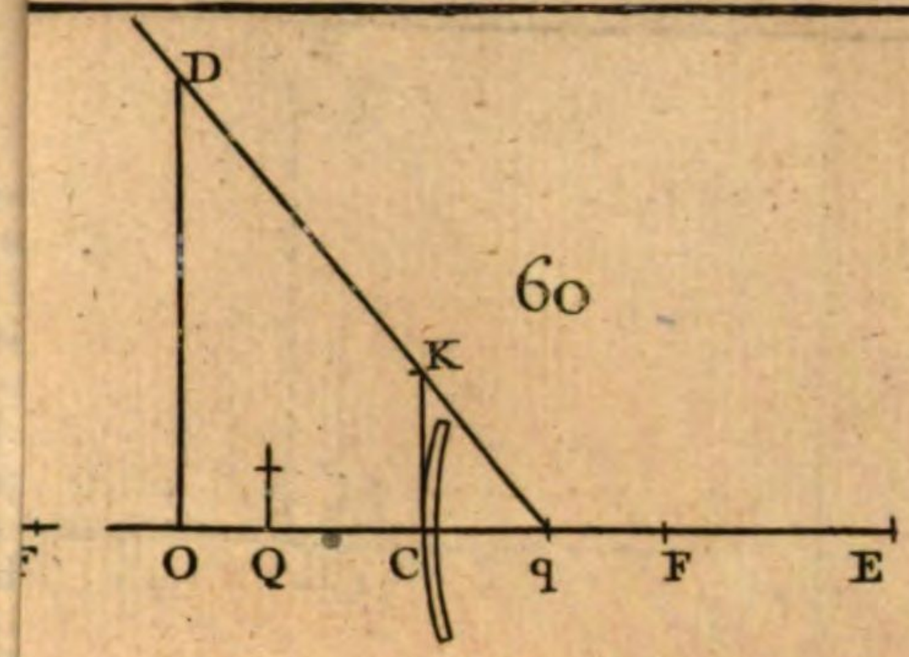

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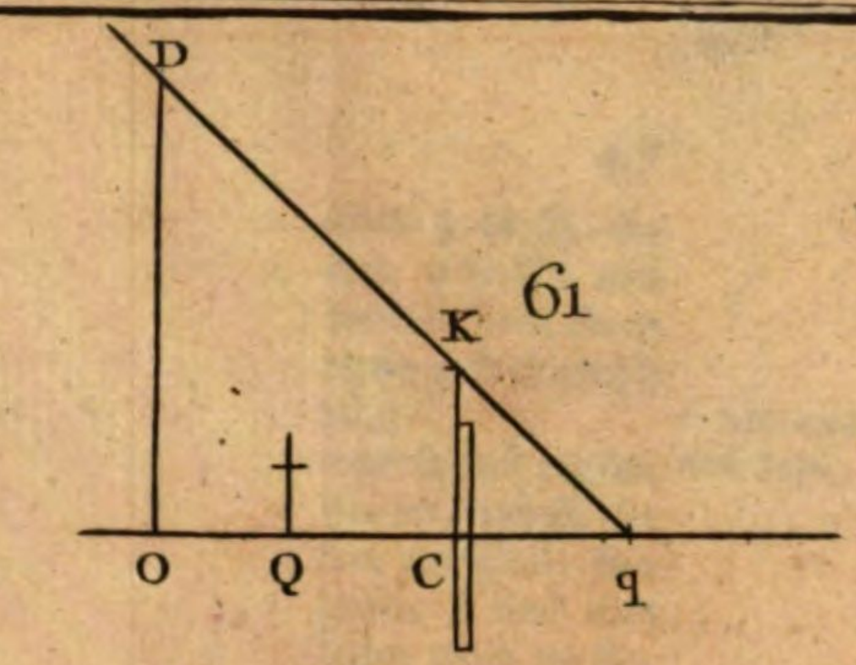

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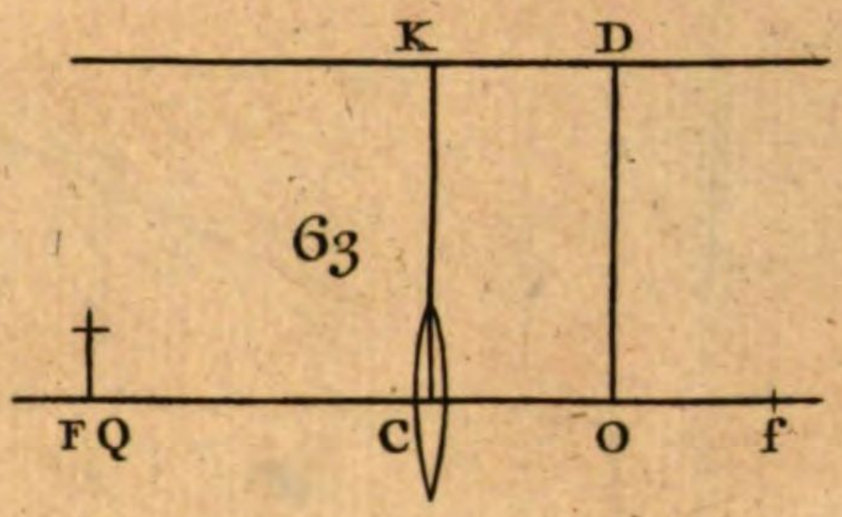




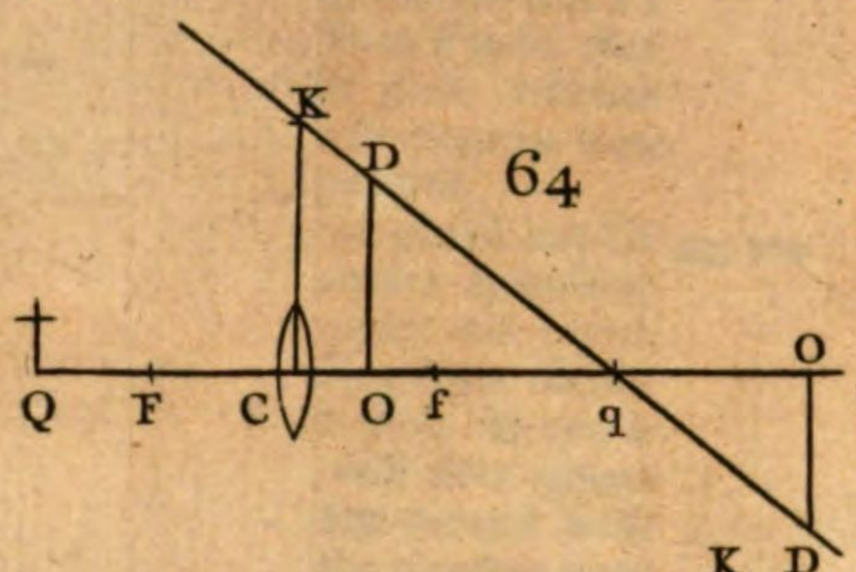
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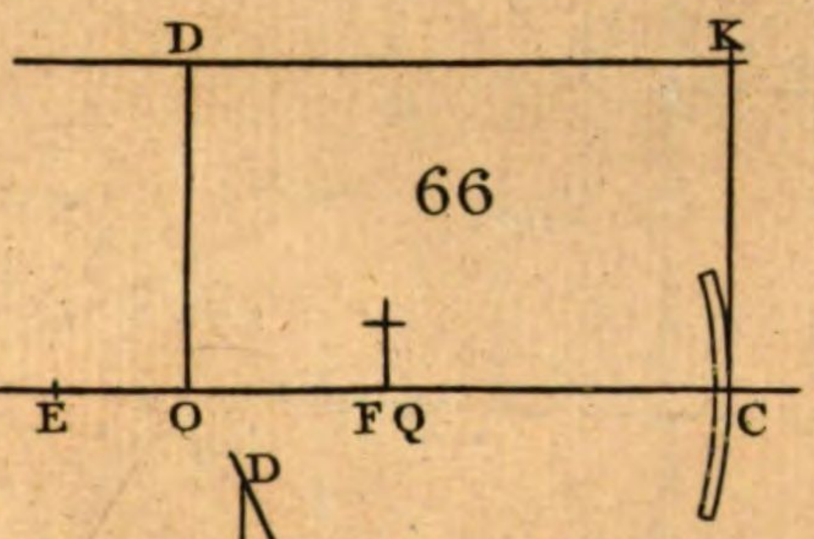
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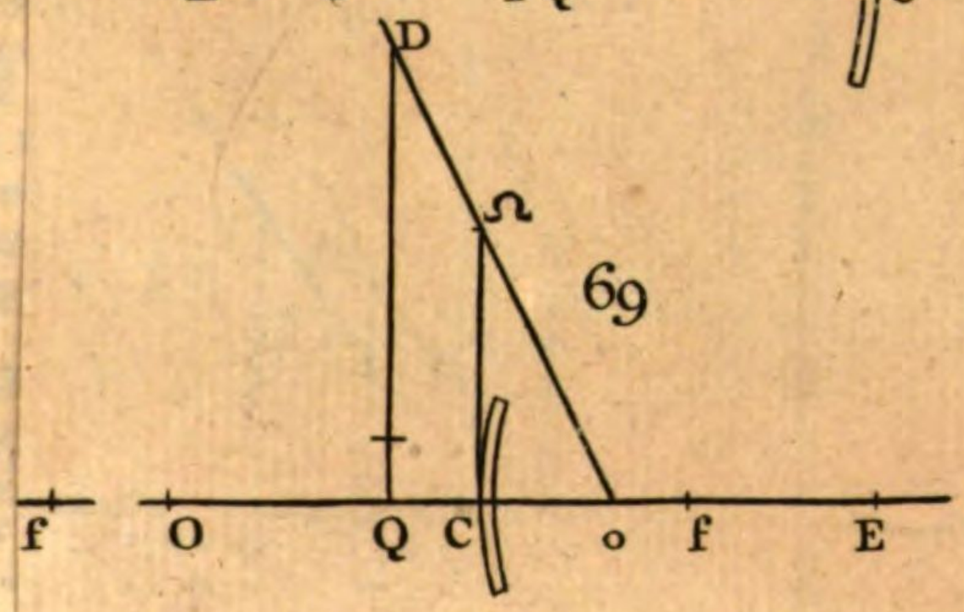
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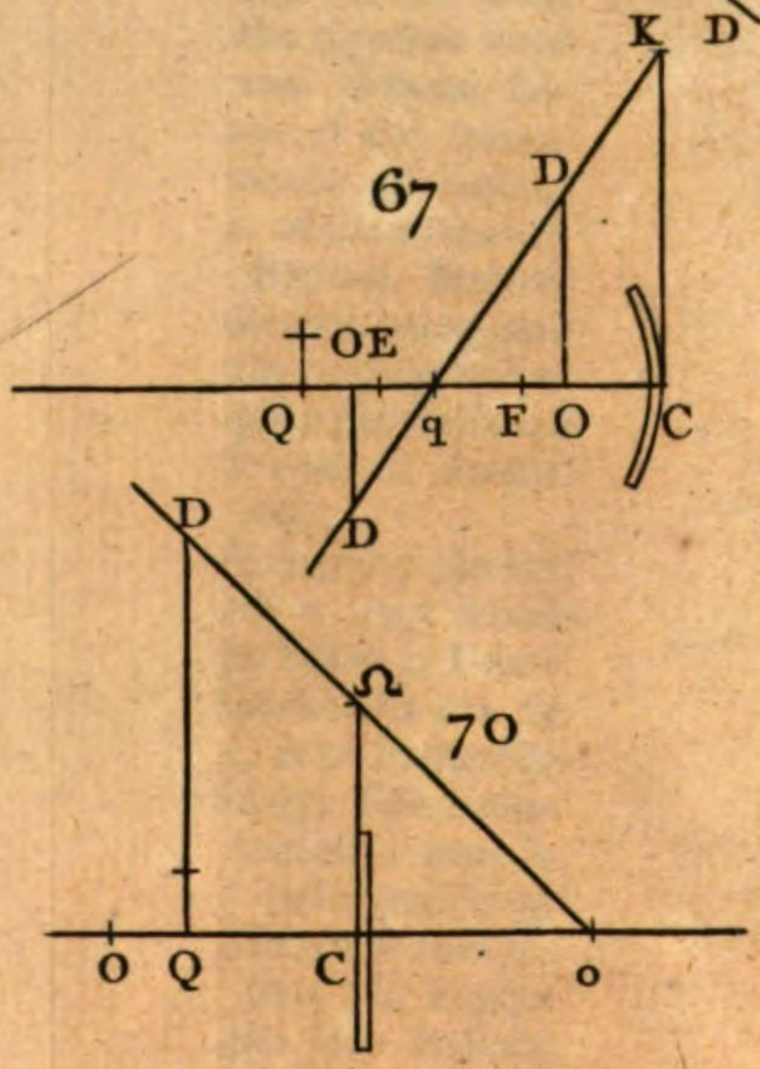
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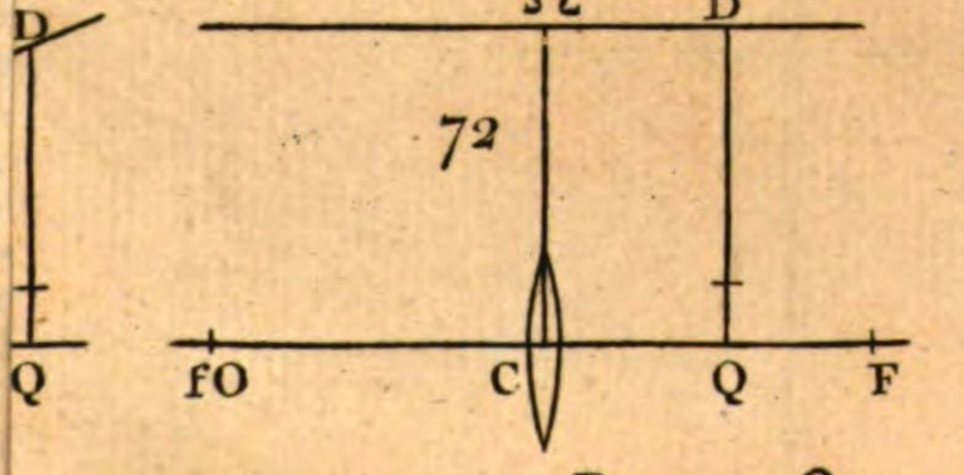


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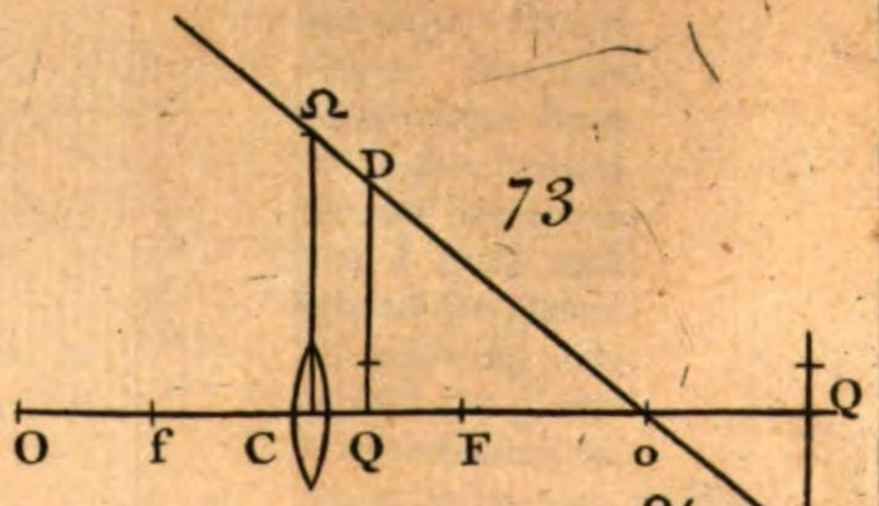


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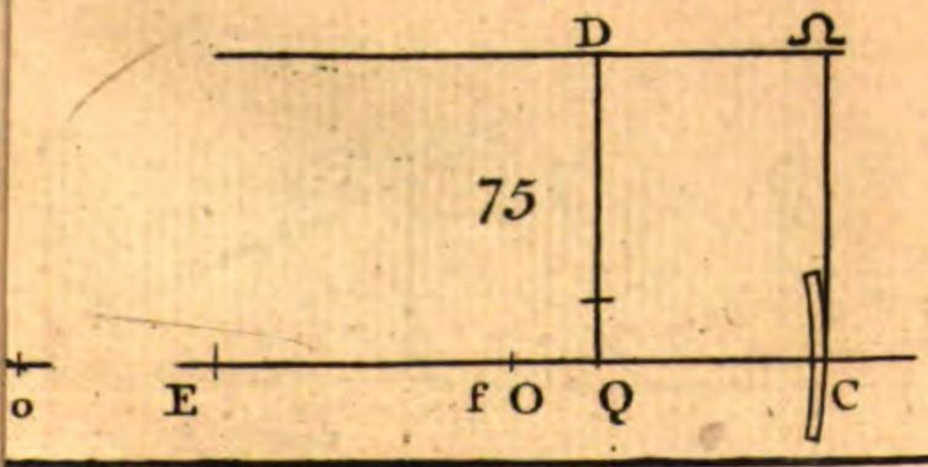
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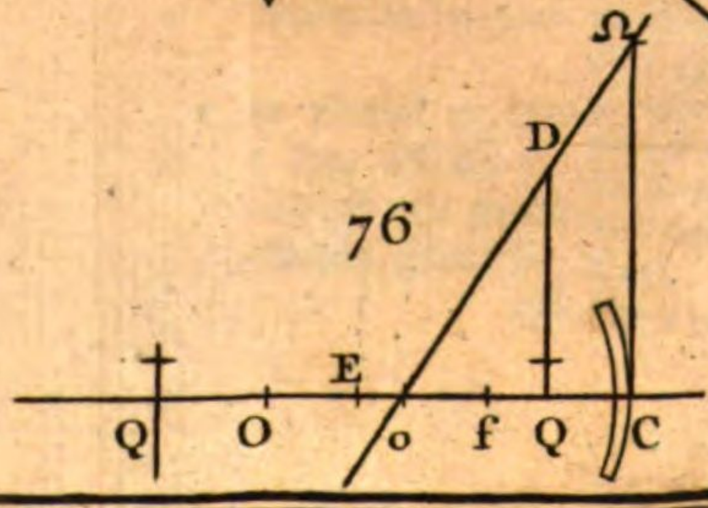
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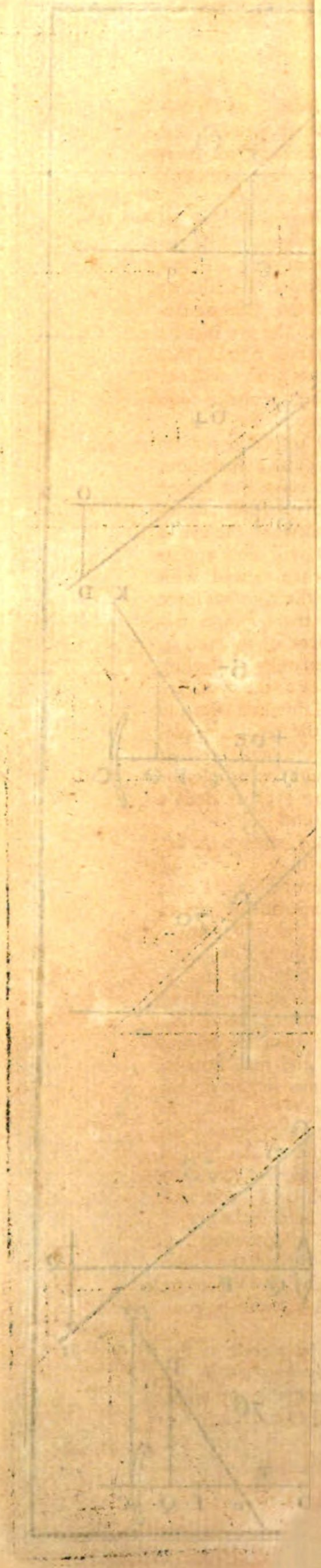
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ſeen through a convex lens drawn from the eye. And therefore upon ſeveral accounts, theſe experiments will ſucceed the beſt in the open fields, where the objects being remoter and ſubtending ſmaller angles at the eye, are more compleatly comprehended within the compaſs of the glaſs; where the imagination has more room to work in; and where the mind being leſs prepoſſeſſed with a knowledge of the diſtances and ſituations of objects than in a room, is more at liberty to perceive the difference between the ideas ſuggeſted by the glaſſes and by the naked eye.

291. I obſerved in Art. 144, that an object ſeen by refractions through any convex lens or by reflections from a concave ſpeculum, ought to appear behind it, or at it, or before it, according as the real diameter of the object is bigger, equal to or leſs than the diameter of the part of the glaſs in which it appears. Hence it follows, that the object PQR placed at any diſtance CQ from the glaſs AC, will appear at the glaſs, when the eye is placed at its principal focus F*; and, that if the object be placed at a greater diſtance from the glaſs, than twice the focal diſtance CF or Cf, it will appear again at the glaſs when the eye is brought to another point O; to be found by biſecting the diſtance CQ in o, and by ſuppoſing the point o to be the focus of incident rays, and by finding their conjugate focus O after refractions or reflections. For then conceiving any ray OA to flow back from the eye at O, and, after its reflection or refraction at any point A and its paſſage through o, to fall upon the object PQR at R; the point R will be ſeen by a ray[a] returning back in the ſame lines RA, AO. And ſince oQ was made equal to OC, the leg QR of the right angled triangle oQR will be equal to the leg AC of the equal right angled triangle oCA; and therefore the part QR of the object PQR will appear in the ſame place by reflections or refractions, as if it were put into the place AC, and were viewed there by the naked eye at the ſame point O*.

292. Hence it is plain, if CQ be equal to $2Cf$, that the point o will coincide with the principal focus f; and ſo the conjugate focus O will be removed to an infinite diſtance; and if CQ be leſs than $2Cf$, and conſequently Co be leſs than Cf, the point O will fall behind the glaſs, where the eye cannot come: and laſtly if the object Q be very remote, the point O will come very near to F.

293. Now let q be the place of the image of the object PQR, to be found by Art. 236; then while the eye is receding from C to F the apparent object ſhould follow it, till it touches the glaſs; and while the eye is receding from F to q, the apparent object ſhould come through the glaſs, ſtill following the eye till it touches it when they both come to q; and while the eye is receding farther back from q to O, the apparent object ſhould return from q to C; and laſtly while the eye is drawn farther back from O, the apparent object ſhould again paſs through the glaſs and go forwards behind it[c]. c Art. 45 and 139.

294. This ſhould be the progreſs and regreſs of the apparent object by the theory, while the eye is continually moving from the glaſs. But by experiments with convex lenſes, I find that it comes forwards from behind the glaſs no farther than till it touches it, when the eye is at F; and while the eye is receding from F to O, that it ſticks all the while at the glaſs, without coming through it, and begins to return as ſoon as the eye has paſſed over the point O.

295. But when the object is placed not far from the center of a large concave ſpeculum, though it does not appear to come out before the glaſs, while the eye is moving from the focus f towards the image q, yet when the eye is got paſt the image, the object will often appear before the glaſs; eſpecially when viewed with both eyes. Which ſhews that the vigorous force of the united ſenſations of the two pictures falling upon correſponding places of the retina, together perhaps with the ſenſation of feeling the mutual turnings of the axes of the eyes, are helps to our apprehenſion of diſtance. Becauſe we have not theſe helps when the eye, being placed between f and q, ſees the object either faintly though diſtinctly through a pin-hole, or elſe confuſedly with only one eye, or double with both eyes and confuſedly too. Fig. 92.

296. Whether one might not contrive the experiment ſo advantagiouſly with a very broad lens as to make the object appear before it I dare not pronounce. But ſome experiments are ſo oppoſite to common experience, as to be hardly reconciled to this part of the theory. For example, when I place my fingers behind a round-bellyed decanter full of water, either touching its back ſurface or at a ſmall diſtance behind it, they will appear much larger than the fingers of my other hand that touch the fore ſurface, and therefore by theory the former fingers ſhould appear nearer to me than the latter. But this would be repugnant to conſtant experience both of ſeeing and feeling; ſince the fingers behind the decanter do not appear alone but joined to my hand and arm, part of which I know and perceive by direct viſion to be behind the globe. Now whether the prejudice of knowing the place of any other object to be behind the globe or the convex lens, be not an impediment to its appearance before it, I muſt leave to be conſidered.

Obſervations upon an object placed in the focus of a convex lens.

297. If the object PQR be placed at the principal focus f of any convex lens ACB, its apparent diſtance Ox, from the eye at any point O, ſhould be continually equal to the given focal Fig. 93, 94.

cal distance of the lens. This has been shewn before, but for the sake of the following application, I will prove it again. Let E be the center of the lens, and since the angle AOB, or $\varpi O \varrho$, under which the object PR appears, is always constituted under a couple of successive rays OA, OB parallel to the axes PE, QE of their respective pencils; it is continually equal to the given angle PER under the said axes. And since the apparent object $\varpi \varkappa \varrho$ is always equal to the real object PQR, it follows that the moveable triangle $\varpi O \varrho$ is continually equal to the fixt triangle PER; and consequently, that the apparent distance $O\varkappa$ should be continually equal to the focal distance Ef or EF. This agrees perfectly with experiments while the eye is receding from the lens as far as its focus F, but no farther; that is, the object appears to follow the eye till it touches the back side of the lens, and then it sticks there, though the eye recedes never so far back from F.

When it appears to stick at the lens it seems to increase.

298. In this experiment I have observed farther; that while the eye is moving any where between the lens and its focus F, as the object appears to accompany it always at the same distance $O\varkappa$, constantly equal to EQ; so it appears constantly of the same magnitude. But while the eye is receding from F still farther back, and the apparent object seems to stick at the glass, it seems also to increase in magnitude; till the least circular spot upon it, whose diameter we may suppose to be PR, will appear to fill the whole aperture, whose diameter is AB, when EO, the eye's distance from the lens, is to the focal distance EQ, as the said AB is to PR; as I have found by accurate measures: which shews that the object was placed exactly in the focus f, or that the rays OA, OB were respectively parallel to the axes EP, EQ, of their pencils.

A surprising circumstance.

299. But the most surprising circumstance of the experiment is this, *viz.* that though the object be a piece of printed or written paper whose letters are but just legible at the distance EQ, either through the lens, or with the naked eye; yet they may be read through the lens with the same ease from any distances how great soever. But if the letters be rather too small to be read distinctly when the eye is close to the lens, or any where between it and its focus F; though they seem afterwards to increase prodigiously while the eye is receding from F, yet the difficulty in reading them still continues the very same as before; notwithstanding that they do not only appear magnified but also very distinct, the rays in each pencil being parallel as was proved by the experiment.

300. Now though these appearances seem odd at first sight, yet they follow naturally from the apparent sticking of the object at the lens, and from the definition of apparent magnitude. For since the subtense AB of the given visual angle AOB increases as its distance ΘC does, and since we refer the apparent object to the place of the subtense AB, we must needs conceive its real magnitude to increase. But since the picture of the letters upon the retina continues invariably the same wherever the eye is placed, by reason of the given visual angle AOB, whose given parts are also invariable; it must needs suggest to the mind the like invariable perception of the least sensible parts of the letters, to the eye in all places.

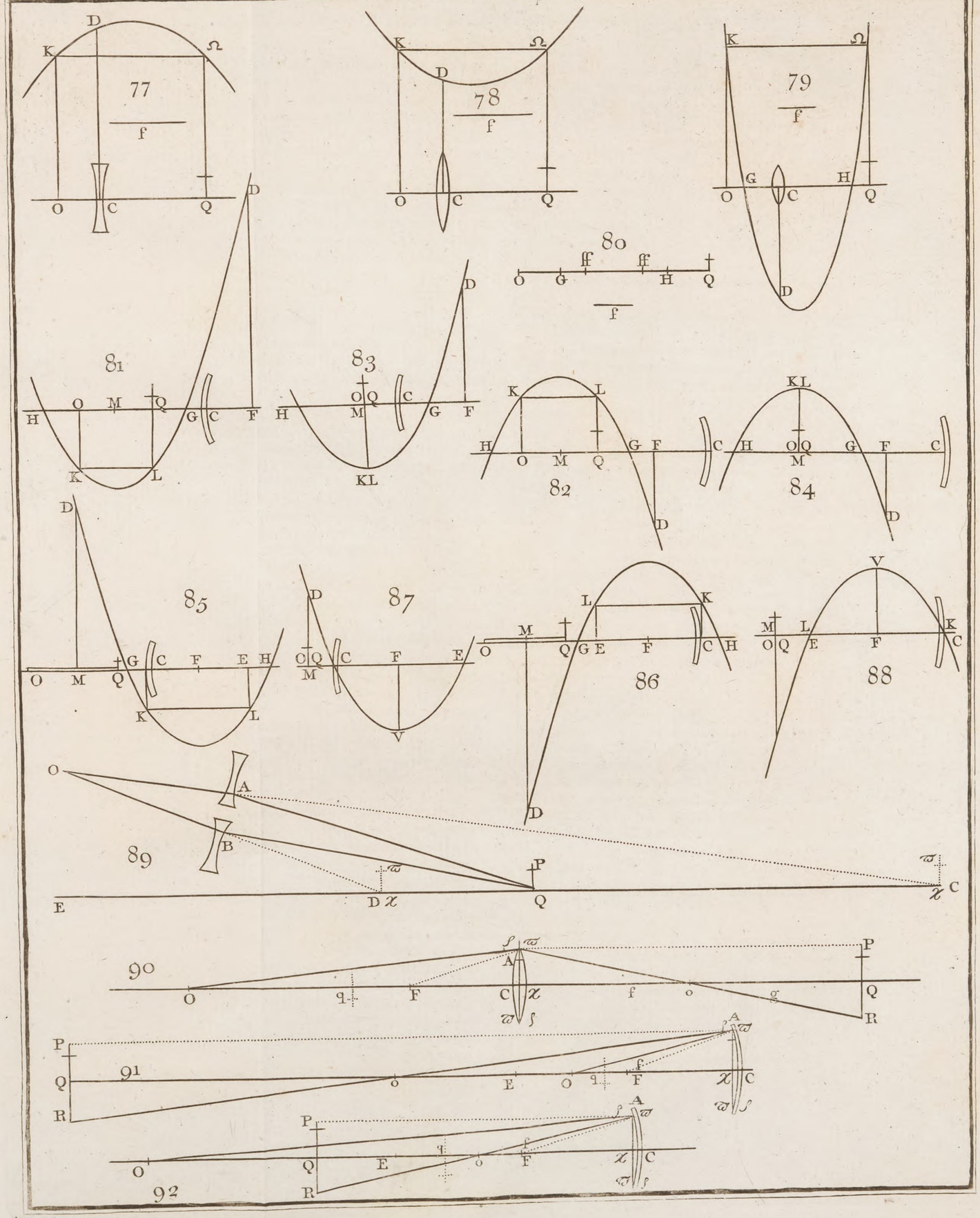
Hence sorts of parent nitude.

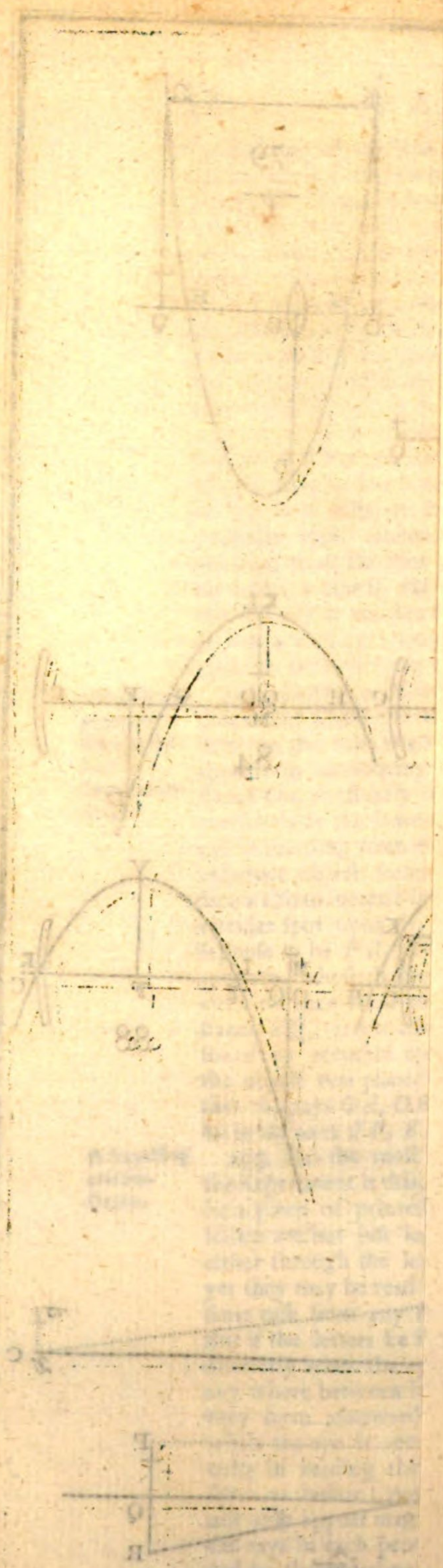
301. It follows then from this experiment and from another that I am going to mention, that apparent magnitude should be distinguished into two sorts: the one, being proportionable to the magnitude of the visual angle or of the picture upon the retina, and consequently to the number and variety of the least sensible parts perceivable in an object, is unalterable either by the power of the imagination, or by any circumstances whatever that do not affect this picture. But the other, we have seen, is varied by the imagination in proportion to the distance at which we conceive the object to appear, whose picture upon the retina is invariable. For when the eye was at O and CO was double of CF, we imagined the real diameter of the object PQR, to be in the place AB, and to be double of what we thought it to be at the same place when the eye was at F.

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302. I have dwelt the longer upon this experiment, in order thereby to illustrate and confirm the solution that I have given in Art. 164. &c. of that famous and perplexing phænomenon of the various apparent magnitudes of the sun, moon, and constellations, at various heights above the horizon. Let the object PQR, now intended to represent the moon, be a wafer stuck upon a card or white paper fixt upon a piece of a board; or it may be any round distinct spot, whose diameter is at least three or four times less than the diameter of the aperture of the lens AB. Then let this object and the lensbe both fixt, upon a longer board MN, at an interval from each other exactly equal to the focal distance of the lens, to be found by Art. 63 or otherwise. And when the board MN is firmly fixt upon a level with the eye of the observer, let him recede for the space of four or five focal distances from the lens, till the wafer, seen just behind it, seems to fill its aperture or near it. Then let him imagine this apparent magnitude and distance of the wafer from his eye, to represent the apparent magnitude and distance of the horizontal moon; and while he is returning gradually towards the lens, the apparent magnitude of the wafer will seem to decrease in proportion to the decrease of its apparent distance from his eye; in like manner as the apparent magnitude of the moon seems also

Fig. 93.


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alſo to decreaſe in proportion to the decreaſe of her apparent diſtance, while ſhe is aſcending higher and higher, through ſeveral parts of the apparent concavity of the ſky; which parts appear ſucceſſively nearer and nearer to the eye while it is directed higher and higher above the horizon, as I have ſhewn in Art. 162, 163.

The two appearances compared.

303. Beſides, as the moon either high or low, appears always at the ſame diſtance from us as thoſe clouds or parts of the ſky do that lye about her, ſo the wafer appears always at the ſame diſtance from the eye as the lens does with the frame about it, ſo long as the eye keeps farther from it than its principal focus. Theſe two phænomena agree alſo in the only circumſtance that creates the difficulty in ſolving the moon's appearances; namely, that each of the viſual angles, under which the wafer and the moon do ſeverally appear, continue invariable; not by means of any refractions of the moon's rays, in paſſing through that imaginary concave ſurface of the ſky, anſwering to the refractions of the other rays in paſſing through the lens, but becauſe the moon's real diſtance varies ſo little, during her aſcent or deſcent, as to cauſe no ſenſible variation of apparent magnitude, in caſe her apparent diſtance did not vary. Nor can I find any one circumſtance immediately perceiveable by ſight, in which theſe two phænomena do not agree; in as much as the refractions of the rays at the lens do no more affect the eye, than if they had come ſtraight through a plane glaſs from an increaſing or decreaſing wafer placed juſt behind it. But if any one ſhould doubt whether the looking higher and higher at the moon, and conſtantly horizontal at the wafer, be a diverſity of circumſtances that can cauſe any difference in theſe appearances; he may eaſily remove this ſcruple by gradually elevating the far end of the board MN with the lens and wafer fixt to it, and by gradually moving his eye towards the lens at the ſame time. I muſt confeſs I have not tryed the experiment in this manner, not having the leaſt ſuſpicion that any diverſity in the appearances can ariſe from this circumſtance; but I find it ſucceeds very accurately with a broad 5 foot-object-glaſs of a teleſcope, while the eye is moving horizontally; and I think it will anſwer tolerably well with a glaſs of a ſhorter focal diſtance, or even with a common ſpectacle glaſs. It will alſo ſucceed well enough when the object is placed in the principal focus of a concave ſpeculum, inſtead of a convex lens.

Fig. 94.

Other remarks on the theory of apparent diſtance. Fig. 76.

304. But to return to my remarks upon ſome other caſes in this theory. When the interval beween my eye and a large concave ſpeculum was given, and was greater than its radius; the apparent diſtance of any ſmall bright object, firſt put cloſe to the ſpeculum, and then gradually drawn from it, ſeemed to decreaſe to nothing and then to increaſe, after the apparent inverſion; according to the theory. But then I took care ſo to withdraw the object, that it always appeared to a ſingle eye over againſt the ſame part of the ſpeculum; otherwiſe when it was ſeen obliquely, and ſucceſſively over againſt ſeveral parts, it ſeemed to move from me behind the ſpeculum. Becauſe the place of its image, from which the rays diverge upon the eye, at firſt recedes from behind the ſpeculum; and thereby affects the eye by rays flowing obliquely and ſucceſſively in different directions from different parts of the ſpeculum. And when both eyes are open, one of them at leaſt muſt be affected in this manner by oblique rays in different directions; and beſides when the two images fall on correſponding places of the retina, the appearance muſt be ſtronger and remoter than to a ſingle eye according to the experiment in Art. 977. The object here uſed was a ſmall lighted candle fixt to the end of a long ſtraight ſtaff, which I drew back by the ſide of my eye, having ſometimes a pin-hole before it.

305. Again, in viewing my own eye or face, in a large reflecting concave, through a pin-hole; while I recede from touching it to its principal focus, the apparent magnitude of my face decreaſed and its apparent diſtance increaſed; and the contrary happened in going ſtill back from the focus to the center, according to the theory. But without the pin-hole, the decreaſe of apparent magnitude was ſcarce ſenſible in going from the glaſs to its focus. Fig. 88.

306. As to the neceſſity of theſe and the like reſtrictions, I need only obſerve in general, that without the pin-hole our judgments of apparent magnitude are not ſo certain and determinate as with it, the appearances without it being frequently too much confuſed; and that our judgments of it are alſo more uncertain with both eyes than with one in many caſes. Becauſe in broad glaſſes we frequently ſee two appearances, which ſometimes recede from, and ſometimetimes accede towards each other, with tranſverſely oppoſite motions; and when all things are fixt at reſt and theſe appearances are but partially united, (as if they lapped over each other) we ſometimes unwarily take them for a ſingle appearance, much larger than it would be to a ſingle eye; and ſometimes the common middle part of both is miſtaken for the whole, by not attending to the fainter and weaker extremities on each ſide of the middle; and then this middle part appears to be ſmaller than the whole would do to a ſingle eye. Thus your face put near to a large concave ſpeculum appears laterally larger to both eyes than to one; and being put near to a common looking-glaſs it appears laterally narrower. Alſo the wriſt of your hand,

held upright and close to your forehead, appears much thinner to both eyes than to one, or even to both eyes viewing it somewhat farther off. And the near approach of a broad image formed by a concave speculum may cause the like appearance. I mention these things as some general hints, by which the disagreement of the theory with experiments in a few particular cases may possibly be explained; and I leave them to be farther considered by those that have leisure and inclination.

307. Very remote objects, that appear contiguous and at equal distances from the naked eye, do generally appear contiguous and equidistant through glasses, though their real distances from the eye be never so unequal. The truth of this proposition is known to those that with a telescope have observed the appulses of the moon to the planets and fixt stars; and may be examined every day by viewing very remote mountains, towers or buildings extant one above another. For all objects so remote that their pictures are projected very nearly upon the same plane at the focus of a telescope, are equally magnified by it, that is, in the same given ratio [a]. And consequently whatever real distance is assumed to represent the same apparent distance of these objects from the naked eye, will be diminished by the telescope in the same given ratio in which the objects are equally magnified. For further illustration, let KL and MN be any two objects that appear to the naked eye contiguous to each other at one and the same apparent distance, represented by any given line OQ; and let certain parts KL, MN of these two objects appear under the two angles KOL, MON severally equal to any given angle POQ; and in the telescope let them appear under two other angles severally equal to any other given angle $\varpi O \varkappa$; then their common apparent distance in the telescope, suppose $O\varkappa$, will be to OQ, in the given ratio of the angle POQ to the angle $\varpi O \varkappa$, by Art. 141.

a Art. 120.

Fig. 95.

General conclusion.

308. Hence it is manifest that the apparent distance of an object seen in glasses, cannot be otherwise determined, than by assuming some certain measure of its apparent distance from the naked eye. But since we usually judge the apparent distances of known and familiar objects to be proportionable to their real distances; because we find they increase and decrease together to a degree of accuracy sufficient for common purposes; it follows, in these cases if we represent the apparent distance of an object from the naked eye by its real distance, that the apparent distance in the glasses will answer the measure of it determined by theory, as exactly as in vision with the naked eye. In other cases where we know there is a great disparity between the real and apparent distances of an object, we must assume a certain quantity of real distance for the measure of its apparent distance from the naked eye, and proceed with it in glasses as above: which is all the accuracy that can be expected.

Upon ART. 155.

Appearances through a drinking-glass full of liquor.

309. If this theory be examined, when the eye is placed within the caustick vpx, that is, between its cusp p and the center of the sphere or concave speculum; it will be necessary to apply a small pin-hole close to the eye, to procure distinct vision. And if it be examined by looking at a lighted candle or at any small, bright object, seen through a drinking-glass full of any clear liquor, the object will appear to the eye, at any place whatever, always upright properly speaking; and inverted only with respect to the right hand and left when the cusps p, q are before the eye. Because the refractions upwards, of the rays passing through the drinking-glass, follow the same laws as if they had passed through the edge of a prism held downwards [b]; (to which the conical figure of the glass may be compared;) but the refractions sideways follow the laws of refractions through a sphere; which last is manifest by the apparent contrary motions of the eye or the glass and of the object seen through it, though it appears upright.

b Art. 54

Theory of apparent shapes postponed.

310. The chasm in the numeration of the figures between FIG. 253 and 266, was occasioned by reprinting this article and by changing the 19th plate for another. For at first I attempted in this place to explain the apparent shape and curvity of objects seen by reflections and refractions; but finding it could not be done with sufficient accuracy, without more geometry than what I proposed to use in the Popular Treatise, I chose to consider it more at large in the Remarks upon Art. 612. &c.

Upon ART. 160.

311. Since the printing of this article I was informed by the Honourable Mr. *North* that this apparent divergency of the rows of trees planted upon the rising ground at the end of his vista, is very manifest through a large perspective as well as to the naked eye.

Dr. Jurin's remarks upon fallacies in vision, &c.

312. "In shaving my self by a glass hung in a window, which looks into a garden, it has several times happened to me, that a fly has settled upon the window near my glass, and in creeping along has raised in me the idea of a large bird flying in the air, and sometimes that of a dog, or cat, running cross the garden, according as the situation of the fly, with regard to my eye, has projected its image either upon the air, or upon the ground. The reason of this appearance

appearance I take to be this. My eyes were then fixt upon my own image reflected from the glaſs, and were ſo adapted, as to ſee that image diſtinctly in a place as much beyond the glaſs, as my face was then diſtant from it; and the fly being much nearer me than that image, and likewiſe being ſeen obliquely, muſt conſequently appear very indiſtinct, as well as larger than it would have done, had my eyes been fixt upon it. And as I could not apprehend, that any creature anſwering to ſo large an image, could be ſo near me, immediately my fancy ſuggeſted to me that it was at a greater diſtance, and a much larger animal, and moving with a greater velocity. So true it is, that in the ſuddeneſt judgments we make, the idea of a greater diſtance raiſes in us the idea of an object of greater magnitude, and moving with a greater velocity; the apparent ſwiftneſs of its motion being magnified in the ſame proportion with the diameter of its bulk.

313. It has likewiſe happened to me, that upon uſing a ſort of fence hung upon my head, to ſhade my eyes from the light of the candles in reading, a fly has ſettled upon the inſide of that fence, within three or four inches of my eyes, and by creeping along the edge of it, has ſuddenly raiſed in me the appearance of a rat running along the floor. Which I account for in the ſame manner as in the other phænomenon; the book, on which my eyes were fixt, being conſiderably farther diſtant from them, than the edge of that fence; and conſequently the picture of the fly upon the retina being larger and leſs diſtinct, and its apparent velocity greater than it would otherwiſe have been.

314. Children of three or four years of age, not having as yet learned how to allow for the leſſer apparent magnitude of diſtant objects, take every thing ſeen at a diſtance to be leſs than it really is. If they ſee a man or a woman thirty or forty yards off, they take the one for a boy and the other for a girl. And though experience teaches them in a little time to correct this error in objects ſeen upon the level, or from a moderate height, yet if they happen to look upon them from any high building, they are apt to make the ſame miſtakes as before.

315. Let a boy, who has never been upon any high building, go up to the top of the Monument, and look down into the ſtreet; the objects ſeen there, as men and horſes, will appear ſo ſmall as greatly to ſurprize him. But ten or twenty years after, if in the mean time he has uſed himſelf now and then to look down from that and other great heights, he will no longer find the ſame objects to appear ſo ſmall. Becauſe now, having had frequent experience of the leſſening of the apparent magnitude by the increaſe of diſtance, he does by a quick and imperceptible judgment, conceive the objects to be of the ſame magnitude as if they were leſs remote: which judgment of the mind not being diſtinguiſhed from the perception of the eye, he thinks he actually ſees them larger than before. And if he were to view the ſame objects from ſuch heights, as frequently as he ſees them upon the ſame level with himſelf in the ſtreets, I ſuppoſe, they would appear to him juſt of the ſame magnitude from the top of the Monument, as they do from a window one ſtory high. For, in the ſtreet, a man at a hundred yards diſtance appears of the ſame height as another at ten, though the image of the former in the eye is but a tenth part of the length of the other; the ſmalneſs of the image being compenſated by the knowledge of the greatneſs of the diſtance. And this allowance is inſtantaneouſly and imperceptibly made in ſuch objects as are conſtantly and perpetually ſeen in any one ſituation.

316. But the famous Monſieur *Perrault* * ſeems to be greatly miſtaken, when he applies this principle to the overthrowing the received opinion of all architects and ſculptors ſince the time of *Phidias*; viz. that ſtatues placed upon very high buildings ought to be made of a larger ſize, and to be heightened with bolder ſtrokes than thoſe which are to be ſeen at a nearer diſtance. He ſays the judgment of the eye is ſo perfect, and ſo accuſtomed to make the proper allowance for the diſtance of the object, that it appears of the ſame magnitude to us, whether at a greater, or a leſſer diſtance. In things very familiar to us he is certainly right: but ſtatues upon very high buildings are not ſo conſtantly and familiarly ſeen as men and horſes in the ſtreets; and therefore we cannot make ſo juſt an allowance for the diſtance in one caſe, as in the other. I believe very few ſpectators are able to make a near eſtimate of the length of the dragon upon *Bow* ſteeple, or of the height of the St. *Michael* at *Bruſſels*, except ſuch as ſaw thoſe figures before they were put up. Beſides that all perſons, except architects, are apt to take the height of a lofty building to be be much leſs than it really is. Therefore I ſee no reaſon for that learned gentleman's ridiculing the celebrated ſtory of the *Minerva* wrought by *Phidias*, which is ſaid to have appeared exceedingly deformed, when ſeen near at hand in his work-houſe, but incomparably beautiful when placed at the height it was deſigned for. Something like this may be ſeen in pictures every day; many of which appear hard and courſe near hand, but perfectly ſoft and natural when viewed at a proper diſtance. The paintings in the cupola of St. *Paul*'s, if they were to be ſeen from the diſtance of a few feet, would have a much leſs agreeable effect than

* Ordonn. des 5 col. d'Architect. P. II. Ch. 7.

when they are viewed from the pavement.' So far Dr. *Jurin*.

Fallacy in the view of *Stilton* Church.

317. Travelling northwards in the great road to *Stilton* in *Huntingtonſhire*, when I came within the diſtance of a mile or two from it, I obſerved by chance that the inns in the road and the church, on the left hand of them, appeared ſo far aſunder, that I remember I aſked my ſervant whether the church before us belonged to *Stilton* or to ſome other town, which he could not anſwer. We then judged them to be near a mile aſunder. But ſoon after, as I came pretty near to the inns, happening again to look at the church, I was greatly ſurpriſed to find it ſo near them. This raiſed my curioſity to know the real diſtance between them; and upon pacing it over, I judged it to be not above 200 yards, or near an eighth part of a mile. About a month after, when I came back through the ſame village, I found the ſame deception in that apparent diſtance as ſtrong upon me as before, notwithſtanding that now I knew what it really was. I was therefore the more curious to obſerve the ſituation of the village and of the fields about it, in order to find out the cauſe of this fallacy.

318. The open field through which the road paſſes on the ſouth ſide of the town, is almoſt level for a mile or two; and from this diſtance there appears to be a gentle riſing of the fields on each ſide of the town for about half a mile or a mile; and the town appears in the horizon (nothing beyond it being viſible) though ſomewhat lower and remoter than the parts of the horizon adjoining to each ſide of it. And this I take to be the cauſe of the deception. For when any object, or interval between two objects, is judged to be farther off, than according to our common conceptions, we judge it alſo to be larger than we ſhould do if we conceived it to be nearer.

319. At the diſtance of half a mile from the north ſide of the town, the road begins to aſcend northwards; and from the bottom of this aſcent the ſaid interval between the inns and the church appears alſo very large: the tops of the inns and of the ſteeple being ſeen in the horizon beyond a long ridge of corn-lands riſing gently from the right hand of the road. And from this lower ſtation the ſaid interval appears much greater than from the top of the aſcent above it, though but little farther from the town. Becauſe from this higher ſtation, thoſe intermediate ridges do not intercept the view of the little field between them and the town, which field from the lower ſtation was imagined too great; and alſo becauſe the town does not now appear in the horizon; the fields beyond it being plainly viſible.

320. I have ſince taken notice of the like deceptions in ſome other places; as of the interval between the church and the caſtle at *Scarborough*, ſeen from ſome parts of the road between it and *Hacknes*.

Upon ART. 162.

Figure of the ſemi-horizon at Sea ſeen from the ſhore.

321. Since the printing of this explanation of the apparent concavity of the ſky, I have had an opportunity or two of making an obſervation that ſeems to be no ſmall confirmation of it. Standing upon the ſea-ſhore under the cliffs, that were extended on both hands, pretty ſtraight, as far as I could ſee, and looking at the horizon of the ſea, it did not appear to be a ſemicircle having my eye for its center; but the diſtance of its parts ſeen on each hand by the ſides of the cliffs, ſeemed to be the greateſt of all; and the diſtances of the intermediate parts ſeemed to decreaſe gradually, while I carried my eye from the cliffs towards the middle of the horizontal arch. I alſo took notice of a certain diſtant ſhip, which appeared almoſt to biſect one half of the horizontal arch intercepted between the line of the cliffs on one hand, and the perpendicular to it on the other. And yet the angle under the line of the cliffs and a line drawn to the ſhip, appeared much ſmaller than its complement, contained under the ſaid line drawn to the ſhip and the ſaid perpendicular to the line of the cliffs: which agrees with the like obſervations upon the ſky mentioned in the next article; and ſhews that our idea of diſtance is inlarged by a variety of objects.

In the 272d FIGURE the line of the cliffs is AOD, the perpendicular to it is OC; the ſtation of the obſerver is O; the horizon of the ſea is $ABCD$; the apparent place of the diſtant ſhip is B, which I judged to biſect the arch ABC in B pretty nearly, though the angle AOB appeared much ſmaller than the angle BOC.

322. At another time and place I made the like obſervation, ſtanding upon the top of ſome very high cliffs; and then the margin of the ſea ſeemed to riſe up a little towards the clouds in the horizon, like the margin of a tea-ſaucer, eſpecially where it appeared the fainteſt and darkeſt.

Upon ART. 163.

Mr. Folkes's remarks upon the conchoidal figure of the ſky, &c. Fig. 96.

323. Upon communicating this determination of the figure of the ſky with my highly eſteemed friend *Martin Folkes* Eſquire, he was pleaſed to approve of it; and added that he had frequently obſerved the ſky to appear of a conchoïdal ſhape, as repreſented by the back line $ABCD$ in fig. 96. Which I find alſo is frequently very evident; but it is eaſy to underſtand, that neither the proportion of the apparent diſtances of its vertical and horizontal parts

parts, nor the method of determining it, is ſenſibly altered thereby.

324. He alſo obſerved to me, that the apparent diſtance of the ſun or moon at any given place *B*, might be computed within certain limits, by conceiving a perpendicular *BP* to fall upon the ground *OA*, and by taking notice of ſome mark near *P*, where it is imagined to fall; (upon which I find that ſeveral perſons ſtanding together at *O* will agree in their judgments very nearly;) then having meaſured the diſtance *OP* and taken the ſun's altitude or the angle *BOP*, the ſide *OB* of the triangle *OBP* may be found ſeveral ways. The angle *BOP* may be had exactly enough for this purpoſe by erecting a ſtaff *pb* and by meaſuring the length of its ſhadow *pO*.

325. Hence from the figure of the concave
rt. 163. *ABC* determined above[a], we have the apparent diſtance of the ſun at any other altitude, and alſo the apparent height of the vertical clouds or ſky at *C*.

326. He alſo informed me, that the parallel
97. ſides of a very long and broad walk, avenue, road, or the like, do not appear to converge like two ſtraight lines tending to a very diſtant point; but rather like the legs of two hyperbola's *abc*, *def* converging towards each ſide of an aſſymptote *op* drawn from the eye at *o* parallel to the ſides of the walk, as repreſented in the figure.

327. He has alſo told me, when he rides very faſt by the ends of ſeveral long and ſtraight ploughed lands, and looks ſideways upon them; that they appear convex towards him, before he comes over againſt them; then ſtraight, and after that convex again towards his eye moved paſt them. I never had an opportunity of obſerving this phænomenon, but this I think is the deſcription that he gave of it. So far Mr. *Folkes*.

Upon ART. 164, &c.

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328. In theſe articles I haveſhewn in general why the moon appears always larger in the horizon than in the meridian, and have confirmed the reaſon of it by an experiment deſcribed in Remarks 297, &c. I ſay in general, becauſe it is agreed, that at different times the horizontal moon appears of different magnitudes even in the ſame horizon, and now and then of a ſize extraordinary large. This, I am inclined to believe, is chiefly owing to an extraordinary largeneſs of her picture on the retina at thoſe times; which picture in the preſent theory was ſuppoſed invariable. This might beſt be examined by taking its diameters with a micrometer; or, becauſe this inſtrument is ſeldom at hand, by regiſtring the year and day of the month, together with the heights of the barometer and thermometer. For if it ſhould appear by many ſuch obſervations, that the largeſt horizontal moons happen generally at her perigee in the warmeſt ſummer-evenings, the barometer being low and the thermometer high; and on the contrary, that the ſmalleſt horizontal moons happen generally at her apogee in the coldeſt winter-mornings, the barometer being high and the thermometer low; ſince theſe cauſes are independent on one another, and all conſpire to increaſe the moon's picture in the former caſe, and to diminiſh it in the latter, we might reaſonably conclude that thoſe extraordinary moons are chiefly owing to the concurrence of theſe three circumſtances.

329 In my remarks upon Art. 170, relating to aſtronomical refractions, I have ſhewn why the horizontal full moon ſeldom appears oval like the horizontal ſun, (notwithſtanding the ſame refractions of their rays) but generally of a roundiſh figure, to which the refracted oval is reduced by a defect of illumination at one end of it, for a day or two before or after the full, eſpecially in the evenings of the autumnal months. Conſequently in comparing together the areas of theſe roundiſh diſks of different full moons, we may ſuppoſe them equal to circles whoſe diameters are equal to the vertical diameters of the diſks: becauſe theſe laſt diameters are ſubject to ſmaller and more regular changes than any other, and about the full are as often bigger as they are leſs than the horizontal, (or rather the ecliptical diameter,) according as the horizontal moon happens at different diſtances from the exact full moon. Now ſince theſe apparent vertical diameters, and the pictures of them, are varied both by the moon's diſtance from the earth and by her horizontal refractions depending upon the air's denſity, that is upon its gravity and warmth together; I find by a courſe computation of their greateſt variations in our climate, that the ſaid apparent diameters of the greateſt and leaſt full moons are nearly in the ratio of 36 to 25, or almoſt as 3 to 2.

330. But it muſt be conſidered that we dont compare in our minds an extraordinary large moon with an extraordinary ſmall one, (the latter being ſeldom ſeen, and even then but little regarded, as being not different from every common moon ſomewhat elevated,) but with an eſtabliſhed idea acquired from the moſt frequent horizontal moons; whoſe mean diameter muſt therefore be aſſumed about a geometrical mean between the ſaid diameters of the greateſt and leaſt moons, rather than an arithmetical mean; becauſe we ſee more moons in cold weather than in hot, and therefore more of the ſmaller than of the larger ſize, by reaſon of a longer continuance of greater refractions which contract thoſe diameters. Therefore the diameter of the

largeſt moon, is to that of the commoneſt middle ſized moon, in a ſubduplicate ratio of 36 to 25 abovementioned, that is (ſince 36, 30 and 25 are continual proportionals) in the ratio of 6 to 5; which is nearly the ratio of the diameters of a ſhilling and a ſixpence or of a crown and a half-crown; by which we maybetter judge, how near this hypotheſis will anſwer our idea of thoſe extraordinary large moons. And this hypotheſis I find is ſomewhat farther confirmed by the only obſervation of the kind that I ever took down in writing; viz. of an extraordinary large moon of the common colour like that of bright copper, which I ſaw on the 27th of July 1732, riſing about 8 o' clock in the evening over the town of *Cambridge* from the diſtance of half a mile, the wind being weſt and the barometer ſomewhat above changeable. Now by the almanack it appears that the moon was near her perigee at that time, and the wind being weſt, in that month, is a ſufficient argument for the warmth of the air and ſmalneſs of its refractive power. For I did not take notice of the thermometer, not thinking of this hypotheſis at that time. Here then we have two of the three cauſes conſpiring with the theory, and the third (viz. the barometer) nearly at a medium; but this I reckon has rather a leſs effect upon the moon's apparent magnitude than either of the other two.

The moon's apparent diameters at various heights are in a leſs ratio than that of her apparent diſtances.

331. Since the picture of the horizontal moon is conſtantly more or leſs contracted by refractions, and that of the meridional moon at her greateſt height with us, of 60 or 65 degrees, ſcarce ſenſibly, it follows, that in theſe two poſitions, the diameters of her diſks conſidered as portions of the concave ſky, muſt be in a leſſer ratio than that of their apparent diſtances in the ſky. Becauſe in the preſent theory in Art. 164, the moon's picture was ſuppoſed invariable during her aſcent or deſcent. The ratio of thoſe apparent diſtances by the table in Art. 164 is about 1 to 3 or 3 to 9; and this would be the ratio of the diameters of her circular diſks in the concave but for refractions; by which alone I reckon the horizontal diſk is generally contracted in the ratio of two circles whoſe diameters are nearly as 9 to 8. Thereforethe higheſt meridional diſk is generally to the horizontal one but as 3 to 8 in diameter. Theſe diſks are repreſented by the circles 3 and 8 below FIG. 174 in Plate 20, for the eye to judge of their proportion. In this compariſon the defect of illumination from the full, is not conſidered, ſeeing it affects the meridional moon as much as the horizontal, taking one time with another.

Objection anſwered.

332. Since the horizon, ſeen from the top of a mountain or any elevated place, appears remoter than it does from below, it ſeems to follow, from the preſent theory, that the horizontal moon ſhould alſo appear larger from above than from below; which I believe is not at all agreeable to experience and I think it ought not to be. For it muſt be conſidered that any preſent proſpect of the moon and horizon together cannot ſo much ſuggeſt a new idea of a certain determinate magnitude of her diſk (the power of eſtimation by the eye being too weak and inaccurate for this purpoſe) as it revives an old eſtabliſhed idea of it; reſulting from a long experience of innumerable proſpects of various moons and horizons ſeen from various ſituations. Our idea of the magnitude of the horizontal moon is therefore about a medium taken from all our obſervations, and is ſo rooted in our minds by repeated views as not to be altered but only confirmed by any ſingle one. The ſame may be ſaid of our idea of the magnitude of the meridional moon; and of the moon at intermediate degrees of altitude. Which ideas, though gained originally from our ideas of the moon's diſtances in the ſky, may yet become connected with and ſuggeſted by our ideas of her correſponding altitudes. For which reaſon the former ideas may continue much the ſame though the preſent view of the horizon be quite intercepted, by the interpoſition of any near objects, and even that of the ambient ſky too, by the ſides of an empty tube, when the moon is ſeen through it. For by memory we retain very diſtinct ideas of the moon's magnitudes for a longer time. Nevertheleſs in a general courſe of experience, if it happens very rarely that the magnitude of the picture of the horizontal moon is conſiderably altered by any means whatever[a]; this powerful cauſe will immediately alter our eſtabliſhed idea of her uſual magnitude, as any one will find by viewing the moon through any ſpectacle-glaſs, moved a little from his eye.

a Rem. 328.

Various ſolutions of the horizontal moon conſidered.

333. That the moſt ancient aſtronomical obſervations upon the poſitions and diſtances of ſtars were not made with inſtruments but barely by eſtimation, or judgment of the eye, is ſufficiently evident not only from the nature of things, but from hiſtory too. And ſince the firſt aſtronomers were not aware that the intervals of ſtars ſeem much greater near the horizon than in the meridian, no doubt their aſtronomy was much embarraſſed by ſo great a fallacy in their eſtimates. How long it was diſcovered before *Ptolemy*'s time I have not enquired; but that he was aware of it, is very plain by the caution he gives to allow ſomething for it, upon every recourſe to thoſe ancient obſervations[b]. But whenever it was that the fallacy was diſcovered, this is certain that the cauſe of it has ever ſince been dubious and diſputed; and therefore I have been the fuller upon it, to place it, if poſſible, beyond diſpute. The variety of authors and opinions

b Almageſt lib. 2. cap. 9.

nions upon it, is so very great, that to avoid all needless prolixity, I chuse to refer the curious to *Ricciolus's* Almagest *, where many of them are mentioned and referred to; and also to a curious dissertation upon them written by the iugenious Mr. *Will. Molyneux* †, with an intent, as he tells us, "not so much to establish any thing of his own that may be satisfactory in solving this admirable appearance, as to detect the errors of those that have offered at a solution thereof, and yet have come short of being satisfactory: that thereby he might again set the minds of Philosophers to work, and rouse them up to enquire a-new after this surprising phænomenon.' Since the publication of these discourses, I do not remember to have met with any farther advances of a different kind, except in the ingenious *Essay*, abovementioned [a], *towards a new Theory of Vision*; in which there is the following solution. Sect. LXVIII.

334. "In order to explain the reason of the moon's appearing greater than ordinary in the horizon, it must be observed that the particles which compose our atmosphere, intercept the rays of light proceeding from any object to the eye; and by how much the greater is the portion of the atmosphere interjacent between the object and the eye, by so much the more are the rays intercepted, and by consequence the appearance of the object rendered more faint: every object appearing more vigorous or more faint in proportion as it sendeth more or fewer rays into the eye. Now between the eye and the moon, when situated in the horizon, there lyes a far greater quantity of atmosphere, than there does when the moon is in the meridian. Whence it comes to pass that the appearance of the horizontal moon is fainter; and therefore by sect. LVI. it should be thought bigger in that situation than in the meridiân, or in any other elevation above the horizon.'

335. In the LVIth section here referred to by the author, he tells us by what means the tangible magnitude of objects is perceived by sight. "First by the magnitude or extension of the visible object, which being immediately perceived by sight is connected with the other which is tangible and placed at a distance. Secondly, by the confusion or distinctness. And thirdly, by the vigorousness or faintness of the aforesaid visible appearance. *Ceteris paribus* by how much the greater or lesser the visible object is, by so much the greater or lesser do I conclude the tangible object to be. But be the idea perceived by sight never so large, yet if it be withal confused, I judge the magnitude of the thing to be but small. If it be distinct and clear I judge it greater; and if it be faint I apprehend it to be yet greater.'

336. After this the Author proceeds to confirm his solution of this phænomenon, by an examination and exclusion of some other causes and opinions *. But to avoid the entering into a particular consideration of his arguments, I chuse to shew from experience only, that these various degrees of the moon's faintness, make no sensible variations of her apparent magnitude: which I think will be evident by the following observations.

* Ibid. sect. 70. &c. item sect. 75. &c.

337. First, that the moon appears much fainter in the day time than in the night, and therefore according to our author's principle, should appear larger by day than by night at the same height: which I could never perceive, though I have often viewed the moon for this purpose.

Is found contrary to experience.

338. Secondly I observe that the horizontal moon, being much fainter than the horizontal sun, viewed by the naked eye, should in consequence of that principle (though the fact is otherwise) appear much larger than the horizontal sun: because these two bodies differ in no other apparent circumstance than this of faintness or brightness. For the angles at the eye subtended by their diameters, are generally as nearly equal to each other as the naked eye can distinguish; as is evident by astronomical tables of these angles and also by total eclipses of the sun: which the intermediate moon sometimes just covers entirely, and sometimes leaves a very small lucid ring of light uncovered; according to the different proportions of the distances of the sun aud moon from the eye. Again the horizontal sun, notwithstanding a very great obstruction of his rays by the atmosphere, is still less faint than the meridional moon; (the eye being generally offended by the brightness of the former, but not at all by that of the latter) and therefore the horizontal sun should appear smaller than the meridional moon, if the author's principle were true. But these consequences from it are manifestly contrary to experience.

339. Thirdly, I observe, that the moon, when totally eclipsed, appears much fainter than she does at the same elevation when not eclipsed; but does not appear larger than usual, as I am fully satisfied by the total eclipse of the moon on Nov. 20. 1732. For though I did not think of this comparison when I saw this eclipse, yet I was put in mind of it the very next morning (which was full as well) upon seeing the moon but a few degrees high just before her setting. She then appeared to me much larger than she did 8 or 9 hours before, when eclipsed very high and near the meridian; at which time she appeared of a much fainter and heavier colour, and more inclined to a dull red, than at the time of her setting. Besides, upon asking several persons that had seen her eclipsed over night, whether they took notice of any other unusual circum-

circumſtance of her appearance, beſides that of her duſky colour, they told me, that they did not perceive any. Then I aſked them directly as to her apparent magnitude; to which they replyed, that it ſeemed to them as uſual, that is, not near ſo large as that of the horizontal moon. But had this greater faintneſs of the meridional moon eclipſed, than that of the horizontal moon uneclipſed, produced an idea of magnitude proportionable to it, or even no greater, than that of the uſual horizontal moon; ſo remarkable an appearance could hardly have paſſed unobſerved in this and in many other total eclipſes that have happened at very great altitudes above the horizon: and yet I have never met with any memorandum of it in any obſervations of eclipſes; though the duſky colour of the moon be frequently mentioned and deſcribed. Now the ordinary degrees of the moon's faintneſs are cauſed by the obſtruction of the atmoſphere to various quantities of refracted light after its reflection from the moon, as explained by our author[a]; and the degrees of this extraordinary faintneſs of the moon eclipſed, are cauſed partly by the abſence of the ſun's direct rays intercepted by the earth, and partly by the like obſtruction of the atmoſphere to other refracted rays both before and after their incidence upon the moon, as explained in the remarks on Art. 170; and therefore the unuſual faintneſs differs from the uſual one, in no other ſenſible circumſtance but in degree only; and yet as we have ſeen, it occaſions no difference in the degrees of apparent magnitude. This faintneſs therefore is not liable to thoſe objections raiſed by our author againſt the like degrees of faintneſs of the ſun and moon, produced by the interpoſition of ſmoked or coloured plane glaſſes[b]; which are well known to cauſe no ſenſible alterations of apparent magnitude.

a Rem. 334.

b Ibid. ſect. 72.

340. Laſtly, I obſerve that this hypotheſis of faintneſs can ſcarce be thought ſufficient by any means, to account for the like variety of the apparent magnitudes of the conſtellations; that is, of the intervals of the ſame fixt ſtars at various altitudes; which yet is allowed to be a phænomenon of the ſame kind and degree as that of the ſun and moon, and conſequently to depend upon the ſame cauſe.

This phænomenon of the moon not incapable of computation.

341. To conclude, "if this phænomenon of the horizontal moon be a clear inſtance of the inſufficiency of lines and angles, for explaining the way wherein the mind perceives and eſtimates the magnitude of outward objects, as the author is pleaſed to aſſert[c], I would leave it to be conſidered, whether the ſaid phænomenon be not as clear an inſtance of the inſufficiency of faintneſs for the ſame purpoſe; the degrees of which may yet be defined and computed by lines and angles, to the ſame degrees of exactneſs in all caſes, as the eye can diſtinguiſh them in the moſt ſimple and advantageous ones: which exactneſs would be quite ſufficient either for eſtabliſhing or overthrowing any general arguments and concluſions drawn from faintneſs; as might eaſily be ſhewn in the preſent phænomenon, if ſo much exactneſs were requiſite, and if (for a foundation of a *calculus*) any certain obſervation or experiment could be produced to ſhew, that *cæteris paribus* any two given degrees of faintneſs did really and plainly ſuggeſt any two diſtinguiſhable and given degrees of apparent magnitude.

c Ibid. ſect. 78.

342. But no more of this, becauſe the author is pleaſed to allow ſome uſe of mathematical computation in opticks. Though this in general may, he thinks, be obſerved concerning it; namely "that it can *never* be very preciſe and exact, ſince the judgments we make of the magnitude of external things, do *often* depend on ſeveral circumſtances, which are not proportionable to, or capable of being defined by lines and angles[d].' Upon this I would only obſerve, that had he been pleaſed to recollect, how many various kinds of quantities, (colours not excepted) which not long ago were judged incapable of any meaſures, have nevertheleſs been accurately reduced to the rules of mathematical computation, he might have had leſs reaſon to deſpair of the like reductions of quantities of many other kinds. But I ought to aſk pardon for this digreſſion, were it not incumbent upon philoſophers to remove all difficulties and diſcouragements, that may lye againſt the purſuit of the moſt approved and ſucceſsful, if not the only method of acquiring juſt and true apprehenſions of the nature of things: which are all made *in number, weight and meaſure.*

d Ibid. ſect. 78. Edit.

343. In the ſecond edition of this Eſſay and alſo in a vindication and explanation of it (called *the Viſual Language*) very lately publiſhed, the author has made ſome additions to his ſolution of the ſaid phænomenon; but ſeeing it ſtill involves, and depends upon, the principle of faintneſs, I may leave the reſt of it to the reader's conſideration.

Upon ART. 167.

Oval figure of halo's confirmed.

344. Since the printing of this article I have had many opportunities, for ſome years together, of obſerving the ſhapes of halo's about the moon, which appeared to me always oval, more or leſs according as the moon was lower or higher, without any exception. I have never ſeen a halo about the ſun ſince I had this notion of them; but the Rev. Dr. *Walker*, Fellow of our College, tells me that he had taken notice of theſe ovals for many years; and remembers one or two that he ſaw about the ſun when elevated very high in ſummer time; and conſequently

when

when the oval figure was leſs diſcernible, according to this theory, than if the ſun had been lower.

Upon ART. 170.

345. Among all the phænomena which ſerve to demonſtrate, that very high and remote objects in the heavens do not appear to us in their real ſhapes, places and poſitions, but according to their perſpective projections upon the apparent concavity of the ſky, as above explained; there is none more appoſite and convincing than that beautiful appearance of thoſe broad diverging beams from the ſun's apparent place among the clouds, which happen in a warm ſummer's evening to lye quietly and diſtinctly interſperſed about that region of the ſky. This application of this common phænomenon was firſt ſuggeſted to me by an odd appearance, ſomewhat like it, which I once ſaw upon *Lincoln*-Heath, and will here deſcribe and account for.

346. It was an apparent convergence of long, whitiſh beams towards a point diametrically oppoſite to the ſun. For, as near as I could eſtimate, it was ſituated as much below the horizon as the ſun was then elevated above the oppoſite part of it. This part is here repreſented by the line *tDt*, and the point below it in oppoſition to the ſun is *E*; towards which all the beams *vt*, *vt*, &c. appeared to converge.

347. Obſerving that the point of convergence was oppoſite to the ſun, I began to ſuſpect that this unuſual phænomenon was but a caſe of the uſual apparent divergence of the beams of the ſun from his apparent place among the clouds, as repreſented in fig. 98. I ſay an apparent divergence; for though nothing is more common than for rays to diverge from a luminous body, yet the divergence of theſe beams in ſuch large angles is not real but apparent. Becauſe it is impoſſible for the direct rays of the ſun to croſs one another at any point of the apparent concavity of the ſky, in a greater angle than about half a degree. For the diameter of the earth being ſo extreamly ſmall, in compariſon to the diſtance of the ſun, as to ſubtend an angle at any point of his body of but 20 or 22 ſeconds at moſt[a]; and the diameter of our viſible horizon being extreamly ſmaller than that of the earth; it is plain that all the rays which fall upon the horizon, from any given point of the ſun, muſt be inclined to each other in the ſmalleſt angles imaginable; the greateſt of them being as much ſmaller than that angle of 22 ſeconds, as the diameter of the viſible horizon is ſmaller than that of the earth. All the rays that come to us from any given point of the ſun may therefore be conſidered as parallel to each other; as the rays *eBg* from the point *e*, or *fBh* from the oppoſite point *f*; and conſequently the rays of theſe two pencils that come from oppoſite points of the ſun's real diameter and croſs each other in the ſun's apparent place *B* among the clouds, can conſtitute no greater an angle with each other than about half a degree; this angle of their interſection *eBf* being the ſame as the ſun would appear under to an eye placed among the clouds at *B*, or (which is much the ſame) to an eye at *O* upon the ground. Becauſe the ſun's real diſtance *OS* is inconceivably greater than his apparent diſtance *OB*. Therefore the rays of the ſun as *Bg*, *Bh* do really diverge from his apparent place *B* in no greater angles *gBh* than about half a degree. Nevertheleſs they appear to diverge from the place *B* in all poſſible angles and even in oppoſite directions. Let us proceed then to an explanation of this apparent divergence, which is not ſelf-evident by any means; though at firſt ſight we are apt to think it is, by not diſtinguiſhing the vaſt difference between the true and apparent diſtances of the ſun.

348. What I am going to demonſtrate is this. Suppoſing all the rays of the ſun to fall accurately parallel to each other upon the viſible horizon, as they do very nearly, yet in both caſes they muſt appear to diverge in all poſſible angles. Let us imagine the heavens to be partly overcaſt with a ſpacious bed of broken clouds, *v*, *v*, *v*, &c. lying parallel to the plane of the viſible horizon, here repreſented by the line *AOD*. And when the ſun's rays fall upon theſe clouds in the parallel lines *sv*, *sv*, &c. let ſome of them paſs through their intervals in the lines *vt*, *vt*, &c. and fall upon the plane of the horizon at the places *t*, *t*, &c. And ſince the reſt of the incident rays *sv*, *sv*, are ſuppoſed to be intercepted from the place of the ſpectator at *O* by the cloud *x*, and from the intervals between the tranſmitted rays *vt*, *vt*, &c. by the clouds *v*, *v*, &c.; a ſmall part of theſe latter rays *vt*, *vt*, when reflected every way from ſome certain kind of thin vapours floating in the air, may undoubtedly be ſufficient to affect the eye with an appearance of lights and ſhades, in the form of bright beams in the places *vt*, *vt*, &c. and of dark ones in the intervals between them: juſt as the like beams of light and ſhade appear in a room by reflections of the ſun's rays from a ſmoky or duſty air within it: the lights and ſhades being here occaſioned by the tranſmiſſion of the rays through ſome parts of the window, and by their interruption at other parts.

349. Now if the apparent concavity of this bed of clouds *v*, *v*, to the eye at *O*, be repreſented by the arch *ABCD*, and be cut in the point *B* by the line *OBx* drawn parallel to the beams *tv*; it will be evident by the rules of perſpective, that theſe long beams will not appear

on the arent di- gence d con- gence of ſun's ams. g. 98.

heir appa- nt con- rgence. g. 99.

he real eams are early pa- llel.

Art. 875.

g. 96.

Fig. 98.

Their apparent divergence explained. Fig. 100.

pear in their real places, but upon the concave *ABCD* diverging every way from the place *B**, where the ſun himſelf appears, or the cloud *x* that covers his body, as repreſented ſeparately in full view in fig. 98.

* Art. 156. and 170. pag. 69.

Their apparent convergence explained. Fig. 100.

350. And for the ſame reaſon, if the line *BO* be produced towards *E*, below the plane of the horizon *AOD*, and the eye be directed towards the region of the ſky directly above *E*, the lower ends of the ſame real beams *vt*, *vt*, will now appear upon the part *DF* of this concave; and will ſeem to converge towards the point *E*, ſituated juſt as much below the horizon as the oppoſite point *B* is above it: which is ſeparately repreſented in full view in fig. 99.

351. For if the beams *vt*, *vt* be ſuppoſed to be viſible throughout their whole lengths, and the eye be directed in a plane perpendicular to them, here repreſented by the line *OF*; they and their intervals will appear broadeſt in and about this plane, becauſe theſe parts of them are the neareſt to the eye; and therefore their remoter parts and intervals will appear gradually narrower towards the oppoſite ends of the line *BE*. As a farther illuſtration of this matter, we may conceive the ſpectator at *O* to be ſituated upon the top of ſo large a deſcent *OHI* towards a remote valley *IK*, and the ſun to be ſo very low, that the point *E*, oppoſite to him, may be ſeen above the horizon of this ſhady valley. In this caſe it is manifeſt that the ſpectator at *O* would now ſee theſe beams converging ſo far as to meet each other at the point *E* in the ſky it ſelf.

Why converging beams are leſs common than diverging ones,

352. I do not remember to have ever ſeen any phænomenon of this kind by moon-light; not ſo much as of beams diverging from her apparent place. Probably her light is too weak, after reflections from any kind of vapours, to cauſe a ſenſible appearance of lights and ſhades ſo as to form theſe beams. And in the unuſual phænomenon I well remember, that the converging ſun-beams towards the point below the horizon, were not quite ſo bright and ſtrong as thoſe uſually are that diverge from him; and that the ſky beyond them appeared very black, (ſeveral ſhowers having paſſed that way) which certainly contributed to the evidence of this appearance. Hence it is probable that the thinneſs and weakneſs of the reflected rays from the vapours oppoſite to the ſun, is the chief cauſe that this appearance is ſo very uncommon in compariſon to that other of diverging beams. For as the region of the ſky round about the ſun, is always brighter than the oppoſite one; ſo the light of the diverging beams ought alſo to be brighter than that of the converging ones. For, though rays are reflected from rough unpoliſhed bodies in all poſſible directions, yet it is a general obſervation that more of them are reflected forwards obliquely, than are reflected more directly backwards. Beſides, in the preſent caſe, the incident rays upon the oppoſite region to the ſun, are more diminiſhed by continual reflections from a longer tract of the atmoſphere, than the incident rays upon the region next the ſun.

353. The common phænomenon of diverging beams, I think, is more frequent in ſummer than in winter, and alſo when the ſun is lower than when higher up; probably becauſe the lower vapours are denſer and therefore more ſtrongly reflective than the higher; and becauſe the lower ſky-light is not ſo bright as the upper, and becauſe the air is generally quieter in the mornings and evenings than about noon-day; and laſtly becauſe many ſorts of vapours are exhaled, in greater plenty in ſummer than in winter, from many kinds of volatile vegetables; which vapours when the air is cooled and condenſed in the mornings and evenings, may become denſe enough to reflect a ſenſible light.

Upon aſtronomical refractions.

354. I will conclude my remarks upon theſe optical appearances in the heavens with a popular explanation of the refractions of rays through the atmoſphere, and of the principal phænomena thence reſulting.

Hiſtory of the diſcovery.

355. *Alhazen* the Arabian, who is reckoned to have lived about A. D. 1100, ſeems to have been more inquiſitive into the nature of refractions than the more ancient writers; inſomuch that having made experiments upon them at the common ſurface betwixt air and water, air and glaſs, water and glaſs or cryſtal, and being prepoſſeſſed with the ancient opinion of cryſtalline orbs in the regions above the atmoſphere, he had the boldneſs to ſuſpect a refraction there alſo. Which he tells us[a] may be proved by taking the diſtance of a ſtar from the pole of the equator (with an Armillary ſphere,) both when it is very low and when very high near the zenith; and affirms that the former polar diſtance will be found leſs than the latter by reaſon of the refraction of the rays. And this, *Fryar Bacon* tells us[b], was taken from *Ptolemy*'s 5th Book of Aſpects; where he attributes it to a refraction of the rays at the common ſurface betwixt the air and æther. If this be ſo, his Book of Aſpects (which I never could meet with) muſt have been written after his Almageſt. For here he tells us[c], that *Hipparchus*, in order to determine the times of the equinoxes, by obſerving at what day and hour the ſun's rays coincided exactly with the plane of his equatoreal *Armilla*, (or braſs circle fixt in the plane of the equator,) was aſtoniſhed to find that it happened twice in the ſame day at the interval of 5 or 6 hours, the ſun being high at one

a Opt. lib. 7. prop. 15.

b Opus Majus p. 398. *Lond.* 1733.

c Lib. 3. cap. 2. See alſo *Riccioli* Almageſt. Tom. I. p. 113.

one time and low at the other. He alſo tells us, that he himſelf had obſerved the ſame thing; and he attributes it to the inaccuracy or unſteadineſs of the *Armilla:* ſo that he had then no ſuſpicion that this effect was cauſed by a refraction of the ſun's rays.

356. Bnt however this may be, it is certain that *Alhazen* proceeds to deduce ſeveral properties of this kind of refraction; as that it increaſes the altitudes of all objects in the heavens; that it contracts their diameters and diſtances from each other; and that it cauſes the twinkling of ſtars. But ſtill we do not find that he or his follower *Vitello,* knew any thing of its juſt quantity; which being but ſmall could not be determined but by very accurate inſtruments; the want of which may be the reaſon that we hear little more of it till about the year 1500; at which time it was revived and cultivated by *Bernard Walther, Mœſtlin* and others, but chiefly by the noble *Tycho Brahe*; who perceiving the importance of it to the perfection of Aſtronomy, could not reſt till after incredible diligence in contriving inſtruments and obſerving with them, he had ſettled the quantity of theſe refractions at all altitudes to a tolerable degree of exactneſs.

357. But ſtill he could not diſcover where and in what manner the rays were refracted, nor even *Kepler,* who wrote ſo much upon this ſubject[a]. *Tycho* thought the refraction was chiefly cauſed by denſe vapours very near the earth's ſurface; and *Kepler* places it wholly at the top of the atmoſphere, which he took to be uniformly denſe; and thence he determines its altitude, from the obſerved quantity of refraction, to be but little more than that of the higheſt mountains. But the true conſtitution of the denſity of the atmoſphere, deduced afterwards from the *Torricellian* experiment, afforded a juſter idea of theſe refractions, eſpecially after it appeared by a repetition of Mr. *Lowthorp*'s experiment that the air's refractive power is proportionable to its denſity.

a Paralipom. in Vitellionem

Refraction of a ray through the atmoſphere explained. Fig. 101.

358. In order to give an idea of the courſe of a ray refracted through the atmoſphere, let the plane of this figure be conceived to paſs through it, and through Y the earth's center; about which let innumerable circles Oo, Pp, Qq, &c. be drawn at any ſmall intervals from one another, continued from the bottom to the top of the atmoſphere. Now though it be evident from the conſideration of the air's gravity and elaſticity, that its denſity decreaſes continually in aſcending upwards, (the ſpring of every higher particle being leſs preſſed upon by the weight of a ſhorter column of air incumbent upon it) yet at firſt we may conceive it to be interrupted in a manner; or, that between the two loweſt circles Oo, Pp, the air is every where equally denſe, but ſomewhat denſer than the air next above it; and that this air, between the circles Pp, Qq, is alſo uniformly denſe, but ſomewhat denſer than the air next above it; and ſo on to the top of the atmoſphere: whoſe expanſion will be limited, where its denſity is ſo much diminiſhed, and conſequently its ſpring ſo much weakened, as to be juſt balanced by the weight of a ſurface of ſingle particles.

359. Now let O be the eye of a ſpectator, YOZ a line drawn up to his zenith, $OPQRS$ the courſe of a ray, which came from any ſtar S, and which we may conſider as going backwards from the eye to the ſtar, becauſe it will deſcribe the very ſame courſe over again[b]. Then the ray OP in emerging at P out of a denſer air into a rarer, will be refracted from the perpendicular YP produced *; and in emerging again at Q out of this laſt air into another ſtill rarer, it will again be refracted from the next perpendicular YQ produced; and ſo on, till at laſt it emerges from the top of the atmoſphere into empty ſpace.

b Art. 11.

* Art. 12.

All ſtars appear elevated by refractions.

360. Let the laſt emergent ray RS be produced backwards, till it meets the vertical line YOZ in V and the viſual ray OP, produced, in X; and it is manifeſt, from what has been ſaid, that the ſtar's apparent diſtance from the zenith, meaſured by the angle XOZ *, is leſs than its true diſtance, meaſured by the angle XVZ; and therefore that OXV, the difference of theſe two angles[c], is the total effect of all the refractions; whereby the ſtar appears ſo much higher than it would have done without them. Becauſe what has been proved of theſe diſcontinued refractions at P, Q, R, through this fictitious atmoſphere, will hold good in the true atmoſphere, by conceiving the intervals of the circles Oo, Pp, Qq to be diminiſhed and their number to be increaſed indefinitely, till the ray becomes continually refracted at every point; and thereby changes its courſe from a polygon to a continued curve, to which the lines SX, OX will then become tangents at the top and bottom of the atmoſphere.

* Art. 101.

c Eucl. I. 32.

361. I have here ſuppoſed the ſtar's true diſtance from the zenith to be meaſured by the angle SVZ, becauſe a ſtraight line ON drawn from the eye to the ſtar, would make an inſenſible angle with the ray SRV, the diſtance of the ſtar being exceeding great, and the curvity of the ray but very ſmall, eſpecially near the top of the atmoſphere. If its curvity were circular, I reckon that the greateſt diſtance OV, between the lines ON, VS, even then would ſcarce be three miles.

The total refraction of a ray decreaſes in aſcending.

362. The total refraction of an horizontal ray is the greateſt of all, and while the ſtar aſcends, the refraction is continually diminiſhed, till it vaniſhes at the zenith. For ſuppoſing an aſcend-

aſcending ſtar to be ſeen ſucceſſively in the directions OP, Op, and the rays to return back in the ſame lines $OPQRS$, $Opqrs$ in which they came to the eye; then ſince the ray Op is leſs inclined than OP to the refracting circle pP, the refraction at p will be leſs than at P *; and therefore the next ray pq will alſo be leſs inclined than PQ to the next refracting circle qQ; and conſequently the next refraction at q will be leſs than at Q; and ſo the ſum of all the refractions at p, q, r, &c. will be leſs than the like ſum of all thoſe at P, Q, R, &c.; that is, the angle OXV will be continually diminiſhed till it becomes nothing at the zenith. On the other hand, when the ray OP becomes horizontal, it will be the moſt refracted, as paſſing moſt obliquely through all the refracting circles.

* Art. 14. &c.

The refractions of ſtars, their true and apparent places and intervals, what.

363. The eye being the center of any ſpherical ſurface, to which the relative places of ſtars are referred; the point where it is cut by a ſtraight line drawn from the eye to a ſtar, is called *the ſtar's true place*; and the point where it is cut by a tangent to the viſual ray at its point of incidence upon the eye, is called *the ſtar's apparent place*; and the arch of a great circle lying betwixt the true and the apparent place, or the angle NOX, (at the eye,) meaſured by it, or the equal angle OXV * under the tangents to the curvity of the viſual ray at its two extremities, is called *the ſtar's refraction*. Accordingly the arch of a great circle betwixt the true places of two ſtars, or the angle at the eye meaſured by it, is called *the true interval of the ſtars*; and laſtly the arch of a great circle betwixt their apparent places, or the angle at the eye meaſured by it, is called *the apparent interval of the ſtars*. A great circle is well known to divide the ſpherical ſurface into two hemiſpheres, and the plane of it to paſs through the eye at the center.

Fig. 101. * Rem. 361.

The intervals of all ſtars appear contracted by refractions.

364. The apparent interval between two ſtars ſituated on the ſame ſide of the zenith and in the ſame vertical circle or plane, is ſo much leſs than their true interval, as the refraction of the higher ſtar is leſs than that of the lower. For if the refractions of the ſtars were equal, their apparent interval would continue equal to their true interval. Therefore as much as the higher ſtar is leſs elevated by refraction than the lower, ſo much their apparent interval falls ſhort of the true; that is, by the difference of the two refractions.

365. If the two ſtars lye on oppoſite ſides of the zenith and in the ſame vertical circle their apparent interval will be leſs than the true by the ſum of their refractions.

366. Secondly, the apparent interval between any two ſtars equally high, is always leſs than their true interval. For the ſtars being equally high, are equally elevated above their true places in two vertical circles that converge to the zenith; and therefore their true interval is ſomewhat contracted by the refractions; though but little, becauſe near the horizon, where the refractions are the greateſt [a], the two vertical circles are nearly parallel to each other; and near the zenith, where they converge quickeſt, the refractions are very ſmall.

a Rem. 362.

367. And laſtly, the apparent interval between two ſtars in any oblique poſition, is always leſs than their true interval. Let the true places of the ſtars be a, b, and their apparent places α, β in the two vertical circles $a\alpha z$, $b\beta z$; and ſince the refraction of the lower ſtar, is greater than that of the higher, that is, $a\alpha$ greater than $b\beta$, the apparent interval $\alpha\beta$ will be leſs oblique to the two verticals than the true interval ab; and being higher too, will be ſhorter than ab upon both accounts.

Fig. 102.

Table of refractions of ſtars.

368. The principal uſe of a Table of refractions, is to deduce readily the true place of a ſtar from its apparent place, by deducting the refraction belonging to its apparent altitude obſerved by an inſtrument. The following Table, made by Sir *Iſaac Newton*, is taken from the Philoſophical Tranſactions N°. 368, where Dr. *Halley*, the publiſher, has given ſome uſeful remarks upon ſome allowances to be made in aſtronomical obſervations for the refractions of the air, without the trouble of trigonometrical calculations.

A Table

A Table of the Refractions of Stars adapted to their apparent altitudes.

Appar. Alt. deg. m.	Refraction. m. sec.	Appar. Alt. deg.	Refraction. m. sec	Appar. Alt. deg.	Refraction. m. sec.
0 0	33 45	16	3 4	46	0 52
0 15	30 24	17	2 53	47	0 50
0 30	27 35	18	2 43	48	0 48
0 45	25 11	19	2 34	49	0 47
1 0	23 7	20	2 26	50	0 45
1 15	21 20	21	2 18	51	0 44
1 30	19 46	22	2 11	52	0 42
1 45	18 22	23	2 5	53	0 40
2 0	17 8	24	1 59	54	0 39
2 30	15 2	25	1 54	55	0 38
3 0	13 20	26	1 49	56	0 36
3 30	11 57	27	1 44	57	0 35
4 0	10 48	28	1 40	58	0 34
4 30	9 50	29	1 36	59	0 32
5 0	9 2	30	1 32	60	0 31
5 30	8 21	31	1 28	61	0 30
6 0	7 45	32	1 25	62	0 28
6 30	7 14	33	1 22	63	0 27
7 0	6 47	34	1 19	64	0 26
7 30	6 22	35	1 16	65	0 25
8 0	6 0	36	1 13	66	0 24
8 30	5 40	37	1 11	67	0 23
9 0	5 22	38	1 8	68	0 22
9 30	5 6	39	1 6	69	0 21
10 0	4 52	40	1 4	70	0 20
11 0	4 27	41	1 2	71	0 19
12 0	4 5	42	1 0	72	0 18
13 0	3 47	43	0 58	73	0 17
14 0	3 31	44	0 56	74	0 16
15 0	3 17	45	0 54	75	0 15

369, As to the methods taken by aſtronomers for making a table of refractions, it is here ſufficient to ſay in ſhort, that they chiefly conſiſt, firſt in finding the latitude of the obſerver's place by obſervations made ſo near the zenith, as not to be affected by a ſenſible refraction; ſecondly in taking the greateſt altitude of a ſtar paſſing ſo near the zenith as to ſuffer no ſenſible refraction, whereby its true diſtance from the pole is given; thirdly in obſerving as many other altitudes, as you pleaſe, of that ſame ſtar, noting the ſidereal times of each obſervation, reckoned from its tranſit over the meridian; and laſtly, from theſe (or the like) *data*, in computing its true altitudes at thoſe given times of obſervation; which, being ſeverally deducted from the obſerved altitudes, will give the corresponding refractions. An obſerver living under the Equator might make a table of refractions with much leſs trouble by means of any ſtar that goes over his zenith.

370. I will only obſerve farther, that though an accurate aſtronomy cannot be obtained without a correct table of refractions, yet that no one table can ſerve preciſely either for the ſame place, the ſame ſeaſon or even the ſame time of the day [a], eſpecially when the objects obſerved are low; it being well known that the air's denſity and refractive power are increaſed by cold and diminiſhed by heat [b], not to mention the changeable mixture of vapours and exhalations with the air near the horizon. There is a famous obſervation of this kind made by ſome *Hollanders* that wintered in *Nova Zembla* in the year 1596, and were ſurpriſed to find, that after a continual night of three months, the ſun

Inconſtancy of theſe refractions.

[a] Art. 691.

[b] See Dr. *Jurin's* Notes on *Varenius's* Geography, Chap. 9. prop. 21.

ſun began to riſe ſeventeen days ſooner than according to computation, deduced from the altitude of the pole obſerved to be 76°: which cannot otherwiſe be accounted for, than by an extraordinary quantity of refraction of the ſun's rays paſſing through the cold and denſe air in that climate. *Kepler* computes that the ſun was almoſt five degrees below the horizon when he firſt appeared [a]; and conſequently the refraction of his rays was about nine times greater than it is with us. On the other hand, it has been obſerved that the horizontal refractions at places near the equator are near a third part leſs than at *Paris*. But in aſcending equally their difference diminiſhes gradually, till it becomes inſenſible at the height of about 60 degrees. Reg. Sci. Acad. Hiſtor. per *du Hamel*. p. 600.

[a] Paralipom. in Vitellio. p. 138.

Oval figures of the horizontal ſun and moon.

371. Since the apparent vertical diameters of the horizontal ſun and moon (by reaſon of the unequal refractions of the higheſt and loweſt rays) are much more contracted than their horizontal ones, their pictures upon the retina and conſequently their apparent figures become oval; their longeſt and ſhorteſt apparent diameters being frequently as 5 to 4 [*], eſpecially in the mornings when the rays are moſt refracted through a colder, denſer, and moiſter air. The circle *adce*, whoſe center is *b*, repreſents the ſun's true diſk projected upon the ſpherical ſurface abovementioned [b], and the oval $\alpha\delta\gamma\varepsilon$ its apparent one. Now ſuppoſing $b\beta$, the refraction or elevation of its center, to be equal to its true diameter *ac*, the elevation $a\alpha$ of its loweſt point will be greater, and $c\gamma$ that of its higheſt point will be leſs, than $b\beta$; which will cauſe $\beta\alpha$ to be leſs than $\beta\gamma$, and the lower ſegment of the oval to be flatter than the upper. Their proportions may be eaſily computed by the table of refractions and obſerved through a ſmoaked glaſs or with a teleſcope [*]. But unleſs the horizontal moon be exactly in the full, the defect of the illumination of her diſk, towards the right hand or left will partly deſtroy that oval figure and reduce it to thoſe rounder, though ſomewhat irregular, ſhapes with which it frequently appears. And the riſing full moons about the time of the autumnal equinox, and the ſetting full moons about the vernal, will be the roundeſt of all, becauſe the ecliptick is then very oblique to the horizon; ſo that the defect of illumination falls fuller upon one end of the horizontal diameter. For theſe reaſons the horizontal moon appears oval but very rarely, eſpecially in the evenings of warm weather, the refractions being then ſmaller.

[*] *Balthaſaris* micrometria p. 101.

Fig. 103.

[b] Rem. 363.

[*] Rem. 128.

Viſible effects of refractions in the moon's eclipſes.

372. The refraction of an horizontal ray being generally equivalent to the apparent diameter of the ſun or moon, it is manifeſt that all the heavenly bodies are entirely viſible when in reality they are below the plane of the horizon produced. For this reaſon ſeveral total eclipſes of the moon have been ſeen in the horizon, when the ſun was alſo viſible in the oppoſite part of it. The ancient philoſophers that knew nothing of the air's refractive power, were very much perplexed with this ſtrange phænomenon; and alſo to give a reaſon why the moon ſhould be viſible at all, even at any altitude, when totally immerſed within the earth's ſhadow, as ſhe always is; appearing generally of a duſky, red colour, not unlike that of tarniſhed copper, or of iron almoſt red hot. This they thought was the moon's native light, by which ſhe became viſible when hid from the brighter light of the ſun. *Plutarch* indeed, in his Diſcourſe upon the Face in the Moon, attributes this appearance to the light of the fixt ſtars reflected to us from the moon; but this is much too weak to produce that effect. The true and only ſenſible cauſe of it, is the ſcattered beams of the ſun bent into the earth's ſhadow by refractions through the atmoſphere, after the following manner

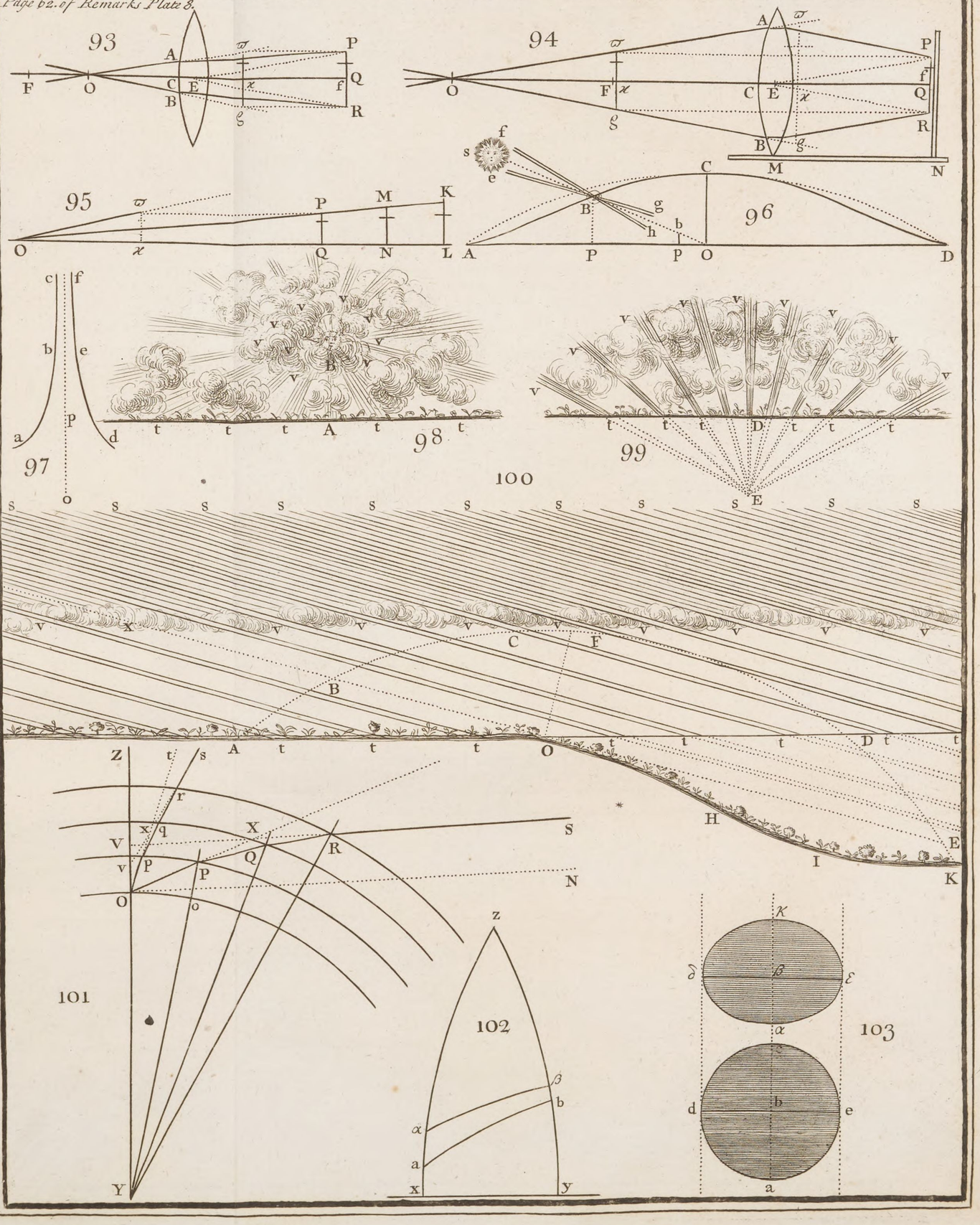
The earth's ſhadow how much contracted by refractions.

373. Let the body of the ſun be repreſented by the greater circle *ab*, and that of the earth by the leſſer one *cd*; and let the lines *ace* and *bde* touch them both on their oppoſite ſides and meet in *e* beyond the earth; then the angular ſpace *ced* will repreſent the conick figure of the earth's ſhadow, which would be totally deprived of the ſun's rays, were none of them bent into it by the refractive power of the atmoſphere. Let this power juſt vaniſh at the circle *hi*, concentrick to the earth, ſo that the rays *ah* and *bi*, which touch its oppoſite ſides may proceed unrefracted and meet each other at *k*. Then the two neareſt rays to theſe, that flow within them, from the ſame points *a* and *b*, being refracted inwards through the margin of the atmoſphere, will croſs each other at a point *l*, ſomewhat nearer to the earth than *k*; and in like manner, two oppoſite rays next within the two laſt will croſs each other at a point *m*, ſomewhat nearer to the earth than *l*, having ſuffered greater refractions, by paſſing through longer and denſer tracts of air lying ſomewhat nearer to the earth. The like approach of the ſucceſſive interſections *k*, *l*, *m*, is to be underſtood of innumerable couples of rays; till you come to the interſection *n*, of the two innermoſt; which we may ſuppoſe juſt to touch the earth at the points *o* and *p*. It is plain then, that the ſpace bounded by theſe rays *on*, *np*, will be the only part of the earth's ſhadow wholly deprived of the ſun's rays. Let *fmg* repreſent part of the moon's orbit, when it is neareſt to the earth at a time when the earth's dark ſhadow *onp* is the longeſt; in this caſe I will ſhew that the ratio of *tm* to *tn* is about 4 to 3, and conſequently that the moon, though centrally eclipſed at *m*, may

Fig. 104.

yet

Page 62. of Remarks. Plate 8.



yet be viſible by means of thoſe ſcattered rays abovementioned, firſt tranſmitted to the moon by refractions through the atmoſphere, and from thence reflected to the earth.

Demonſtration. Fig. 105.

374. For let the incident and emergent parts *aq*, *rn* of the ray *aqorn*, that juſt touches the earth at *o*, be produced till they meet at *u*, and let *aqu* produced meet the axis *st* produced in *x*; and joining *us* and *um*, ſince the refractions of an horizontal ray paſſing from *o* to *r* or from *o* to *q* would be alike and equal, the external angle *nux* is double the quantity of the uſual refraction of an horizontal ray; and the angle *aus* is the apparent meaſure of the ſun's ſemidiameter ſeen from the earth; and the angle *ust* is that of the earth's ſemidiameter *tu* ſeen from the ſun (called his horizontal parallax); and laſtly the angle *umt* is that of the earth's ſemidiameter ſeen from the moon, (called her horizontal parallax;) becauſe the elevation of the point *u* above the earth, is too ſmall [a] to make a ſenſible error in the quantity of theſe angles; whoſe meaſures by aſtronomical tables are as follow.

[a] Rem. 361.

		′	″
The ſun's leaſt apparent ſemidiameter	= ang. *aus* =	15 —	50
The ſun's horizontal parallax ———	= ang. *ust* =	00 —	10
Their difference [b] is ———	= ang. *txu* =	15 —	40
Double the horizontal refraction ——	= ang. *nux* =	67 —	30
Their ſum [c] is ——— ———	= ang. *tnu* =	83 —	10
The moon's greateſt horizontal parallax	= ang. *tmu* =	62 —	10

[b] Eucl. I. 32.

[c] Eucl. I. 32.

Therefore we have *tm* : *tn* :: (ang. *tnu* : ang. *tmu** :: 83′ — 10″ : 62′ — 10″ ::) 4 : 3 in round numbers, which was to be proved. It is eaſy to collect from the moon's greateſt horizontal parallax of 62′ — 10″, that her leaſt diſtance *tm* is about 55 $\frac{1}{3}$ ſemidiameters of the earth, and therefore the greateſt length *tn* of the dark ſhadow, being three quarters of *tm*, is about 41 $\frac{1}{2}$ ſemidiameters.

*, Art. 60.

Illumination of the moon eclipſed how great.

375. The difference of the laſt mentioned angles *tnu*, *tmu* is *mun* = 21′, that is, about two thirds of 31′ — 40″, the angle which the whole diameter of the ſun ſubtends at *u*. Whence it follows that the middle point *m* of the moon centrally eclipſed, is illuminated by rays which come from two thirds of every diameter of the ſun's diſk, and paſs by one ſide of the earth; and alſo by rays that come from the oppoſite two thirds of every one of the ſaid diameters, and paſs by the other ſide of the earth. This will appear by conceiving the ray *aqorn* to be inflexible, and its middle point *o* to ſlide upon the earth while the part *rn* is approaching to touch the point *m* [*]; for then the oppoſite part *qa* will trace over two thirds of the ſun's diameter [d]. The true proportion of the angles *num*, *aus* could not be preſerved in the ſcheme, by reaſon of the ſun's immenſe diſtance and magnitude with reſpect to the earth.

* Art. 44.

d Art. 59.

Images of the ſun how formed by the atmoſphere. Fig. 106.

376. Having drawn the line *atα*, it is obſervable that all the incident rays as *aq*, *ax* flowing from any one point of the ſun, to the circumference of the earth, will be collected to a focus *α*, whoſe diſtance *tα* is leſs than *tm* in the ratio of 62 to 67 nearly; and thus an image of the ſun will be formed at *αβ*, whoſe rays will diverge upon the moon. For the angle *tαu* is the difference of the angles *xuα*, *uαt* found above; and *tα* : *tm* :: ang. *tmu* : ang. *tαu** :: 62′ — 10″ : 67′ — 30″.

* Art. 60.

How they increaſe in magnitude and brightneſs.

377. The rays that flow next above *aq* and *ax*, by paſſing through a thinner part of the atmoſphere, will be united at a point in the axis *atα* ſomewhat farther from the earth than the laſt focus *α*; and the ſame may be ſaid of the rays that paſs next above theſe, and ſo on; whereby an infinite ſeries of images of the ſun will be formed, whoſe diameters and degrees of brightneſs will increaſe with their diſtances from the earth.

The colour of the moon eclipſed explained.

378. Hence it is manifeſt why the moon eclipſed in her perigee is obſerved to appear always duller and darker than in her apogee. The reaſon why her colour is always of the copper kind between a dull red and orange, I take to be this. The blue colour of a clear ſky ſhews manifeſtly that the blue-making rays are more copiouſly reflected from pure air than thoſe of any other colour; conſequently they are leſs copiouſly tranſmitted through it among the reſt that come from the ſun, and ſo much the leſs as the tract of air through which they paſs is the longer. Hence the common colour of the ſun and moon is whiteſt in the meridian and grows gradually more inclined to diluted yellow, orange and red as they deſcend lower, that is, as the rays are tranſmitted through a longer tract of air; which tract being ſtill lengthened in paſſing to the moon and back again, cauſes a ſtill greater loſs of the blue-making rays in proportion to the reſt; and ſo the reſulting colour of the tranſmitted rays muſt lye between a dark orange and red; according to Sir *Iſaac Newton*'s rule for finding the reſult of a mixture of colours *. We have an inſtance of the reverſe of this caſe in leaf-gold, which appears yellow by

* Opt. p. 134. 8°.

by reflected and blue by tranſmitted rays. The circular edge of the ſhadow in a partial eclipſe appears red, becauſe the red-making rays are the leaſt refracted of all others a, and conſequently are left alone in the conical ſurface of the ſhadow, all the reſt being refracted into it.

a Art. 172.

Other effects of the reflective power of the air.

379. The reflective power of the air is the chief cauſe that enlightens objects ſo uniformly on all ſides. The abſence of this power would occaſion a ſtrange alteration in the appearances of things; their ſhadows would be ſo very dark and their ſides enlightened by the ſun ſo very bright, that probably we could ſee no more of them than their bright halves; ſo that for a view of their other halves we muſt turn them half round, or if immoveable, muſt wait till the ſun could come round upon them. Such a pellucid unreflective atmoſphere would indeed have been very commodious for aſtronomical obſervations upon the courſe of the ſun and planets among the fixt ſtars, viſible by day as well as by night; but then ſuch a ſudden tranſition from darkneſs to light and from light to darkneſs immediately upon the riſing and ſetting of the ſun, without any twilight, would have been very inconvenient and offenſive to our eyes.

Twilight explained. Fig. 107.

380. The better to underſtand the beginning, increaſe and ending of twilight, let the ſun's rays coming in the direction *sab*, enlighten a ſegment of the atmoſphere repreſented by the ſhaded ſegment *abga*, and terminated below by the line *ab*, that touches the earth's ſurface at *d*, and above by the arch *agb*. From the end *b*, oppoſite to the ſun, draw a line *be*, touching the earth at another point *e*; and ſuppoſing no refractions of rays, a ſpectator at *e* might juſt perceive a faint light in his horizon *eb*, reflected to him from the air or vapours at *b*. By the earth's diurnal rotation let the ſpectator be carried from *e* to *f*, and his horizon *be* into the poſition *gf* cutting *db* in *h*; and from *f* he will ſee the part *bgh* of the luminous ſegment *bgab*, by rays reflected every way from every point of it *. And laſtly when the earth has carried him to *d*, the whole luminous ſegment *agba* will become viſible to him, together with the ſun in his horizon *da*.

* Art. 56.

The old way of finding the height of the atmoſphere.

* *Alhazen* prop. ult.

381. From this conſideration the ancient mathematicians determined the heigth of the atmoſphere to be about 50 miles *, as follows. Immediately upon perceiving the firſt and fainteſt light in the eaſtern horizon *eb*, they obſerved the altitudes or poſitions of ſome of the brighteſt ſtars; from which they computed how many degrees the ſun was then below the horizon, and found them at a medium to be about 18 degrees; which being the meaſure of the angle *dbm* between the two horizons *db*, *eb* produced to *m*, or between their perpendiculars *cd*, *ce*, drawn from the earth's center, they rightly concluded, that the enlightened vapours at *b*, were ſituated in the line *cb* which biſects the ſaid angle *dce* of 18 degrees. Now in the right-angled triangle *cdb*, the radius *cd* is to the ſecant *cb*, of the angle *dcb* of 9 degrees, as 10000 to 10125, or (multiplying both by 4 and dividing by 10) as 4000 to 4050. Therefore, the earth's ſemidiameter *cd* being about 4000 miles, we have *cb* equal to 4050; and conſequently the height of the vapours at *b* above the earth's ſurface is 50 miles: ſuppoſing as I ſaid that the horizontal rays *db*, *be* were not refracted; which *Alhazen* did not conſider, not knowing how much to allow for it, as I obſerved above.

A correction of it by refractions. Fig. 108.

382. But ſince thoſe rays *db*, *be* do each of them ſuffer a continual refraction inwards along the curves *dβ*, *βe*; the utmoſt height of the reflective matter at *β* above the earth's ſurface, will be reduced to about 44 ½ miles, according to the following rule given by Dr. *Halley* *. From the aforeſaid angle *dce* of 18 degrees, ſubſtract twice the uſual refraction of an horizontal ray, or about a degree; and half the remainder will be about 8 ½ degrees; whoſe ſecant being 10111, it will follow that as 10000 to 111, ſo is the ſemidiameter of the earth or 4000 miles to 44, 4 miles.

* Phil. Tranſ. N°. 181.

Demonſtration of it.

383. For ſuppoſing two rays to ſet out from *d* and *e* along the horizontal lines *db*, *eb*, and after they have deſcribed the curves *dβ*, *eβ*, to croſs each other at *β* near the top of the atmoſphere; they will ſoon depart from it in ſtraight lines *βγ*, *βδ*, each declining from the reſpective horizons *db*, *eb* in an angle of about half a degree. And therefore the perpendiculars *cp*, *cq* to the lines *βγ*, *βδ* produced back, will ſeverally decline from the perpendiculars *cd*, *ce*, to the ſaid horizons, in the like angles *pcd*, *qce* of half a degree; and will be nearly equal to the ſemidiameter of the earth; becauſe the curvity of the rays *βd*, *βe* near *β* is exceeding ſmall *. Conſequently *cβ* is the ſecant of the angle *βcp*, to the radius *cp* or *cd* nearly, that is, of the angle *dcb* diminiſhed by *dcp*, the refraction of an horizontal ray. It is farther to be obſerved, that the ray *αdβ* came from the ſun not ſituated in the horizontal line *adb*, but in the tangent *sα* of the curve *αd*, inclined to *adb* in an angle of half a degree. Therefore ſince the tangent *βδ* to the reflected ray *βe*, is alſo inclined to the other horizon *eb* in the like angle, the angle under the tangents *sα*, *βδ* muſt be equal to the angle *abm* under the two horizontal lines *bd*, *be*.

* Rem. 361.

Dimenſions of the viſible ſegment of the atmoſphere

384. Hence it follows, that the height of the reflective atmoſphere, (being about 44 ½ miles,) is about $\frac{1}{90}$ part of the earth's ſemidiameter; and that

that a ray of light $\alpha d\beta$, passing horizontally by any place d on the earth's surface, is so refracted along the curve $d\beta$, as, at the point β where it departs from the reflective atmosphere, to sink about $5\frac{1}{2}$ miles below the tangent db to the place d; and that the distance $d\beta$ is about 600 miles; and consequently that any place d is constantly enlightened in the day time by rays reflected from every part of a segment of the atmosphere, whose height is about $44\frac{1}{2}$ miles and whose circular base is about 1200 miles in diameter.

The air's refractive power. * Rem. 410.

385. The sine of incidence in vacuo is to the sine of refraction into common air, as 1000000 to 999736*; and therefore when the angle of incidence is a right one or the nearest to it, the greatest angle of deviation, contained under the refracted ray and the incident one produced, is almost 79 minutes. And this angle being but small will be diminished very nearly in a subduplicate ratio of the air's density as Sir *Isaac Newton* shews in his Opticks[a]; and in his *Principia* he shews[b], that at the height of one semidiameter of the earth above her surface, if the air reaches so high, it must become rarer than the air with us, in a far greater ratio than that of all the space within the orbit of Saturn to a globe of space of an inch in diameter.

a pag. 247. 8o.

b pag. 512. Edit. 3.

All the heavenly bodies are equally raised by refractions.

386. Hence we may reasonably conclude that the sun and all the planets, at any given altitude, appear equally raised by refractions; because they are all far above that rarefied air, and because their lights, as being all derived from the sun, are all equally refrangible. And it appears by astronomical observations that the rays of the fixt stars are also equally refracted with those of the sun and planets. *Tycho Brahe* was a long time of a different opinion, holding the refractions of the fixt stars to be less than those of the sun and moon; but at last he was sensible of his mistake, occasioned by making the parallaxes of the sun and moon too great; by which he depressed them too much among the fixt stars, and was therefore obliged to raise them too much by too great an allowance for refractions.

Twinkling of stars owing to refractions.

387. The twinkling of stars is another effect of the refractive power and trembling motion of the air and interspersed vapours; which cause the successive rays to fall upon the eye in different directions, and consequently upon different parts of the retina at different times, and also to hit and miss the pupil alternately. These tremors of the air are manifest to the eye by the tremulous motion of shadows cast from high towers; and by looking at objects through the smoke of a chimney or through steams of hot water, or at objects situated beyond hot sands, especially if the air be moved transversely over them[a]. But when stars are seen through telescopes that have large apertures, they twinkle but little and sometimes not at all. For (as Sir *Isaac Newton* has observed) "the rays of light which pass through divers parts of the aperture, tremble each of them apart, and by means of their various and sometimes contrary tremors, fall at one and the same time upon different points in the bottom of the eye, and their trembling motions are too quick and confused to be perceived severally. And all these illuminated points constitute one broad lucid point, composed of those many trembling points confusedly and insensibly mixed with one another by very short and swift tremors, and thereby cause the star to appear broader than it is, and without any trembling of the whole."

a *Hook's* Micrography p. 232. Opt. pag. 98.

Upon Chapter 6. Concerning the origine and cause of colours.

Design.

388. Sir *Isaac Newton* has not only obliged us with his own discoveries of the true origine and causes of colours, but also with an account of the notions which the world had formerly of them[a]. But as these are very unsatisfactory, I chuse to pass them by, and to transcribe some of those practical methods by which he determined the constant Ratio of refraction both in fluids and solids; and thereby confirmed this fundamental property of light, from which all the certainty and accuracy of our knowledge in opticks is chiefly derived.

a Lectiones Optic. p. 146.

Description of an instrument for finding the ratio of refraction in fluids. Fig. 109.

389. Let HK be a square beam of wood two or three yards long, or rather a hollow tube, to prevent its bending by its own weight; and let its opposite sides be perfectly plane and parallel; and let HI and KL be two square boards fixt perpendicular to one of its sides; the one KL at the very end of the beam, and the other HI about four inches from the other end. Then let the base of a small deep vessel CF, of any shape and substance, be fixt upon the board HI by hard cement, and rest against the end of the beam continued beyond HI; and let a small hole about a tenth of an inch broad be bored through the middle of the bottom of the vessel at F, and also through the board it rests upon; and the opposite board being painted white, let a mark R be made upon it at the same distance from the beam as the center of the hole F, so that the line FR may be exactly parallel to the sides of the beam; and let a piece of glass ground equally thick and well polished be applyed to the under side of the board HI, and be fastened to it with cement dawbed round about the hole F, to prevent any of the fluid contained in the vessel from running out; and let this glass be placed exactly perpendicular to

the ſides of the tube by means of a ſquare. Then let two cylindrical pins of braſs or iron be fixt in the middle of the oppoſite ſides of the beam, and be lodged in two angular notches cut in two parallel boards placed upon a firm pedeſtal, that the beam may turn eaſily upon the pins, like the beam of a ballance, and be readily fixt at any given inclination to the horizon.

390. The inſtrument being thus prepared and the beam being placed in a vertical plane paſſing through the ſun, let the veſſel CF be about half filled with water; and when the beam is ſo inclined that the refracted rays ſhall paſs through the hole F and fall on the board KL, by moving the beam a little it will be eaſy to make any colour you pleaſe fall upon the mark R; then let the inclination of the beam to the horizon be taken with a large quadrant, whoſe ſide ek, being applyed to the under ſide of the beam, will give the angle of refraction ekr of that ray, and its ſine er; becauſe the plumb-line kr is perpendicular to the ſurface of the water. Then let the ſun's altitude be immediately taken; and its complement AkD to 90 degrees will be the angle of incidence, and AD its ſine. Which ſines being compared together, after ſeveral trials of the experiment at different heights of the ſun, will always be found to be in the ſame ratio, when a ray of the ſame colour falls upon the mark R. If it be deſired to make many repetitions of the experiment in a little time, and for ſmaller angles of incidence than thoſe of the ſun's neareſt diſtance from the zenith, a piece of looking-glaſs may be inclined over the mouth of the veſſel, to reflect the rays downwards, partly into the veſſel and partly by the ſide of it, to be examined as before inſtead of the ſun's direct rays. Lect. Opt. Par. I. §. 2.

How to place a priſm ſo as to refract rays equally at its ſides.

391. But the moſt accurate way of all for finding the exact quantity of the ratio of refraction, is this other of Sir *Iſaac Newton*[a]. It was obſerved in Art. 171 that when the axis of the priſm is held perpendicular to the ſun's rays, and the refractions are made upwards, that by turning the priſm ſlowly about its axis, the refracted light, or the ſun's coloured image caſt upon a wall, will firſt deſcend and then aſcend during the ſame continued rotation of the priſm. Between the aſcent and deſcent when the image is ſtationary, let the priſm be ſtopt and remain fixt in this poſture; and then the refractions of the rays at their ingreſs and egreſs on each ſide of the priſm will be equal.

392. For during the deſcent of the image, it is plain that the ſum of theſe two refractions continually decreaſes, and then increaſes again during its aſcent; and therefore there are two poſitions of the priſm, before and after the image is ſtationary, wherein the ſum of the refractions at its ſides are equal; and cauſe the image to fall upon one and the ſame place of the wall. In theſe two poſitions, the ray DE in the one and $\delta\varepsilon$ in the other, lying within the refracting angle ABC, are equally inclined to its ſides AB, BC, but contrary ways; that is, the triangles BDE, $B\varepsilon\delta$ are equiangular. For ſuppoſing it ſo, and the rays to go both ways, along the lines DE, $\delta\varepsilon$, the refractions in going out at D and ε will be equal to each other, and alſo in going out at E and δ; and conſequently the ſum of the unequal refractions at D and E will be equal to the ſum of thoſe at δ and ε: and hence it is that the image falls upon the ſame place on the wall in both poſitions of the priſm. But according as this common place of the image comes nearer to the limit of its contrary motion, experience ſhews that the two poſitions of the priſm come nearer to that intermediate poſition which throws the image to its limit. Therefore at the ſame time the angles at the baſes DE, $\delta\varepsilon$ of thoſe ſimilar triangles BDE, $B\varepsilon\delta$ approach gradually to equality, and become equal when the image is at its limit; and conſequently the refractions at D and E are then equal. This will alſo appear by the rays STV reflected from the top of a glaſs-priſm provided the legs AB, BC of the refracting angle be equal, as they generally are; for then, as the incident rays ST and SD are parallel, ſo the reflected ray TV and the refracted one EP will alſo be parallel in this poſition of the priſm; as will be evident to the eye by the ſmall diſtance or rather coincidence of the two images V, P on the wall, which in all other poſitions of the priſm will be widely diſtant. Sir *Iſaac* has proved the whole mathematically, Lect. Opt. Par. I. §. 3. prop. 25.

Fig. 112. Fig. 112a.

393. In this poſition of the priſm the angle of refraction at the ingreſs of a ray is equal to half the refracting angle ABC. For let LDK be perpendicular to AB, and ſince the line BQ, that biſects the vertical angle B of the iſoſcelar triangle DBE, is perpendicular to its baſe DE; in the right angled triangle BQD, its two acute angles QBD, QDB will be equal to its right angle[b], or to the right angle BDK, conſiſting of the angles QDB and QDK. Therefore by taking away the common angle QDB, there remains QBD equal to QDK, that is, half the refracting angle of the priſm is equal to the angle of refraction.

The angle of refraction is then half the refracting angle of the priſm.

Fig. 113.

394. Now the refracting angle of the priſm may be meaſured by laying two rulers croſs each other upon the ſide of a ſmooth table; and by altering their inclination till the edges of the parts that project over the table, coincide with the ſides of the refracting angle of the priſm interpoſed between them. For then two lines drawn upon the table, by the ſides of each ruler, will give the refracting angle, to be meaſured by a ſector; as appears by the figure, where the rulers are ab, cd; and the priſm is e.

How to meaſure it.

Fig. 114.

395.

a Ibid. Par. I. §. 2.

b Euc. I. 32.

How to find the angle of incidence. Fig. 113.

395. The prism being placed as above, let the altitudes of the incident and emergent rays, SD, EP, be taken with a quadrant; and the angle of incidence SDL, will be equal to half the sum of these altitudes. For let these rays be produced back till they intersect one another in I, and any horizontal line in M and N; and the angles at M and N will be the altitudes of these rays above the horizon; and both together will be equal to the external angle MIE*, which is equal to the two internal angles of the triangle IDE; and therefore half the sum of the altitudes is equal to one of these equal angles IED or IDE, which added to the angle of refraction EDK found before, gives the angle of incidence IDK or SDL.

* Eucl. I. 32.

396. If the sun be higher elevated, till the emergent ray EP becomes parallel to the horizon, then the angle at N will vanish; and if the sun be still higher, the emergent ray will tend downwards; and then the angle at N will become negative: therefore in this last case, half the difference of these altitudes must be added to half the refracting angle of the prism, to get the angle of incidence.

An example. * Opt. p. 71.

397. Sir *Isaac Newton* has given us the following example of this method*. In a glass-prism whose refracting angle was $62\frac{1}{2}$ degrees, the half of it 31°. 15′ is the angle of refraction into the prism, whose sine is 5188 the radius being 10000. When the axis of the prism was parallel to the horizon, and the sun's image upon the wall was at its limit of regression, he observed, with a quadrant, the angle which the mean refrangible rays (that is those which went to the middle of the coloured image) made with the horizon; and by adding this angle to the sun's altitude observed at the same time, he found the angle PIM, which the emergent rays contained with the incident ones, to be 44°. 40′; whose half 22°. 20′, being added to the angle of refraction 31°. 15′, makes the angle of incidence 53°. 45′., whose sine is 8047; and the ratio of these sines in round numbers* is 20 to 31.

* See *Cotes's* Harmonia Mensurarum p. 7. Schol. 3.

The excellencies of the method.

398. The excellency of this method appears from these considerations, that it requires no other instruments but a quadrant and a prism; that the refraction of the ray being doubled, an error in the practice is but half of what it would be in a single refraction; that it is very easy to place the prism in the required position; and that a small deviation from it does not alter the place of the image, or the sum of the two refractions, as is evident upon trial and because this sum is then the least of all others. For it is a known thing that the variations of quantities generated by motion are generally insensible, when the quantities become the greatest or least possible, that is, in the moment between their increase and decrease.

Properties of refractions thro' several parallel planes. a Lect. Opt. Par. I. §. 2. No. 32.

399. Sir *Isaac* informs us of some other properties of refractions well worth observing[a]; to wit, *if a ray of light goes out of air through several contiguous mediums terminated by parallel planes, as through water and glass, and thence goes out again into air, the emergent ray will always be parallel to the incident one.* For let a piece of coach-glass of an equal thickness, be smeared over with a little water, or any other fluid; and be held parallel to the horizon, that the water may be of an equal thickness; then you will find that the rays of the sun, transmitted through both mediums, will go parallel to his unrefracted rays; as will appear by observing the places where they fall upon any distant plane beyond the glass.

400. Hence if a ray passes through many refracting mediums terminated by parallel planes, it will have the same inclination to the surfaces of the last medium as if it had suffered but a single refraction in passing immediately out of the first medium into the last. For example let Aa, Bb, Cc be the parallel surfaces of water lying upon glass, and let the ray DE be refracted into the line EF in water, and then into FG in the glass; and let another ray PQ parallel to DE, fall immediately upon the glass and be refracted into the line QR; then the refracted rays FG, QR will be parallel. For let these rays emerge into air in the lines GH, RS; and because GH is parallel (to DE* or to PQ* by hypothesis or) to RS*, it follows that the refractions at G and R are equal*; and consequently the rays FG and QR are parallel and therefore equally inclined to the incident rays DE, PQ; that is, the sum of the refractions of the one at E and F is equivalent to the single refraction of the other at Q.

Fig. 115.

* Rem. 399.

* Art. 36.

* Art. 11. 12.

401. The conclusion will be the same though the number of different mediums be increased to infinity and the intervals of their parallel surfaces be as much diminished; so that the ray may be continually refracted into a curve line.

402. Hence the proportion of the sine of incidence to the sine of refraction of one and the same sort of rays out of one medium into another, is compounded of the proportion of the sine of incidence to the sine of refraction out of the first medium into any third medium, and of the proportion of the sine of incidence to the sine of refraction out of the third medium into the second*. So that by this Theorem the refraction out of one medium into another is gathered as often as you have the refractions out of them both into any third medium.

* Newt. Opt. pag. 113.

403. For let the perpendiculars to the surfaces through the points of refraction E, F, G be IK, LM, NO. Then the sine of the angle

EFL or *FEK*, is to the ſine of *DEI*, as 3 to 4 *; and the ſine of the ſame angle *DEI* or *HGO* * is to the ſine of *FGN* or *GFM* as 31 to 20 *; and by compounding theſe proportions the ſine of *EFL*, is to the ſine of *GFM*, as 3 × 31 to 4 × 20, as 93 to 80; which meaſures the refraction out of water into glaſs.

* Rem. 404.
* Rem. 399.
* Rem. 397.

The exacteſt way of finding the refractions of fluids.

404. If a priſmatick veſſel be made of wood, and two oppoſite holes, for light to paſs through, be made any where in the ſides containing the refracting angle, and if pieces of broken looking-glaſs unfoiled be cemented over the out ſides of the holes, and be ſet exactly to a right angle, as being eaſieſt to be examined by a ſquare, and the veſſel be filled with water, poured into a hole made in its third ſide, or with any fluid whoſe refractive power is to be found out, and the hole be ſtopped up with a cork; and the like experiment be repeated with this priſm as was tryed before with that of glaſs *, you will have the ratio of the refraction of water. For the incident and emergent rays in the air will be inclined to the intermediate rays within the water, in the very ſame angles, as they would have been if the water had been contiguous to the air a. Sir *Iſaac Newton* by this accurate method, found the ratio of refraction of red rays out of air into water to be 4 to 3.

* Rem. 395.

a Rem. 400.

Refraction of air determined by experiment.

405. While the rays are paſſing through the priſm to the place *P* upon the oppoſite wall, if you ſuppoſe the water to be let out, the image at *P* will immediately ſink down to *M*, where the line *SD* continued ſtraight on would fall upon the wall. Becauſe the refractions at the outward and inward ſurfaces of the glaſſes correct one another b. In like manner ſuppoſing the priſm to be full of condenſed air, and that this air has a greater refractive power than the outward air, the like appearance would happen in proportion to that power, excepting that the image *P* in this caſe would not ſink all at once but gradually, as the air within becomes thinner, by eſcaping gradually. Conſequently if we ſuppoſe the remaining air to be exhauſted from the priſm, the refractions would now be made downwards; and while the air was ſuffered to re-enter the priſm, the image *P* would appear to aſcend gradually. During theſe experiments if we ſuppoſe the rays to go backwards to the eye of a ſpectator placed at *S*, at firſt he would ſee the place *P* upon the wall, and during the time that the condenſed air was eſcaping from the priſm in the firſt caſe, or the outward air was re-entering in the ſecond, he would ſee all the points of the line *PM* ſucceſſively appearing in the ſame direction *SD*: and laſtly if you ſuppoſe the wall *PM* to be very remote and the rays *DS* to come through a fixt teleſcope to his eye, he would ſee the ſame appearances more plainly and diſtinctly, eſpecially if croſs hairs be placed in the focus of the teleſcope for him to take aim by.

Fig. 113.

b Rem. 400.

Fig. 116.

406. An experiment of this kind was firſt tryed by Mr. *Lowthorp* * who made his vacuum between two glaſs planes by the help of quick-ſilver, and found the ratio of the ſine of incidence to the ſine of refraction out of air into the vacuum as 100000 to 100036; and the Royal Academy at *Paris* having afterwards attempted the ſame without ſucceſs *, it was again repeated by order of our Royal Society at *London*. The apparatus was now made by the direction of Dr. *Halley* and conſiſted of a ſtrong braſs priſm, two ſides of which, had ſockets to receive two plane glaſſes; and the third ſide had a pipe and ſtop-cock, whereby the air in the priſm might either be exhauſted or condenſed; the priſm had alſo a mercurial gage fixt to it, to diſcover the denſity of the contained air; and was contrived to turn upon its axis in order to make the refractions equal on each ſide when it was fixt to the end of the teleſcope. The refracting angle was near 64 degrees and the length of the teleſcope about 10 feet, having a fine hair in its focus.

* Phil. Tranſ. N°. 257.

* Hiſt. An. 1700. p. 144. &c.

407. The event of the tryals is related by Mr. *Hauksbee* in theſe words *. "Having choſen a proper and very diſtinct erect object whoſe diſtance was 2588 feet (June 15. S. V. 1708. in the morning, the barometer being then at 29, 7 $\frac{1}{2}$ and the thermometer at 60) we firſt exhauſted the priſm; and then applying it to the teleſcope, the horizontal hair in the focus covered a mark on our object diſtinctly ſeen through the vacuum, the two glaſſes being equally inclined to the viſual rays. Then admitting the air into the priſm, the object was ſeen to riſe above the hair gradually, as the air entered, and in the end the hair was found to hide a mark 10 $\frac{1}{4}$ inches below the former mark. This often repeated as often ſucceeded.

* Phyſ. Mech. Exper. p. 225. &c.

408. "This done we applyed the condenſing engine to the priſm; and having pumped in another atmoſphere, ſo that the denſity of the included air was, by the mercurial gage, double to that of the outward, we again placed it before the teleſcope and then letting out the air by the cock, the object, which before ſeemed to riſe, now appeared gradually to deſcend; and the hair at length reſted on an object higher than before by the ſame interval of 10 $\frac{1}{4}$ inches. And this likewiſe often repeated never failed.

409. "We again crouded in another atmoſphere, and upon diſcharging the condenſed air, the object was ſeen near 21 inches lower than the hair. But in this the great preſſure forcing the cement, would not permit us to make ſo frequent repetitions as in the former."

116.

410. "Now the radius being 2588 feet the interval $10\frac{1}{4}$ inches ſubtended an angle PIM of 68″. The half of which gives 34″ for the angle QDI; which being taken from the angle QDK or QBD of 32°, gives the angle KDI or LDS equal to 31° — 59′ — 26″. And ſo the ſine of incidence in vacuo, is to the ſine of refraction into common air as 1000000 to 999736." So far Mr. *Hauksbee*.

411. It appears by theſe experiments that the ſubtenſes of the angles of deviation PIM, and conſequently the angles themſelves [a], generated by the air's refractive power, were proportionable to its denſities. And ſince the denſity of the atmoſphere is as its weight directly and as its heat inverſely, the ratio of its denſity at any given times may be had by the heights of the barometer and of Mr. *Hauksbee*'s thermometer, as deſcribed in pag. 220. ibid. And thence he concludes that this will alſo be the ratio of the air's refractions, that is, of the ſaid angles of deviation at thoſe given times. But before we can depend upon the accuracy of this concluſion, I think it ought to be examined whether heat and cold alone may not alter the refractive power of air while its denſity continues the ſame. This may be tryed by heating the condenſed or the rarified air ſhut up in the priſm, juſt before it is fixed to the teleſcope; and by obſerving whether the hair in its focus will continue to cover the ſame mark all the while the air is cooling.

412. A Table of the ſines of incidence and refraction through a great variety of ſolid and fluid bodies may be ſeen in Sir *Iſaac Newton*'s Opticks pag. 247. from which he computes another table of the Forces by which thoſe bodies refract and reflect light; and finds them nearly proportionable to the denſities of the ſame bodies; excepting that unctuous and ſulphureous bodies refract more than others of the ſame denſity. Mr. *Hauksbee* has alſo given another Table of the ratio of the refractions of many other liquors chiefly chymical [b]; and when he ſays "that bodies do not refract light in proportion to their ſpecifick gravities or denſities," he means only that when the ſines of incidence upon different bodies are the ſame, the ratio of the ſines of refraction is not the ſame as the inverſe ratio of their ſpecifick gravities; which is very true, but not at all contrary to Sir *Iſaac Newton*'s rule for the ratio of their forces. By which rule and by the foregoing experiments I find that the refractive force of air is as its denſity, its degree of heat being given.

Tables of refractions referred to.

b Ibid. pag. 292.

Upon Chapter 7. Concerning the cauſe of the refraction and reflection of light.

ſign.

413. Having explained Sir *Iſaac Newton*'s ſentiments upon this ſubject, in purſuit of my general deſign I muſt here ſay ſomething upon the opinions of other philoſophers, ſome of which may be ſeen in Dr. *Barrow*'s and in Dr. *Gregory*'s opticks. But as moſt of them are too precarious and arbitrary to require a formal confutation, I will content my ſelf with the mention of Mr. *Leibnitz*'s theory as having been received with great approbation. This gentleman being diſſatisfied with *Des Cartes*'s theory, built upon an hypotheſis that the motion of light is more reſiſted in a rarer medium than in a denſer, advances another principle, by which he ſays the direct, reflected and refracted motion of light may be accounted for even to a degree of mathematical exactneſs. His principle and theory built upon it, is thus tranſlated from the *Leipſick* Acts [c] by Mr. *Molyneux* [d]. "*Light proceeds from the radiating point to the point to be enlightened, that way, which is of all the moſt eaſy*; and this is firſt to be determined in reſpect to plane ſurfaces, and then to be accommodated to concave and convex ſurfaces, by conſidering the planes that are tangents to theſe ſurfaces.

414. "*Hence in plane or ſimple opticks, the direct ray proceeds from the point* C *to the point to be illuminated* E, *by the ſhorteſt direct way; the ſame medium continuing all along, that is, in the right line* CE. Fig. 117.

415. "*In catoptricks the angle of incidence* CEA *and of reflection* DEB *are equal.* Let C be the radiating point, D the point to be illuſtrated, and AB a plane ſpeculum; it is required to find in the ſpeculum the point E that reflects the ray to D. I ſay it ſhall be ſuch a point, that the whole progreſs, way or journey of the ray $CE+ED$ may be the leaſt or ſhorteſt that is poſſible; or leſs than $CF+FD$ ſuppoſing F any other point of the ſpeculum. And this ſhall be obtained, if E be taken ſuch, that the angles, CEA, DEB may be equal, as is manifeſt from geometry. For produce DE to the point Z in the circumference CDZ whoſe center is E, and join FZ; then the arch $AZ = (DB =)$ AC; and $FZ = FC$; therefore $CE + ED$ $(= DZ)$ is leſs than $CF+FD$ or $ZF+FD$ *. * Eucl. I. 20.

416. "*In dioptricks the ſines* CI *and* GK *of the angles of incidence and refraction* CEI *and* GEK, *are to each other reciprocally as the reſiſtances of the mediums.* Let IE be air and EK water.

Anni 82. p. 5. Dioptr. 3. 192.

water or glaſs, more denſe than air, *C* a radiating point in air, *G* the point to be illuſtrated within the glaſs. It is required by what way or path the point *C* ſhall radiate to *G*, or it is required to determine, in the ſurface of glaſs *AB*, the point *E* which refracting the ray that comes from *C* ſends it to *G*. Here the point *E* muſt be taken ſuch that the way which the ray takes may be of all ways the *eaſieſt*. But now in different mediums the difficulties of the way or progreſs are in a ratio compounded of the length of the way and of the reſiſtance of the mediums. Let the right line *m* repreſent the reſiſtance that light finds in its paſſage through air, and *n* the reſiſtance of its paſſage through glaſs; the difficulty of the way from *C* to *E* ſhall be as the rectangle *CE* and *m*, and from *E* to *G* as the rectangle under *EG* and *n*. Therefore that the difficulty of the whole way *CEG* may be the leaſt poſſible, the ſum of the rectangles $CE \times m + EG \times n$ ought to be the leaſt of all poſſible, or leſs than $CF \times m + FG \times n$ ſuppoſing any other point *F* taken beſides *E*. The point *E* is now required." So far Mr. *Leibnitz*.

417. Then by arguing mathematically he finds the point *E* will be ſo ſituated, that *CI* the ſine of incidence in air, will be to *GK* the ſine of refraction into glaſs, as *m* to *n*, that is in the aſſumed ratio of the reſiſtance in glaſs to the reſiſtance in air.

418. This is Mr. *Leibnitz*'s theory, who tells us farther that the catoptrical demonſtration abovementioned was inſiſted upon by *Ptolemy* and other ancients, and is ſtill extant in *Heliodorus Lariſſæus* and elſewhere. But had he proceeded the leaſt ſtep farther than they did, to accommodate this principle to concave and convex ſurfaces, as he talked of doing at firſt; he might ſoon have perceived the inſufficiency of it, and that a ray takes ſeveral other courſes from one point to another beſides the ſhorteſt.

Fig. 118. 419. For example let *AEBF* be a great circle of a concave ſphere or ſpeculum whoſe diameter is *AB* and center *C*; produce the radius *CB* to any point *D*, and let *CD* be the diameter of any other circle *CEDF* cutting the former circle in *E* and *F*; and in the arches *CE*, *CF* let *G* and *H* be any two points equidiſtant from *C*; I ſay that rays flowing from *G* will be reflected to *H* from every one of the four points *A*, *E*, *B*, *F*; and conſequently three of their courſes will be of different lengths. For joining *EG*, *EC*, *EH*, the angles of incidence and reflection *CEG*, *CEH* will be equal, becauſe they ſtand upon equal arches *CG*, *CH* of the circumference *EGH* *. The reaſon of the other two caſes is very plain; and alſo that three of the courſes of the rays are unequal; becauſe they all approach towards equality as the points *G* and *H* approach to *C*; and become moſt unequal when *G* and *H* approach neareſt to *E* and *F*.

* Eucl. 1. 27.

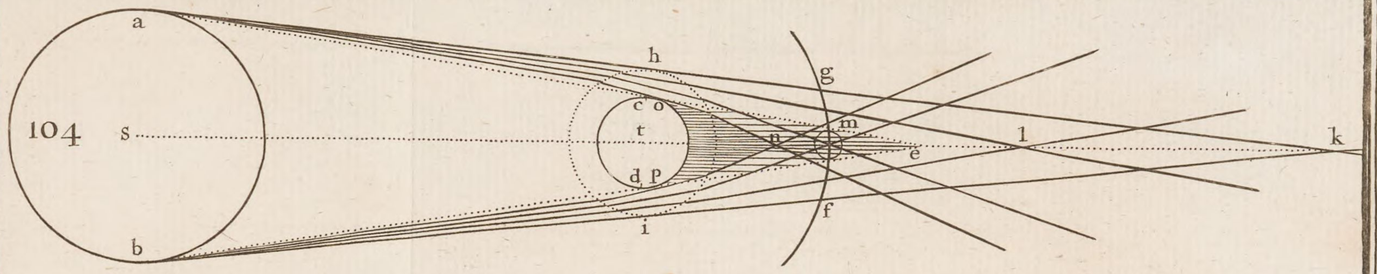
420. As to the dioptrical demonſtration, Fig. 118. where Mr. *L.* ſays "the difficulty of the way from *C* to *E* ſhall be as the rectangle under *CE* and the aſſumed reſiſtance *m*", his notions of difficulty and reſiſtance are not eaſy to be underſtood. In all known caſes the motion of a body and the reſiſtance it ſuffers do conſtantly decreaſe together, action and reaction being always equal; and therefore the difficulty of the way from *C* to *E* can ſcarce be rightly expreſſed by the rectangle under *CE* and a conſtant reſiſtance *m*. But be the difficulty and reſiſtance what you pleaſe, no doubt of it, both of them muſt be nothing at all in *vacuo*; and therefore the eaſieſt way for a ray to paſs from a given point in any reſiſting medium into a vacuum, is to go in the perpendicular to the refracting ſurface, as being the ſhorteſt way through any difficulty or reſiſtance whatever; which being quite over at the refracting ſurface, it may then take any other courſe in vacuo without any farther difficulty: and on the contrary, in returning back from the vacuum into the denſe medium, it muſt take the ſhorteſt courſe through the ſame perpendicular as before: and thus when the ſun ſhines upon the atmoſphere all his rays ſhould be refracted into lines tending to the earth's center as being the ſhorteſt and eaſieſt way through the atmoſphere; and then we ſhould ſee the ſun exactly over our heads in all places and at all times. But it is no wonder that ſtrange conſeqnences ſhould follow from an arbitrary hypotheſis. I will only obſerve farther, that all other Theories for ſolving the reflection and refraction of light, except Sir *Iſaac Newton's*, do alſo ſuppoſe that it ſtrikes upon bodies and is reſiſted by them; which yet has never been proved by any deduction from experience. On the contrary it appears by Art. 183, &c. and might be ſhewn by Mr. *Molyneux*'s and Profeſſor *Bradley*'s obſervations upon the parallax of the fixt ſtars *, that their rays are not at all impelled by the rapid motion of the earth's atmoſphere, nor by the object-glaſs of the teleſcope through which they paſs. And by Sir *Iſaac Newton*'s theory of refraction, which is grounded upon experience only, it appears that light is ſo far from being reſiſted and retarded by refraction into any denſe medium, that it is ſwifter here than in vacuo in the ratio of the ſine of incidence in vacuo to the ſine of refraction into the denſe medium; and on the contrary *.

* Phil. Tranſ. N. 406.

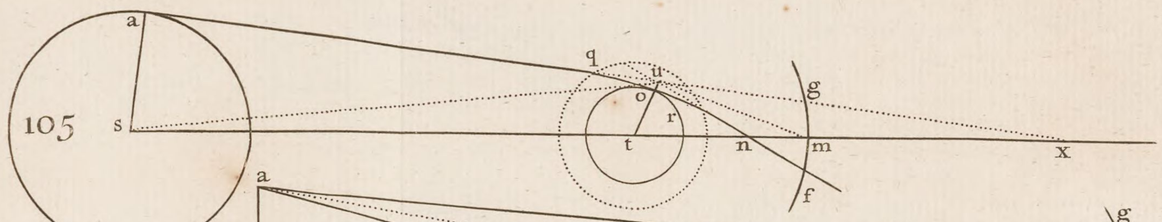
* Philoſ. Princip. lib. I. prop. 95.

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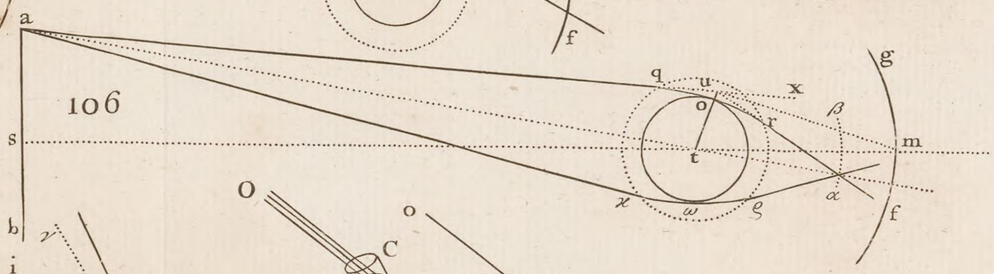
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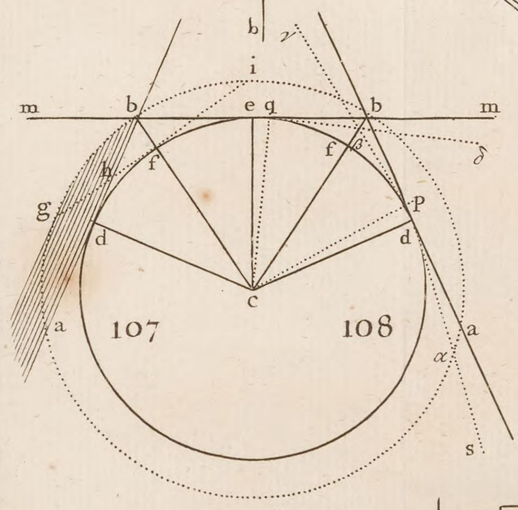

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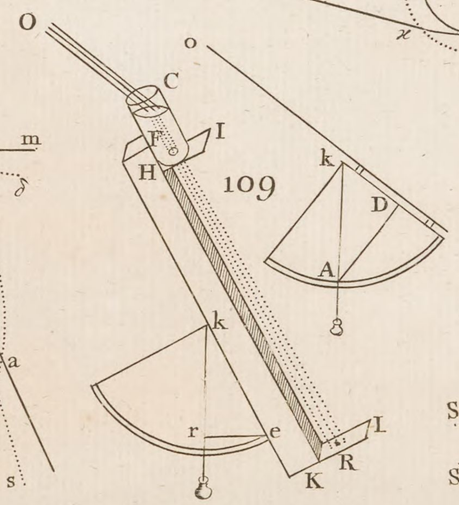
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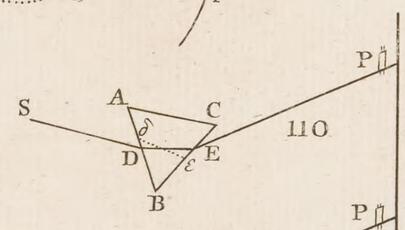

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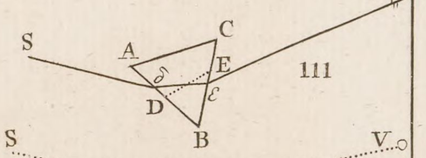
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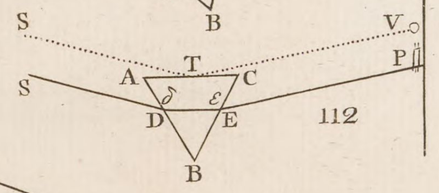

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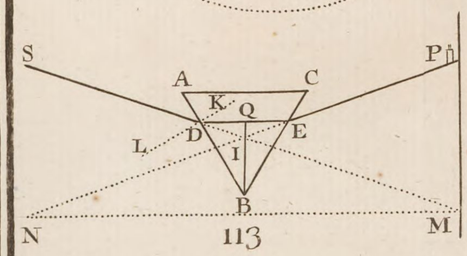

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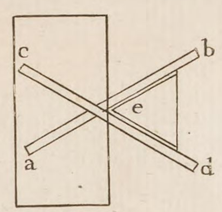

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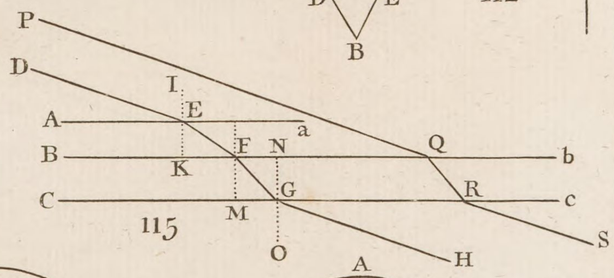

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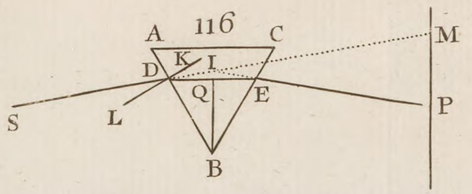

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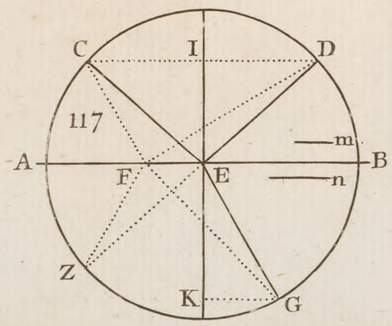

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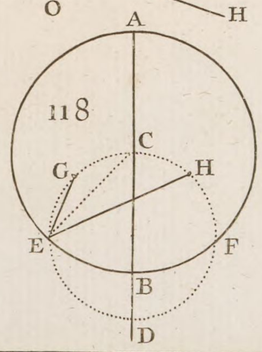

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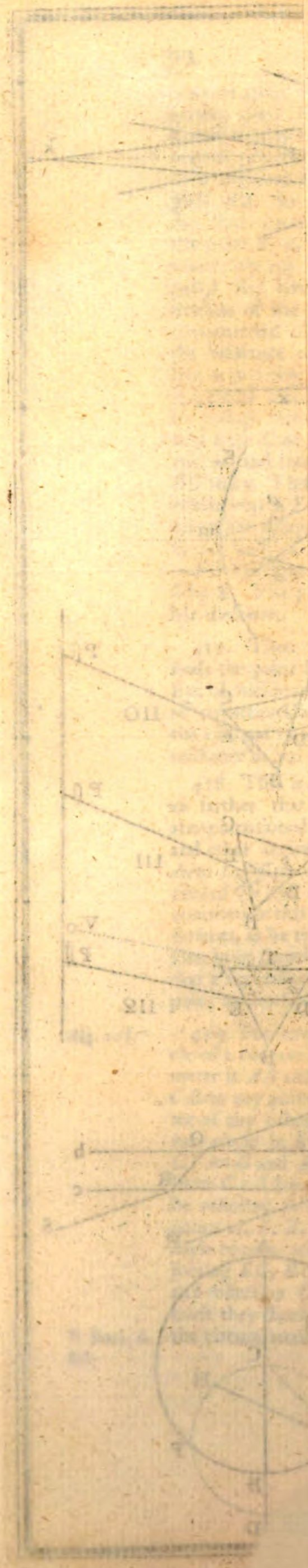

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THE
AUTHOR'S REMARKS
UPON BOOK II.

Upon Chapter 1. 2. 3. 4. *Containing the Geometrical Elements of Opticks.*

421. THE elementary propoſitions for finding focuſs's and images, are contained in the four firſt chapters of this Book, and are taken partly from Sir *Iſaac Newton*[a] and partly from Mr. *Huygens*[b]. But as Sir *Iſaac* has given us no demonſtrations at all, and as thoſe by Mr. *Huygens* are generally very tedious and intricate, by reaſon of too many compoſitions and reſolutions of ratio's, I was therefore obliged to contrive others in a ſhorter and eaſier way. Now though the proportions expreſſed in theſe propoſitions and their corollaries, are the beſt rules for determining the place of a focus to the utmoſt exactneſs, it will yet be of ſome uſe to do the ſame thing by geometrical Conſtructions, that is, by drawing lines only, and laſtly by a general algebraick Theorem or two comprehending the whole of theſe Elements. For as to the method uſed by ſome authors, of computing the place of a focus by trigonometry, it is not only the leaſt ſcientifick, but of the leaſt uſe too; unleſs to determine the courſe of a ray when the angles of incidence are very great. But for this I have given ſome rules in Lem. 3 and 4 Book 2.

[a] Ax. [b] 8. optr. 1. to

Upon ART. 207.

422. The focus $\mathcal{Q}$ of incident rays being given, their focus after reflection from a ſpherical ſurface C, whoſe center is E, may be found by this conſtruction. Through the given points $\mathcal{Q}$ and E draw a line $\mathcal{Q}E$ cutting the concave or convex ſurface in C, then biſect its radius CE in $\mathcal{T}$, and at the points $\mathcal{T}$, C erect the perpendiculars $\mathcal{T}G$, CH, cutting any line drawn through $\mathcal{Q}$ in the points G, H; join the points G, E and draw a line Hq parallel to the line GE, and it will cut the axis $\mathcal{Q}E$ in q the focus of the reflected rays.

metri- con- ſtions finding focus reflected . 119, 120.

423. For the triangles $\mathcal{T}\mathcal{Q}G$, $C\mathcal{Q}H$ being equiangular, and alſo $G\mathcal{Q}E$ and $H\mathcal{Q}q$; we have $\mathcal{T}\mathcal{Q}:\mathcal{T}E$ or $\mathcal{T}C$ $(::G\mathcal{Q}:GH)::E\mathcal{Q}:Eq$*; and diſjointly we have $\mathcal{T}\mathcal{Q}:\mathcal{T}E::\mathcal{T}E:\mathcal{T}q$, the ſame proportion as was proved in the Article it ſelf.

Euclid. VI.

424. But this next conſtruction is ſtill ſimpler. In a perpendicular IEK to the axis $\mathcal{Q}EC$, take any two points I, K equidiſtant from E, and draw $\mathcal{Q}I$ cutting the perpendicular CH in H; then draw the line KH and it will cut the axis in the focus q.

425. For let the perpendicular $\mathcal{T}G$ cut $\mathcal{Q}I$ in G and join G, E; then ſince $\mathcal{T}C=\mathcal{T}E$, we have $GH=GI$*; and conſequently, ſince we took $EK=EI$, we have KH parallel to EG*, as in the foregoing conſtruction.

* Euclid VI. 2.

426. Hence when the focus $\mathcal{Q}$ is infinitely remote, the line IH is parallel to EC, and conſequently the focus q coincides with $\mathcal{T}$.

Fig. 122.

Upon ART. 211.

427. For a proof of this Article ſee the Remarks upon Art. 339.

Upon ART. 216.

428. The determination of the conick ſections which I promiſed in this article may be ſeen with ſome other determinations in the Remarks upon Art. 246.

Upon ART. 236.

429. In order to make the ſolution of this propoſition ſerve for a ſingle ſurface as well as for a lens and a ſphere, I forgot to alter part of it in the following manner. In the 4th line of the Roman print read thus; *then taking* E*f equal to* EF in the lens or ſphere, but equal to CF in the ſingle ſurface; ſay as &c. And in the 9th line read thus. *For with the center* E *and ſemidiameters* EF and Ef deſcribe &c.

Upon ART. 239.

Geometrical conſtructions for finding the focus of rays refracted through a ſphere or lens. Fig. 122. 123.

430. The focus $\mathcal{Q}$ of incident rays being given, the focus of the refracted rays through a ſphere or a thin lens, whoſe center is E, may be found in this manner. At the principal focus F of rays that came parallel to the axis $\mathcal{Q}E$, the contrary way to the incident ones that belong to $\mathcal{Q}$, and alſo at the center E, erect the perpendiculars FG, EI to the axis, cutting any line drawn through $\mathcal{Q}$, in G and I reſpectively. Join E, G and the line Iq drawn parallel to it, will cut the axis in the focus q of the refracted rays. For the triangles $\mathcal{Q}FG$, $\mathcal{Q}EI$ being equiangular

angular, and alſo $\mathcal{Q}GE$, $\mathcal{Q}Iq$, we have $\mathcal{Q}F : \mathcal{Q}E :: (\mathcal{Q}G : \mathcal{Q}I^* ::) \mathcal{Q}E : \mathcal{Q}q^*$, the ſame proportion as was proved in the article it ſelf.

** Eucl. VI. 4.

And thro' a ſingle ſurface.

Fig. 124.

431. The focus q of rays refracted through a ſingle ſpherical ſurface C may be found by erecting one of the perpendiculars FG at its principal focus of rays that came parallel to $\mathcal{Q}E$, the contrary way to thoſe that belong to $\mathcal{Q}$, and the other perpendicular CH at its vertex C, cutting any line $\mathcal{Q}G$ in H, and by joining GE, and drawing Hq parallel to it. For we have $\mathcal{Q}F : \mathcal{Q}C :: (\mathcal{Q}G : \mathcal{Q}H^* ::) \mathcal{Q}E : \mathcal{Q}q^*$, the ſame proportion as was proved in Art. 238.

** Eucl. VI. 4.

432. Or the focus q may be found by this conſtruction. Let the perpendiculars CH, EI cut any line drawn through $\mathcal{Q}$ in H and I, and in EI take EK to EI as the ſine of incidence to the ſine of refraction of any ray belonging to $\mathcal{Q}$ the focus of incident rays. And the line HK produced will cut the axis in the focus q of the refracted rays. For ſince $EK : EI ::$ (as the ſine of refraction, to the ſine of incidence, that is, $:: FC : FE^* ::) GH : GI^*$; it follows that the line HK is parallel to the baſe GE of the triangle IGE^*, as it ought to be by the foregoing conſtruction.

* Art. 224. ** Eucl. VI. 2.

And thro' any two ſurfaces.

Fig. 125.

433. Hence having $\mathcal{Q}$ the focus of incident rays we may find their focus $\varkappa$ after refraction through any two ſurfaces whoſe vertex's are C and c and centers E and e. For having made the foregoing conſtruction for the firſt ſurface C, let qKH cut the perpendicular at c in h, and the perpendicular at e in i, in which taking ek to ei as EI to EK and drawing kh, it will cut the axis Cc produced in the focus $\varkappa$. For this ſecond operation is only a repetition of the firſt, becauſe q is the focus of incident rays upon the ſecond ſurface c. But in practice it is eaſier to determine the point k by a line drawn through the points q, I. Dr. *Barrow* tells us* that this conſtruction was communicated to him by his friend Mr. *Iſaac Newton*, to whom he ſoon after reſigned his Mathematical Profeſſorſhip in 1669.

* Lect. Opt. pag. 103. and Preface.

Upon ART. 246.

An accurate determination of images by reflections from a ſpherical ſurface.

Fig. 126. to 129.

434. Other things remaining as in Art. 215, now let the object $P\mathcal{Q}$ be a ſtraight line perpendicular to $\mathcal{Q}E$, and let $\mathcal{Q}E$ produced cut the reflecting circle CA continued round in γ oppoſite to C, and the circle TS continued round in τ oppoſite to T; and let the incident rays which diverge from or converge towards $\mathcal{Q}$, belong to the focus $\varkappa$ after reflection from the neareſt points to γ; and according as the perpendicular $E\mathcal{Q}$ is longer or ſhorter than ET, with the Focus E and tranſverſe axis $q\varkappa$ let an ellipſis or an hyperbola $qp\varkappa\varpi$ be deſcribed, and let it cut any line EP produced in p and ϖ, and the reflecting circle in A and α; then the conick arch pq will be the image of the object $P\mathcal{Q}$, formed by rays reflected from the arch AC; and the conick arch $\varpi\varkappa$ the image of the ſame object $P\mathcal{Q}$, formed by rays reflected from the oppoſite arch $\alpha\gamma$: and the whole conick ſection will be the image of the infinite line $\Omega P\mathcal{Q}Z$ formed by reflections from the whole circle.

435. When the perpendicular $E\mathcal{Q}$ is equal to ET, the ellipſis or the hyperbola will be changed into a parabola, whoſe focus is E and whoſe vertex is $\varkappa$; and when $E\mathcal{Q}$ is increaſed to infinity, the ellipſis will coincide with the circle whoſe diameter is $T\tau$ or $z\omega$; which is the parameter of all the curves.

436. Thoſe that are converſant in the analyſis of geometrick Places may ſoon be ſatisfied of the truth of theſe conſtructions and of all that follow, by ſuppoſing a focus of incident rays P to move along the line $P\mathcal{Q}$, and by ſeeking the geometrick place deſcribed by its conjugate focus p, while the line PE is revolving about the center E; or they may ſee a demonſtration of the like conſtructions of moſt of theſe caſes in the 17th and 18th of Dr. *Barrow*'s Optick Lectures, who firſt diſcovered this remarkable figure of the image of a ſtraight line.

437. Other things remaining as in Art. 245, now let the object $P\mathcal{Q}$ be a ſtraight line perpendicular to a line $\mathcal{Q}Eq$ drawn through E the center of a refracting ſphere; and let q be the focus of a ſlender pencil of rays which before their refractions through the ſphere diverged from $\mathcal{Q}$, and $\varkappa$ the focus of another pencil of rays which before their refractions through the ſphere converged towards $\mathcal{Q}$; and according as the perpendicular $E\mathcal{Q}$ is longer or ſhorter than EF, the focal diſtance of the ſphere, with the focus E and tranſverſe axis $q\varkappa$ let an ellipſis or an hyperbola $qp\varkappa\varpi$ be deſcribed, and let it cut any line EP produced in p and ϖ; and the conick arch pq will be the image of the object $P\mathcal{Q}$, formed by rays diverging from $P\mathcal{Q}$; and the oppoſite conick arch $\varpi\varkappa$ the image of the ſame $P\mathcal{Q}$, formed by rays converging towards $P\mathcal{Q}$; and the whole conick ſection will be the image of the infinite line $\Omega P\mathcal{Q}Z$.

And by refractions. Fig. 130. 131. 132.

438. When the perpendicular $E\mathcal{Q}$ is equal to EF, the ellipſis will be changed into a parabola, whoſe focus is E and whoſe vertex is $\varkappa$; and when $E\mathcal{Q}$ is increaſed to infinity, the ellipſis will coincide with the circle whoſe diameter is Ff or $z\omega$; which is the parameter of all the curves.

439. Hence when the angle $PE\mathcal{Q}$, which a ſtraight object ſubtends at the center of a thin lens, is but ſmall, the image of it coincides very nearly with the arch of a conick ſection, determinable in the ſame manner as for a ſphere.

Becauſe

Becauſe the relation of the conjugate focus's $\mathcal{Q}$, q is given by the very ſame proportion in both caſes.

g. 133. 440. Now let the rays that diverge from $\mathcal{Q}$ be but once refracted through a ſpherical ſurface AC, and then belong to the focus q; and ſuppoſing this ſurface continued round till it cuts the axis again in c, let other rays which converge towards $\mathcal{Q}$ be refracted at c only, and then belong to the focus $\varkappa$; and according as $E\mathcal{Q}$ is longer or ſhorter than EF, with the focus E and tranſverſe axis $q\varkappa$ let an ellipſis or an hyperbola $qp\varkappa$ be deſcribed, and let it cut any line PE produced in p; and the conick arch pq will be the image of the perpendicular $P\mathcal{Q}$, formed by rays diverging from it and once refracted at the arch AC.

g. 133. 134. 441. When $E\mathcal{Q}$ is equal to EF the ellipſis will become a parabola whoſe focus is E and vertex $\varkappa$; and when $E\mathcal{Q}$ is infinite the ellipſis will coincide with the circle whoſe diameter is $2Ef$ or $f\varphi$ or $\varkappa\omega$; which is alſo the parameter of all the curves.

PROPOSITION I.

Having the focus of rays falling almoſt perpendicularly upon a given ſpherical ſurface, to find their focus after refractions.

lgebraick heorems r finding ocus's.

ig. 135. 442. Let OI be the given ſurface whoſe center is S, and in any radius OS produced let $\mathcal{Q}$ be the given focus of the incident rays as $\mathcal{Q}I$; it is required to find the focus q of the refracted rays. Call OS, $O\mathcal{Q}$, Oq reſpectively S, $\mathcal{Q}$, q; and let the given ratio of the ſines of refraction be m to n, and let m be bigger than n. Then joining SI, ſince very ſmall angles are very nearly proportionable to their ſines, we Art. 221. have the ang. OSI : ang. $SI\mathcal{Q}$ :: $\mathcal{Q}$: $\mathcal{Q}-S$ *, and the ang. $SI\mathcal{Q}$: ang. SIq :: m : n. And by compounding theſe proportions we have the ang. OSI : ang. SIq :: $m\mathcal{Q}$: $n\mathcal{Q}-nS$; and Eucl. I. 2. disjointly we have the ang. OSI : ang. SqI *, Art. 221. 04. that is, q : S * :: $m\mathcal{Q}$: $m-n$, $\mathcal{Q}+nS$. Whence (putting $\theta=m-n$) we have this firſt Theorem,

$$q=\frac{m\mathcal{Q}S}{\theta\mathcal{Q}+nS}=\frac{\frac{m}{\theta}S\mathcal{Q}}{\mathcal{Q}+\frac{n}{\theta}S}.$$

443. This is the value of q in the given caſe where the lines $O\mathcal{Q}$, OS, Oq lye all on the ſame ſide of the ſurface OI; and thence the theorem for q may be eaſily adapted to any other given caſe, by conſidering $O\mathcal{Q}$ as being always affirmative, and by changing the ſine of S when OS, and $O\mathcal{Q}$ lye on contrary ſides of their origin O; and by changing the ſign of θ when the ſine of incidence is leſs than the ſine of refraction; and laſtly by placing Oq the contrary way to $O\mathcal{Q}$ when the value of q comes out negative by the theorem ſo changed.

444. *Corol.* 1. When the line S becomes infinitely great, the ſurface OI will become a plane; and then we have $q=\frac{m}{n}\mathcal{Q}$, (becauſe, in the theorem, the limit of the variable ratio of $\frac{m}{\theta}S$ to $\mathcal{Q}+\frac{n}{\theta}S$ is m to n.) Therefore qO : $\mathcal{Q}O$ :: m : n, that is, as the ſine of incidence, to the ſine of refraction; which is the rule in Art. 223.

445. *Corol.* 2. When the line $\mathcal{Q}$ is infinite, let the point q come to t; and then the line q or t $=\frac{m}{\theta}S$, (becauſe, in the theorem, the variable ratio of $\mathcal{Q}$ to $\mathcal{Q}+\frac{n}{\theta}S$ becomes a ratio of equality.) Alſo when q is infinite and conſequently $\mathcal{Q}+\frac{n}{\theta}S=o$ *, let the point $\mathcal{Q}$ come round to $\mathcal{T}$; then the line $-\mathcal{Q}$ or $\mathcal{T}$ $=\frac{n}{\theta}S$. But $t-S=(\frac{m}{\theta}S-S=)\frac{n}{\theta}S$. And ſo the continuations St, $O\mathcal{T}$ of SO to the principal focus's t, $\mathcal{T}$ are equal to each other; and are ſeverally in proportion to SO, as n to θ or $m-n$; which is the rule in Art. 224.

* Rem. 442.

446. *Corol.* 3. In the theorem ſubſtitute $\mathcal{T}$ and t for their values juſt now found, and we have $q=\frac{t\mathcal{Q}}{\mathcal{Q}+\mathcal{T}}$. Hence $\mathcal{Q}-q=(\mathcal{Q}-\frac{t\mathcal{Q}}{\mathcal{Q}+\mathcal{T}}$ $=\frac{\mathcal{Q}+\mathcal{T}-t}{\mathcal{Q}+\mathcal{T}}\mathcal{Q}=)\frac{\mathcal{Q}-S}{\mathcal{Q}+\mathcal{T}}\mathcal{Q}$. Therefore the line $\mathcal{Q}\mathcal{T}$: $\mathcal{Q}O$ :: $\mathcal{Q}S$: $\mathcal{Q}q$, which is the rule in Art. 239.

447. *Corol.* 4. In the Theorem ſubſtitute $-m$ for n, and conſequently $2m$ for θ, and we have $q=\frac{\frac{1}{2}S\mathcal{Q}}{\mathcal{Q}-\frac{1}{2}S}$; which theorem gives the focus of rays reflected from the ſpherical ſurface OI. For the calculation continues the ſame whether the ray goes forwards or backwards in the line Iq; and to change the angle of refraction SIq into an angle of reflection, it (and its ſine n) muſt be diminiſhed to nothing and then become negative and equal to ($-m$ the ſine of) the angle of incidence $SI\mathcal{Q}$.

Theorem for reflected rays. Fig. 136. Fig. 135. 136.

448. *Corol.* 5. Hence when S becomes infinite, the ſurface OI will become a plane; and then we have $q=-\mathcal{Q}$; which is the rule in Art. 202. Fig. 136.

449. *Corol.* 6. When $\mathcal{Q}$ is infinite let the point q come to $\mathcal{T}$; then the line q or $\mathcal{T}$ $=\frac{1}{2}S$, which is the rule in Art. 205.

450. *Corol.* 7. In the theorem of corol. 4. ſubſtitute $\mathcal{T}$ for its value found in the laſt corol.

and we have $q = \frac{TQ}{Q-T}$. Hence $q-T = \frac{TQ}{Q-T} - T = \frac{TT}{Q-T}$; that is, TQ, TS, Tq are continual proportionals; which is the rule in Art. 207.

PROPOSITION II.

Having the focus of rays falling almoſt perpendicularly upon a given lens, to find their focus after refractions.

Fig. 137. 451. Let $OIEo$ be the given lens, whoſe vertex's are O and o; R the center of the firſt ſurface OI; r the center of the ſecond oE; P the given focus of incident rays in the axis $oOrR$ (produced,) and p the focus of the emergent rays required. Let ϖ be their focus after the firſt refraction at the ſurface OI, and m to n the ratio of the fines as above, and call Oo, or, OR, OP, op reſpectively o, r, R, P, p. Then for Q, S, m, n, θ in Theorem 1*, write P, R, m, n, θ, and we have $O\varpi = \frac{mPR}{\theta P + nR}$, to which adding Oo or o, we have $o\varpi$ or $\varpi = \frac{mPR + \theta Po + nRo}{\theta P + nR}$.

* Rem. 442.

* Rem. 442. Again, for Q, S, m, n, θ in Theorem 1*, write ϖ, r, n, m, $-\theta$; and we have $p = \frac{nr\varpi}{-\theta\varpi + mr}$; in which by ſubſtituting the value of ϖ, we have $p = \frac{mnPRr + n\theta Pro + nnRro}{m\theta Pr - m\theta PR + mnRr - \theta\theta Po - n\theta Ro}$.

General theorem for any lens. 452. This theorem for a meniſcus lens, having its concave ſurface expoſed to P, is eaſily adapted to a lens of any given form, by conceiving one or both its ſemidiameters OR, *or* to decreaſe or to increaſe or to become infinite and then negative, till the meniſcus acquires the form of the given lens; and by changing the ſign of R or r when the ſemidiameters lye on oppoſite ſides of their ſurfaces O, o to the focus P; and laſtly by placing p on the oppoſite ſide of o to P, when its value in the theorem ſo changed comes out negative.

453. Thus by writing ∞ (which ſignifies infinite) for R, this theorem is eaſily adapted to a plano-convex lens, having its firſt ſurface plane; and by writing $-R$ for R it is adapted to a double convex lens; and by writing $-R$ for R and ∞ for r, it is adapted to a plano-convex lens whoſe firſt ſurface is convex; and by writing $-R$ for R, and $-r$ for r, and $o+r$ for R it is adapted to a lens of concentrick ſurfaces, whoſe firſt ſurface is convex: and by writing $R+o$ for r, it is adapted to a lens of concentrick ſurfaces whoſe firſt ſurface is concave; and by writing ∞ for r it is adapted to a plano-concave lens whoſe firſt ſurface is concave; and by writing $-r$ for r, it is adapted to a double concave; and by writing ∞ for R and $-r$ for r, it is adapted to a plano-concave whoſe firſt ſurface is plane; and laſtly it is adapted to a ſphere whoſe ſemidiameter is R and diameter Oo, by writing $-R$ for R, and R for r, and $2R$ for o; and by ſubſtituting given numbers for the ratio of refraction, the bigger for m and the leſſer for n, it is adapted to lens's of any given ſubſtances.

454. If all the quantities in the theorem be given in magnitude and poſition except one, this one, whatever it be, may be found by the known rules of algebraick reductions. And thus alſo the ratio of the fines of incidence and refraction may be found, by ſubſtituting an unite either for n or for m, and by ſeeking the value of the other. I think the firſt general theorem of this kind was publiſhed by Dr. *Halley** and looked upon as a notable inſtance of the great uſe and comprehenſiveneſs of algebraick Theorems.

* Phil. Tranſ. No 205.

455. *Corol.* 1. Since the thickneſs of a lens for moſt optical uſes is generally but ſmall in compariſon to the ſemidiameters of its ſurfaces; and ſince the accuracy of this theorem depends on the ſmalneſs of the angles of incidence and refraction, that is, *cæteris paribus*, on the ſmalneſs of the breadth and conſequently of the thickneſs of the lens; the theorem will ſtill be ſufficiently accurate though we reject all its terms multipyed by o, as inconſiderably ſmall in compariſon to the reſt; which being all divided firſt by m and then by $\theta r - \theta R$, will give $p = \frac{nPRr}{\theta Pr - \theta PR + nRr} = \frac{\frac{n}{\theta} \times \frac{Rr}{r-R} \times P}{P + \frac{n}{\theta} \times \frac{Rr}{r-R}}$.

456. *Corol.* 2. When the line p is infinite, let the point P come to F; then the line $F = -\frac{n}{\theta} \times \frac{Rr}{r-R}$: Likewiſe when the line P is infinite, let the point p come round to f; then the line $-p$ or $f = \frac{n}{\theta} \times \frac{Rr}{r-R}$. This will appear by the reaſons given in Rem. 444, 445. Therefore the focal diſtances OF, of are equal; and $r - R : r :: \frac{n}{\theta} R : F$ or f; which is the rule in Art. 232, becauſe $\frac{n}{\theta} R$ is the Continuation of R mentioned in that article.

457. *Corol.* 3. In the theorem of corol. 1. ſubſtitute F for its value found in corol. 2. and we have $p = \frac{-FP}{P-F} = \frac{FP}{F-P}$. Hence $p - P = \frac{FP}{F-P} - P = \frac{PP}{F-P}$; therefore the lines PF, PO, Pp are continual proportionals, of which the

the first and third tend the same way from P; which is the rule in Art. 239.

458. *Corol.* 4. Hence $p+f=\frac{FP}{F-P}+F=\frac{FF}{F-P}$; therefore $PF:FO::of:fp$; which is the rule in Art. 236.

Focus of an oblique pencil considered. Fig. 138.

459. *Corol.* 5. If Q the focus of incident rays be placed at any small distance from either side of the axis of a lens of any thickness, the focus q of the emergent rays may be found in this manner. Through Q and R draw a line QRI perpendicular to the first surface; and the focus x after the first refraction will therefore be in some point of it. Again through x and r drawing xrE perpendicular to the second surface, the focus q after the second refraction will be in some point of this perpendicular. And thus the successive focus's x and q might be found by the theorem in Rem. 442.

460. *Corol.* 6. But supposing two focus's P and Q to be at equal distances from the first surface, their conjugate focus's p and q will be at other equal distances from the second surface very nearly, provided the interval between the focus's P, Q, or rather the angle PRQ, be very small. For since we suppose $OP=IQ$, we have $O\varpi=Ix$ and consequently $R\varpi=Rx$; and by adding Rr to both, we have $r\varpi=(rR+Rx=)rx$ very nearly, by reason of the smalness of the angle ϖRx; and consequently $o\varpi=Ex$ very nearly, and therefore $op=Eq$ very nearly.

461. *Corol.* 7. Therefore a small object PQ perpendicular to the axis of any lens, will also have its image pq very nearly perpendicular to the axis. Images determined.

462. *Corol.* 8. Hence the small object PQ will be to its image pq, in the given ratio of $\overline{\theta P+nR}, r$ to $\overline{\theta p+nr}, R$ very nearly. For joining ϖx, the figures PRQ, ϖRx will be very nearly similar; and also the figures ϖrx, prq. And since we had $OT=\frac{n}{\theta}R$*, and $PT:PO::PR:P\varpi$*, disjointly we have $PT:TO::(PR:\varpi R::)PQ:\varpi x$. Again taking $ot=\frac{n}{\theta}r$, since t will be the focus of parallel rays, after refraction at oE only, that come the contrary way to those which we may suppose to converge towards p, we have also $pt:po::pr:p\varpi$*, and disjointly $ot:pt::(\varpi r:pr::)\varpi x:pq$. And by compounding these two proportions we have $PQ:pq::(PT\times to:pt\times TO::P+\frac{n}{\theta}R\times\frac{n}{\theta}r:p+\frac{n}{\theta}r\times\frac{n}{\theta}R::)\overline{\theta P+nR}, r:\overline{\theta p+nr}, R$. Here I conclude the algebraic Elements of Opticks.

* Rem. 445.
* Rem. 446.
* Rem. 446.

Upon Chapter 5. Appearances through any number of lenses &c.

Upon Art. 256.

463. Line 16. *Hence it is easy to collect* &c.
For we had

$$fA=\frac{OA,a}{\overline{OA+a}}$$

$$gB=\frac{\overline{fA+AB}\times b}{\overline{fA+AB}+b}=\frac{\overline{OA,a+\overline{OA+a},AB}\times b}{\overline{OA,a+\overline{OA+a},AB+\overline{OA+a},b}}=\frac{nb}{m}\text{ for shortness.}$$

$$bC=\frac{\overline{gB+BC,c}}{\overline{gB+BC}+c}=\left(\frac{\frac{n}{m}\overline{b+BC,c}}{\frac{n}{m}b+BC+c}=\right)\frac{\overline{nb+mBC,c}}{\overline{nb+mBC}+mc}$$

Hence

$$\frac{AB}{fA}=\frac{\overline{OA+a}}{\overline{OA,a}}AB$$

$$\frac{BC}{gB}=\frac{m}{nb}BC.$$

$$\frac{CP}{bC}=\frac{\overline{nb+mBC}+mc}{\overline{nb+mBC,c}}CP$$

But we had

$$O\Pi = OA \times 1 + \overline{\frac{AB}{fA}} \times 1 + \overline{\frac{BC}{gB}} \times 1 + \overline{\frac{CP}{bC}} \times \text{\&c. in the present article.}$$

Therefore

$$O\Pi' \text{ or } O\beta = OA \times 1 + \overline{\frac{AB}{fA}} = OA \times 1 + \overline{\frac{OA+a}{OA,a}\,AB} = OA + \frac{OA+a}{a}\,AB = \frac{n}{a}.$$

$$O\Pi'' \text{ or } O\gamma = OA \times 1 + \overline{\frac{AB}{fA}} \times 1 + \overline{\frac{BC}{gC}} = \frac{n}{a} \times 1 + \overline{\frac{BC}{gB}} = \frac{n}{a} \times 1 + \overline{\frac{m}{nb}BC} = \frac{n}{a} + \frac{m}{ab}BC.$$

$$O\Pi''' \text{ or } O\delta = OA \times 1 + \overline{\frac{AB}{fA}} \times 1 + \overline{\frac{BC}{gC}} \times 1 + \overline{\frac{CP}{bC}} = \frac{n}{a} + \frac{m}{ab}\,BC \times 1 + \frac{CP}{bC} =$$

$$= \frac{nb+mBC}{ab} \times 1 + \overline{\frac{nb+mBC+mc}{nb+mBC,c}\,CP} = \frac{nb \times mBC}{ab} + \frac{nb+mBC+mc}{abc}CP$$

$$= \frac{n}{a} + \frac{m}{ab}\,BC + \frac{m}{ab}\,CP + \frac{n}{ac}\,CP + \frac{m}{abc}\,BCP.$$

Reſtore the values of m and n, then

$$O\beta = OA + \frac{OAB}{a} + AB = OB + \frac{OAB}{a}.$$

$$O\gamma = OB + \frac{OAB}{a} + \left(\frac{m}{ab}BC = \right) \frac{OA,BC}{b} + \frac{OA,AB,BC}{ab} + \frac{AB,BC}{b} + \frac{OA,BC}{a} + BC =$$

$$= OC + \frac{OAC}{a} + \frac{OBC}{b} + \frac{OABC}{ab}.$$

$$O\delta = OC + \frac{OAC}{a} + \frac{OBC}{b} + \frac{OABC}{ab} + \left(\frac{m}{ab}\,CP + \frac{n}{ac}\,CP + \frac{m}{abc}BCP = \right) \text{\&c.}$$

Upon ART. 270.

464. The diſtance Ob is here ſuppoſed to be finite in compariſon to OP and bP. For if Cb was not finite, the ratio of the angles AOr, Cbt could not be finite. This and the following article may be demonſtrated ſeveral other ways, which for ſhortneſs I omit.

After ART. 280.

465. That noble and beautiful theorem in Art. 249, from which I deduced all theſe corollaries, was the laſt invention of that great Mathematician Mr. *Cotes*, juſt before his death at the age of 32: upon which occaſion I am told Sir *Iſaac Newton* ſaid, *if* Mr. COTES *had lived we might have known ſomething*. His demonſtration of it is likewiſe ſo elegant and clear, that it highly deſerved to have followed the theorem it ſelf. But ſince the method of it cannot eaſily be extended to the like general Theorems which I have given in the next propoſitions, for the ſake of uniformity I was obliged to ſubſtitute another in its ſtead, and to give Mr. *Cotes*'s by it ſelf, as follows. Where the reader need only obſerve, that what he calls the apparent place of the object, is only the real place of its laſt image, according to the received opinion and language of Opticians at that time.

PROPOSITION.

"466. *To find the apparent magnitude, ſituation, apparent place and degree of diſtinctneſs with which an object is ſeen through any number of glaſſes of any ſort, at any diſtances from each other and from the eye and object.*

467. Let PM be an object viewed by the eye at O through any number of glaſſes at A, B, C whoſe focal diſtances are the lines a, b, c. The diſtance OP may be conſidered as divided by the glaſſes A, B, C into two parts ſuch as PA, AO; PB, BO; PC, CO; or into three parts ſuch as PA, AB, BO; PA, AC, CO; PB, BC, CO; or into four parts ſuch as PA, AB, BC, CO; and ſo on as far as the number of glaſſes permits. All the ſeveral products of ſuch correſponding parts applyed reſpectively to the focal diſtance, or to the product of the focal diſtances Fig. 135.

ſtances of the glaſſes which are placed at the point or points of diviſion, will give ſo many ſeveral lines, which muſt be looked upon as negative if there be an odd number of convex glaſſes at the points of diviſion, otherwiſe as affirmative. Let $P\Omega$ be the ſum of PO and thoſe ſeveral lines according to their ſigns, and let $P\Omega$ and PO lye the ſame way if the ſum be affirmative, but contrary ways if the ſum be negative, and Ω will be the point at which the naked eye being placed ſhall ſee the object under the ſame magnitude with which it appears through all the glaſſes the eye being at O, and therefore the apparent magnitude of the object will bear the ſame proportion to the true * magnitude, as the diſtance PO bears to the diſtance $P\Omega$.

* Art. 100.

468. The apparent ſituation of the object is alſo determined by the point Ω. For if Ω and O be placed on the ſame ſide of the object, it will appear erect, otherwiſe inverted.

469. Imagine now the eye to be removed from O to C, ſo that its diſtance from the laſt glaſs may vaniſh, at the ſame time the point Ω will move to another place ω, which may be found as above. Let $O\gamma$ bear the ſame proportion to OC which ΩP bears to $\Omega\omega$, and let the order of the points O, C, γ be the ſame as the order of the points Ω, ω, P, then will γ be the apparent place of the object viewed at O through all the glaſſes.

470. And from the ſituation of this point γ a judgment may be formed of the degree of diſtinctneſs with which the object appears. For the rays flowing from the point P, by paſſing through the glaſſes, are diſpoſed to fall upon the eye in the ſame manner, as if the glaſſes being removed they tended from the point γ when it is before the eye, or towards the point γ when it is behind the eye.

Demonſtration. Fig. 140.

471. For let αr be the firſt image of the object made by the glaſs A; βs its ſecond image made by the glaſs B; γt its third image made by the glaſs C. It is evident that the object PM and its image αr will be terminated by the ſame lines $PA\alpha$, MAr *; that the image αr and its image βs will be terminated by the ſame lines $\alpha B\beta$, rBs; that the image βs and its image γt will be terminated by the ſame lines $\beta C\gamma$, sCt. Now if the eye be placed at O and γt be the laſt image, it is manifeſt that the object ſeen through all the glaſſes will appear under the ſame angle which is really ſubtended by the image γt *. Draw therefore $M\Omega$ parallel to tO making the angle $P\Omega M$ equal to γOt, and the naked eye being placed at Ω ſhall ſee the object PM under the ſame magnitude with which it appears through all the glaſſes, the eye being at O. And conſequently the apparent magnitude ſhall be to the true magnitude, as the angle $P\Omega M$ to the angle POM *, or as the diſtance PO to the diſtance $P\Omega$ *. Let us at firſt ſuppoſe all the glaſſes to be concaves. The diſtance $P\Omega$ will be to PA, as the angle PAM to the angle $P\Omega M$, or as αAr to γOt, or in the compounded ratio of αAr to αBr, βBs to βCs, γCt to γOt, that is in the compounded ratio of $\alpha A + AB$ to αA *, $\beta B + BC$ to βB, $\gamma C + CO$ to γC. Therefore $P\Omega = PA \times \frac{\alpha A + AB}{\alpha A} \times \frac{\beta B + BC}{\beta B} \times \frac{\gamma C + CO}{\gamma C}$ &c; by which theorem the diſtance $P\Omega$ is given ſo ſoon as αA, βB, γC, can be found. Theſe may be found by Art. 239 as follows, $\alpha A = \frac{PA \times a}{PA + a}$, $\beta B = \frac{\overline{\alpha A + AB} \times b}{\alpha A + AB + b}$, $\gamma C = \frac{\overline{\beta B + BC} \times c}{\beta B + BC + c}$. Whence it is eaſy to conclude * that if the eye at B views the object PM through one glaſs A, $P\Omega$ will be equal to $PB + \frac{PAB}{a}$; that if the eye at C views the object PM through two glaſſes A and B, $P\Omega$ will be equal to $PC + \frac{PAC}{a} + \frac{PBC}{b} + \frac{PABC}{ab}$; that if the eye at O views the object PM through three glaſſes A, B and C, $P\Omega$ will be equal to $PO + \frac{PAO}{a} + \frac{PBO}{b} + \frac{PCO}{c} + \frac{PABO}{ab} + \frac{PACO}{ac} + \frac{PBCO}{bc} + \frac{PABCO}{abc}$; and ſo forwards continually as the ſolution of the problem directs. Now if any of the glaſſes be convex, the focal diſtances of ſuch glaſſes muſt be looked upon as negative, ſince they are contrary to thoſe of concave glaſſes *. and therefore the terms which involve an odd number of convex glaſſes at the points of diviſion muſt be taken as negative. *Q. E. D.* 1°.

* Art. 105.

* Art. 108.

* Art. 222.

* Art. 222.

* As in Rem. 463.

* Art. 230.

472. The point M is ſeen immediately through the glaſſes by the ray tO, which enters the eye at O in a direction parallel to $M\Omega$ by conſtruction. If therefore Ω falls on the ſame ſide of the object as O, the ray tO muſt advance towards the eye from the ſame ſide of the common axis OP as the point M, and conſequently the object will appear through the glaſſes erect. But if Ω falls on the contrary ſide of the object to the point O, the ray tO which advances towards the eye in a direction parallel to ΩM, muſt appear to come from the contrary ſide of the axis to the point M, and conſequently the object will appear through the glaſſes inverted. *Q. E. D.* 2°.

473. By conſtruction the lines $M\Omega$, $M\omega$, MP, $P\Omega$ are reſpectively parallel to the lines tO, tC, $t\gamma$, γO and therefore the figure $M\Omega\omega P$ is

is similar to the figure $tOC\gamma$. Hence it follows that $O\gamma$ the distance of the last image from the eye, or the distance of the apparent place of the object from the eye, is to OC as ΩP is to $\Omega\omega$; and that the order of the points O, C, γ will be the same as the order of the points Ω, ω, P. Q. E. D. 3°.

474. The point γ being the last image of the point P, the rays which flow from P, after they have passed through all the glasses, will flow from or towards the point γ. *Q. E. D.* 4°." So far Mr. *Cotes*.

475. Mr. *Huygens* has demonstrated a good many cases of this general theorem in as many distinct propositions of his Dioptricks* ; which is more than I have found in any other author. But notwithstanding his great invention and accuracy in geometry, he has generally embarrassed his readers with so many formal compositions and resolutions of ratio's, that they cannot have a better idea of the excellency of Mr. *Cotes*'s theorem, than by comparing it with those of this other great Geometer.

* Prop. 36 to 47.

Upon Chapter 6. *Aberrations of rays from the geometrical focus.*

Upon ART. 339.

The density of rays in the focus of a speculum. FIG. 382. Plate 30.

476. Hence supposing the density of the reflected rays in the circle of aberrations to be uniform, it is to the density of the incident rays falling perpendicularly upon a plane AP, as the whole surface of the sphere of which the speculum is a portion, to the area of a circle whose diameter is the versed sine PC of the small arch AC very nearly, and the more exactly as this arch is smaller; supposing also that all the incident rays are reflected.

477. For since the very same rays pass through two circles described by the lines AP and XY turned about EC; their densities in these circles are reciprocally as the circles themselves; that is, the density of the reflected rays, is to the density of the incident rays, as AP^2 to XY^2 * or $\frac{1}{16} FG^2$ * or $\frac{AP^6}{16 \times 4CE^4}$ *; that is, putting D for $2CE$, as $4D^4$ to AP^4; that is, as $4D^2$ to PC^2, (because D, AP, PC are very nearly continual proportionals * ;) that is, as the area of 4 great circles of the sphere, or the whole surface of the sphere *, to the area of a circle whose diameter is PC very nearly.

* Eucl. XII. 2.
* Art. 339.
* Art. 337.
* Eucl. VI. 8. Cor.
* Archim. de Sph. & Cyl.

478. Therefore the greatest density of the reflected rays is at the focus F, considered as a physical point; and is immensely greater than the density of the incident rays. For the proposition above becomes geometrically exact when AP is infinitely diminished, and XY comes to its limit at F; and the density at F is always the same whether a slender pencil falls upon the speculum or a large one, because the outward rays are reflected wide of the focus F.

Density of rays in the focus of a lens. FIG. 381. Plate 30.

479. In like manner when rays fall parallel upon the plane side of a plano-convex lens, (putting m to n for the ratio of majority of the sines of incidence and refraction) their greatest density at their focus F, is to the density of the incident rays, as the whole surface of a sphere whereof the lens is a portion, to the area of a circle whose diameter is $\frac{mm}{nn} PC$, or in glass $\frac{2}{4}$ of the versed sine of the smallest aperture of the lens; that is immensely great. It follows from Art. 332.

480. Therefore the density of reflected or refracted rays in the several points of an image of a very remote object, is also immensely greater than the density of the incident rays of any one pencil. For it would be immensely great, if all the rays of every pencil were rejected, except a few that go near to their axes, and those outward rays being scattered upon points collateral to each point of the image, help to increase the density of the rays in the whole image.

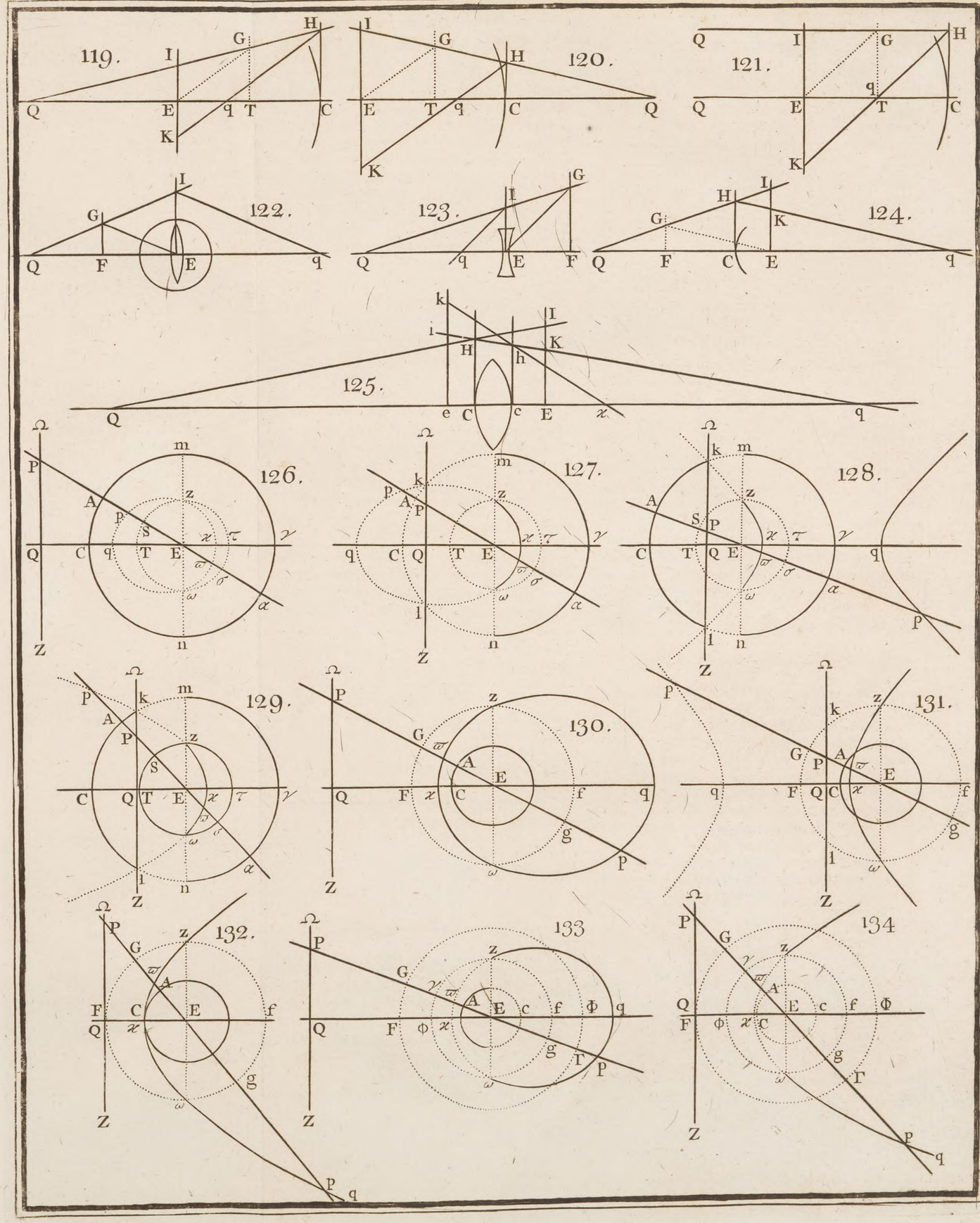
481. I had almost forgot to acknowledge that I borrowed the first and fourth propositions of this chapter from Sir *Isaac Newton*'s Optical Lectures, Part 1. Sect. 4. where the second is demonstrated by his method of infinite series.

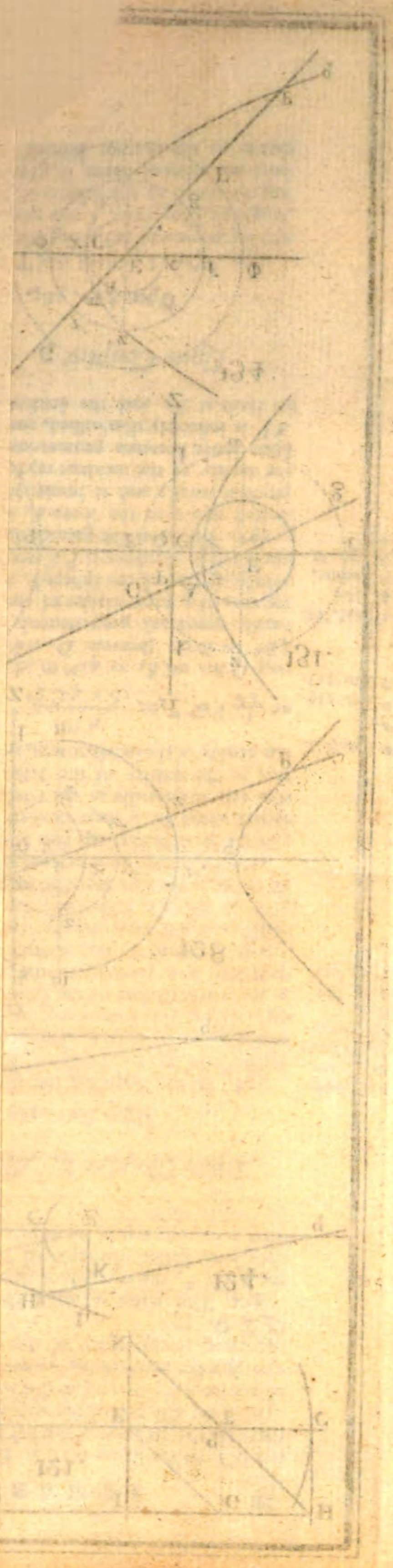
Upon Chapter 7. *Of the magnifying powers of Telescopes &c.*

Upon ART. 364, 365.

482. When I printed this Table of the magnifying powers of refracting telescopes from Mr. *Huygens*'s Dioptricks, I was not aware of its being calculated by the Editors of that work; who have not allowed them so great a magnifying power as the Author himself intended, or as the best object-glasses now made will admit of. For the Author (in his *Astroscopia Compendiaria*) mentions an object-glass of 34 feet focal distance, which in astronomical observations bore an eye-glass of $2\frac{1}{2}$ inches focal distance *, and consequently magnified 163 times. According to this standard, a telescope of 35 feet ought to magnify 166 times, and of one foot, 28 times; whereas

* Art. 909.


119.
120.
121.
122.
123.
124.
125.
126.
127.
128.
129.
130.
131.
132.
133
134



whereas the table allows but 118 times to the former and but 20 to the latter. Now $\frac{166}{118}$ or $\frac{28}{20} = 1,4$; by which if you multiply the numbers in the given column of magnifying powers, you will gain a new column, ſhewing how much thoſe object-glaſſes ought to magnify if wrought up to the perfection of this ſtandard.

The new apertures and eye-glaſſes muſt alſo be taken in the ſame proportions to one another as the old ones have in the table; or the eye-glaſſes may be found by dividing the length of each teleſcope by its magnifying power. And thus a new table may eaſily be made for this or any other more perfect ſtandard when offered.

Admirable performance of reflecting teleſcopes.

483. As an undoubted evidence of the great excellency of reflecting teleſcopes, I will add, that upon a juſt compariſon, made by Mr. *Bradley Savilian* Prof. of Aſtron. and the late Dr. *Pound*, of Mr. *Hadley*'s teleſcope, whoſe focal diſtance is not quite 5 feet and $\frac{1}{4}$, with the *Hugenian* object-glaſs, whoſe focal diſtance is 123 feet; they found that the former magnified objects as many times as the latter, and repreſented them as diſtinct, though not altogether ſo clear and bright; which they attributed partly to the difference of their apertures, (that of the *Hugenian* being ſomewhat the larger) and partly to ſeveral little ſpots in the concave ſurface of the object-metal which did not admit of a good poliſh. But, notwithſtanding this difference in the brightneſs of the objects, they were able, with this reflecting teleſcope, to ſee whatever they had diſcovered by the *Hugenian*: particularly the tranſits of *Jupiter*'s ſatellites, and their ſhades, over the diſk of *Jupiter*, the black liſt in *Saturn*'s ring, (an argument of its being double,) and the edge of the ſhade of Saturn caſt on his ring *, and laſtly the five ſatellites of Saturn; in viewing of which, this teleſcope had the advantage of the *Hugenian* at the time of compariſon; which being in ſummer, and the *Hugenian* teleſcope being managed without a tube * the twilight hindered them from ſeeing in this, ſome of thoſe ſmall objects, which at the ſame time they could diſcern with the reflecting teleſcope. This account was ſent by Dr. *Pound* to Dr. *Jurin*. Phil. Tranſ. No. 378.

See FIG. V. Phil. Tranſ. No. 376.

Art. 892.

484. Notwithſtanding this admirable performance of Mr. *Hadley*'s teleſcope, which magnified between 228 and 230 times, I am well aſſured that an object-metal of 3 feet and $\frac{1}{4}$ focal diſtance was wrought by Mr. *Hauksbee* to ſo great a perfection as to magnify 226 times; and therefore ſeems to be ſcarce inferior to Mr. *Hadley*'s of 5 feet $2\frac{1}{2}$ inch focal diſtance; ſince with the ſame eye-glaſs that gave it this power, it ſhewed not only the minute parts of the new moon exceedingly diſtinct, but alſo the belts of Jupiter and the black liſt or diviſion of Saturn's ring. For the latter objects it bore an aperture of $3\frac{1}{2}$ or 4 inches; and in cloudy weather ſhewed land-objects beſt when the whole ſurface of the ſpeculum was open, whoſe breadth was $4\frac{1}{2}$ inches.

I computed my table of the magnifying powers of reflecting teleſcopes from the module of Mr. *Hadley*'s, uſing its middle ſized eye-glaſs, and aperture, long before I had heard of Mr. *Hauksbee*'s. But if this be taken for a new ſtandard, it follows, from Art. 361, that a ſpeculum of one foot focal diſtance ought to magnify 93 times, whereas our table allowed it but 60. Now $\frac{93}{60} = 1.55$, and the given column of magnifying powers multiplyied by this number gives a new column, ſhewing how much the object-metals ought to magnify if wrought up to the perfection of Mr. *Hauksbee*'s. And thus a new table may eaſily be made for this or any other more perfect ſtandard; taking alſo the new eye-glaſſes and apertures in the ſame ratio's to one another as the old ones have in the preſent table.

To find by experiment how much a teleſcope magnifies.

485. The magnifying power of Mr. *Hauksbee*'s teleſcope was determined by the following experiment made by himſelf, Mr. *Folkes* and Dr. *Jurin*. Having fixt a paper-circle of one inch diameter upon a wall, at the diſtance of 2674 inches from the eye-glaſs of the teleſcope, they viewed it in the teleſcope with one eye, while with the other eye naked, they viewed two parallel lines, drawn upon paper, 12 inches aſunder, moving them gradually to and fro' till they appeared to touch two oppoſite points of the circle ſeen in the teleſcope; and then the perpendicular diſtance of the lines from the eye was found to be 132 inches. The teleſcope being of Sir *Iſaac Newton*'s form, the obſerver was obliged to incline his head and neck in a poſture nearly horizontal and parallel to the length of the tube, that his naked eye might ſee the two lines croſs the under ſide of it.

In this poſition of the objects, the angle at the eye made by the rays that came from the extremities of the diameter of the one inch circle, was equal to the angle ſubtended at the other eye by the 12 inch interval of the parallel lines; and therefore the ratio of this angle to that which the ſaid circle would ſubtend at the naked eye viewing it at the ſaid diſtance of 2674 inches, is the magnifying power of the teleſcope; and is compounded of the direct ratio of the ſubtenſes of theſe angles, and the inverſe ratio of the diſtances of the ſubtenſes from the

the eye, that is, of 12 to 1 and 2674 to 142; which make the ratio of 226 to 1 very nearly.

486. Suppoſing a larger paper-circle had been placed at ſo great a diſtance, that its picture might have been formed by the ſpeculum in its principal focus, the teleſcope would have magnified it more than our one inch circle, in the ratio of the diſtance of this latter circle from the principal focus, to its diſtance from the center of the ſphere of the ſpeculum: becauſe the diameter of the picture of the remoter circle would have been greater, in this ratio, than that of our one inch circle, ſuppoſing theſe circles to ſubtend the ſame angle at the center of the ſpeculum. But this ratio in the preſent experiment being only 2674 to 2671, gives but an inconſiderable increaſe to that of the magnifying power already determined.

487. If this experiment ſhould be further ſuſpected of inaccuracy, becauſe the pictures of the objects upon the two retina's of the obſerver's eyes might poſſibly be unequal, I will ſhew that no inequalities of this kind can affect the concluſion. For let any ſtraight object be viewed with both eyes, and ſuppoſe the viſual angle at one of the eyes to be alſo ſubtended by any nearer object. Then it is plain, that the pictures of both objects in this one eye, will be exactly equal to one another, however they may differ from that of the remoter object in the other eye. And the nearer object will then cover the remoter in appearance. But if the magnitude or diſtance of the nearer object be ſo altered as to alter the viſual angle and picture upon the retina; this will of conſequence be perceived by a neceſſary alteration of the apparent magnitude of this object. Therefore when the apparent magnitudes of the two objects were equal, the viſual angles were alſo equal whether the pictures upon the two retina's were equal or not. The application of this hypotheſis to the experiment with the teleſcope I think is ſufficiently obvious.

Method of examining the goodneſs of teleſcopes.

488. Thus we have an eaſy and accurate method of examining the goodneſs of a teleſcope of any kind. Firſt by giving it the leaſt eye-glaſs that will ſhew the new moon, or rather Jupiter and Saturn with ſufficient light and diſtinctneſs when the air is quiet and pure; and then by finding how much it magnifies by the method above mentioned; by which it will appear how near it approaches to the perfection of thoſe ſtandards. But if ſeveral teleſcopes of the ſame kind have nearly the ſame length, or the ſame magnifying power though of different kinds, thoſe are the beſt in their kind with which you can read a given print at the greateſt diſtance. Experiments of this kind if made publick, would be very uſeful to the buyers of teleſcopes, and would excite workmen to excell one another: ſome few are mentioned, in the next remark.

Teleſcopes made with glaſs-ſpeculums.

489. The method of making reflecting teleſcopes with glaſs ſpeculums quickſilvered over on the convex ſide, was firſt recommended by Sir *Iſaac Newton* *, and has lately been executed with great ſucceſs at *Edinburgh*, as will appear by the following extract of a letter from my honoured friend Mr. *Mac Laurin*, the worthy Profeſſor of Mathematicks in that Univerſity.

* Opt. p. 94. 80.

" Mr. *Short* an ingenious perſon well verſed in the theory and practice of making teleſcopes, has improved the reflecting ones ſo much, that I am fully ſatisfied he has far outdone what has yet been executed in this kind. He has not only ſucceeded in giving ſo true a figure to his ſpeculums of glaſs quickſilvered behind, as to make the image from them perfectly diſtinct, but has made teleſcopes with metal ſpeculums, which far ſurpaſs thoſe I have ſeen of any other workman.

He has made ſix reflecting teleſcopes with glaſs ſpeculums, three of 15 inches focal diſtance and three of nine inches. One of the firſt is at preſent in my Lord *Ilay*'s hands; with which it is eaſy to read in the philoſophical tranſactions at the diſtance of 230 feet. Another of them is in the hands of Mr. *Alexander Bayne* our Profeſſor of Law; with which he eaſily reads the philoſophical tranſactions at the diſtance of 280 feet. I made ſome tryals with one of the ſpeculums of nine inches, and can read with it very eaſily in the philoſophical tranſactions at the diſtance of 138 feet; but at that time had not an opportunity to try it at a greater diſtance; at another time I read with it a much ſmaller print croſs the ſtreet at the diſtance of 125 feet. It coſt him a great deal of trouble to make theſe of a true figure and with parallel ſurfaces*, and ſeveral when finiſhed were found uſeleſs, by reaſon of veins that then appeared in the glaſs.

* Newt. Opt. p. 94, 95. 80.

In the glaſs ſpeculums every thing elſe was very well, only the light was ſomewhat faint compared with that reflected from his metal-ſpeculums. This I take to have been owing to the ſpeculum's not having been well quickſilvered, and partly to the thickneſs of the glaſs. For one of them, I obſerved, had a brighter reflection when fluid quickſilver was applied to its back ſurface than after it was foiled.

After he found the light in theſe glaſs-ſpeculums fainter than he expected, and alſo becauſe of the great difficulty in finiſhing them, he applied himſelf to improve the teleſcopes with metal ſpeculums. By taking care of the figure he finds himſelf able to give them larger apertures than other workmen do; and by adjuſting the ſpeculums and the whole inſtrument he has much improved it. He executes every part himſelf,

ſelf, and takes vaſt pains to make the inſtruments as perfect as poſſible, and has made them of focal diſtances of two inches and ſix tenths, of four inches, of ſix inches, of nine inches, and of fifteen inches. He perforates the large ſpeculum and uſes a concave little ſpeculum. By thoſe of four inches focal diſtance he ſaw the Satellits of Jupiter very well, and read in the philoſophical tranſactions at above 125 feet diſtance. By thoſe of ſix inches focal diſtance he read at 160 feet diſtance. By thoſe of nine inches focal diſtance he read at 220 feet diſtance. By thoſe of fifteen inches focal diſtance he and Mr. *Bayne* have read in the Tranſactions at 500 feet diſtance, and have ſeveral times ſeen the five Satellits of Saturn together, particularly on the 24th of November and the 7th of December laſt; which very much ſurpriſed me till I found that Mr. *Caſſini* had ſometimes ſeen them all with a ſeventeen foot refracting teleſcope.

I have compared ſome of theſe with ſuch as have been brought from *London*; and find one of Mr. *Short*'s of ſix inches focal diſtance, compared with one of the beſt I have ſeen from *London* of nine inches and three tenths focal diſtance, to exceed it in brightneſs, diſtinctneſs and magnifying power; and when I called an indifferent perſon, who knew not who had made the inſtruments, to give his opinion, he very readily preferr'd that of ſix inches focal diſtance. It alſo manifeſtly exceeded another I had from *London* of eleven inches and a half focal diſtance. The ſame was the reſult of ſome other compariſons.

Upon the whole I am convinced he has much improved this excellent Invention and that his inſtruments are by far the beſt of their lengths that have yet been executed. I am *&c.*

Edinburgh Dec. 28. 1734.

Colin Mac Laurin.

Upon Chapter 9. Determinations of the focus's of rays very obliquely reflected or refracted at any number of ſurfaces, and of cauſticks.

490. Before the diſcovery of the law of refraction, according to the given ratio of the ſines of incidence and refraction of any given magnitudes, opticians could only conſider the refractions of ſuch rays as fell almoſt perpendicularly upon the refracting ſurfaces, where the angles of incidence and refraction, being but ſmall, were known by experience to be nearly in a given ratio to each other; and theſe rays they found would all belong to one focus pretty nearly. But Dr. *Barrow*, obſerving that the ſeveral ſmall portions of a large pencil of rays, flowing from a given focus, would diverge after refraction or reflection from ſeveral different focus's, according as they fell with different obliquities upon the ſeveral parts of a ſpherical ſurface, and being of opinion that the eye receiving a certain ſmall portion of theſe rays would judge the object to appear in the place from which they diverged, and conſequently to appear in different places according as the eye received a different portion; took occaſion from thence to determine theſe places geometrically by means of the law of refraction then newly diſcovered *; and conſequently to handle the ſubject of dioptricks and catoptricks in a more extenſive manner than any writer had then done. The focus's of rays obliquely refracted and reflected have alſo been touched upon by Sir *Iſaac Newton* in his Optical Lectures, in order to determine the diameters and breadths of the rain-bows and to make way for his admirable theorems concerning the ſeparations of heterogeneal rays. Theſe are the principal writers on the ſubject of the preſent chapter, in which I have not only comprehended the chief of their diſcoveries, but alſo have made them much more general; by ſhewing that the relation of the focus's of incident and emergent rays, to the focus's of parallel rays coming contrary ways, is always the ſame, after any number of oblique refractions or reflections, as when a pencil of rays is but once refracted or reflected at the vertex of a ſingle ſurface; as will appear more plainly in the propoſitions themſelves.

* Rem. 2.

Upon Art. 412.

491. Having yet ſaid nothing upon the focus of rays obliquely refracted through a plane ſurface, I will here tranſcribe a very elegant determination of it by Sir *Iſaac Newton* *. Let *ABD* be one of the incident rays that diverge from or converge towards the focus *A*; and let the perpendicular *AH* to the refracting ſurface *BH*, cut the refracted ray *EBG* in *G*. To the rays *AB*, *BG* draw the perpendiculars *HI*, *HR*, and in *BG* take *BF* to *BA* as *RG* to *IA*; and *F* will be the focus of the refracted rays, that go the neareſt to *EBG* on both ſides of it.

Focus of a ſuperficial pencil.

* Lect. Opt. Par. I. §. 3. prop. 8. Fig. 141. 142.

492. The author having given no demonſtration of this conſtruction, I will here deduce one from the propoſition in the preſent article. Let any refracting circle *BK*, whoſe center is *C*, touch the refracting line *BH* in *B* on its ſide oppoſite to *A*, and let *CD* and *CE* be the ſines of incidence and refraction common to the

plane and the ſpherical ſurface; and putting m to n for the ratio of $CD \times BE$ to $CE \times BD$, if the rays were refracted at the arch BK, by the preſent article we ſhould have $BF : EF (:: mBA : nDA) :: BA : \frac{n}{m} DA$ or $\frac{n}{m} BA + \frac{n}{m} BD$; and disjointly as $BF : BE :: BA : \frac{n}{m} BA + \frac{n}{m} BD - BA$, or $\frac{n}{m} BD$ when the rays are refracted through the plane BH. For the arch BK will coincide with its tangent BH, when its radius BC becomes infinite, at leaſt before it becomes negative: and then the finite line $\frac{n}{m} BA - BA$ may be rejected from the infinite line $\frac{n}{m} BA + \frac{n}{m} BD - BA$. We have therefore $BF : BA :: mBE : nBD :: CD \times \overline{BE}^2 : CE \times \overline{BD}^2$ (by reſtoring the values of m and n) $:: \frac{\overline{BE}^2}{CE} : \frac{\overline{BD}^2}{CD} :: \frac{\overline{HK}^2}{BR} : \frac{\overline{HI}^2}{BI}$ (becauſe the triangles BEC and HRB, and alſo BDC and HIB are ſimilar) $:: RG : IA$, becauſe RB, RH, RG and alſo IB, IH, IA are continual proportionals *.

* Eucl. VI.

8. Cor. Focus of a ſolid pencil of oblique rays conſidered.

493. Upon this occaſion Sir *Iſaac Newton* makes the following remark. "That F is the focus of thoſe rays only which lye in the plane ABH. For no other refracted rays can cut BF either in F or any where elſe, excepting thoſe which before refraction lay in a conical ſurface deſcribed by the revolution of AB about the axis AH, and which will all cut the ſaid BF in G where the axis AH cuts it. Therefore the rays that go very near to the ray FBE diverge chiefly from two centers or focus's F and G; the point F being the focus of thoſe that lye in the plane ABH, and G of thoſe that lye in the conical ſurfaces, deſcribed by the lines AB, BG turned about AH. All other rays that ſurround AB, will be ſo refracted as to approach the neareſt to FB ſomewhere between F and G. So that with reſpect to an eye, having the center of its pupil at any point O of the ray BE, the place of the image of the point A ought to be diffuſed through the whole ſpace FG; or becauſe the ſpace FG is the image of a ſingle point A, its ſenſible image ought rather to be placed at ſome ſingle point near the middle of that ſpace, and as it were in the middle of all the rays that diverge from it upon the eye. But an accurate determination of that point, regard being had to all the rays that flowed from A before their incidence on the pupil, is a very difficult problem, unleſs its ſolution be rather founded upon ſome probable hypotheſis than upon the rigour of the caſe. For inſtance, ſince the number of rays flowing from G and the adjacent points, ſeems to be equal to the number of thoſe that flow from F and its adjacent points, the place Z of the image of A ought to be ſo ſituated between theſe limits, that the angle under two rays converging from F and G to any given point of the pupil, may always be nearly biſected by a ray flowing from Z to that ſame point of the pupil. Upon this hypotheſis, taking O for the center of the pupil, the point Z may be found by ſaying as $OF + OG : OG :: FG : GZ$. For diſjointly we ſhall have $OF : OG :: FZ : GZ$; and therefore ſuppoſing three lines to be drawn from the points F, G and Z to any point of the pupil, lying very near to its center O, the angle under the two outward lines will always be very nearly biſected by the intermediate line *."

* Eucl. VI. 3.

494 So far Sir *Iſaac Newton*, whoſe doctrine is manifeſtly applicable to rays obliquely refracted through a ſpherical ſurface, taking a line drawn through its center and the focus of the incident rays for its axis. In his application of it to the ſenſible image or apparent place of the point A he follows Dr. *Barrow*'s opinion, upon which ſee the remarks upon Art 138. and Art. 614.

Upon ART. 477.

495. The focus A of incident rays as AB upon a refracting plane BH being given, Dr. *Barrow* has taught us how to draw the refracted rays as BO very expeditiouſly *. In the perpendicular AH to the refracting ſurface, on the ſame ſide of it as A, take HL to HA as the ſine of incidence to the ſine of refraction; and draw LM parallel to BH; then let any incident ray AB cut LM in M; and in the angle BHL inſcribe a line BN equal to BM, and the line NBO will be the refracted ray. For by conſtruction the ſine of incidence is to the ſine of refraction (as HL to HA, or as BM or BN to BA*, that is by Art. 221.) as the ſine of the angle BAH, equal to the angle of incidence, to the ſine of the angle BNH, which therefore is equal to the angle of refraction.

Fig. 143. 144.

* Opt. Lect. IV. Sect. 5.

* Eucl. VI. 2.

496. When parallel rays fall upon a refracting ſpherical ſurface, he has alſo taught us how to draw any number of refracted rays by this conſtruction*. Let I to R be the ratio of refraction, and through the center C of the refracting circle BCN draw the ſemidiameter BC parallel to any incident ray MN, and in BC produced take $BZ : CZ :: I : R$; and divide CZ in F by taking $FZ : FC :: I : R$; and with the center F and ſemidiameter FZ deſcribe a circle ZGE; and through the given points N, C draw

Fig. 145. 146.

* Opt. Lect. XI. Sect. 1.

draw NCG cutting this circle in G; and in the axis CZ take $CK=CG$, and drawing NK it will be the refracted ray. For having joined FG and BG, since $BZ:CZ::(I:R::)\ FZ:FC$ alternately we have $BZ:FZ::CZ:FC$, and disjointly $BF:FZ::FZ:FC$. Therefore since the sides of the triangles BFG, GFC, that contain the common angle GFC, are proportionable, those triangles are similar. Hence we have $BG:GF::GC:CF$ and alternately $BG:GC::GF:CF$, that is $BG:GC::FZ:CF::I:R$. But in the triangles BCG, NCK we have $BC=CN$ and $CG=CK$, and the angle $BCG=NCK$, and consequently $BG:GC::NK:CK$; therefore $NK:CK::I:R$. But in the triangle NCK the angle at C is equal to the angle of incidence or to its supplement, and therefore CNK is the angle of refraction or its supplement by Art. 221.

497. Hence while the point of incidence N approaches to B the point K will approach to Z; and therefore every two contiguous rays will cross each other before they cross the axis CZ. For while BN decreases it is plain that CG or CK must increase till they equal CZ. So far from Dr. *Barrow*.

Upon Chapter 11. *Concerning corona's and parhelia.*

History of the Subject.

498. Upon occasion of a corona observed at *Paris*, 12 May 1667, *Hugenius* gave in his thoughts on that subject to the Royal Academy of Sciences, in a discourse afterwards re-published in our Philosophical Transactions * ; which I need not transcribe as containing little more than the heads and design of the present treatise. There indeed he gives a hint at the cause of an Anthelion, observed by *Hevelius*, in the intersection of two bright arches crossing one another at oblique angles, which here, I find, he intended to explain at large, but did not finish the treatise; for which reason, and especially as I here give a translation of a latin translation from the original dutch, the reader will excuse me, if, upon a critical examination, he should find I have any where mistaken the author's meaning; for now and then I was a little doubtful of it.

* Abr. by *Lowthorp*. Vol. II. p. 189.

499. I judged it best to throw all the mathematical materials into one view, to be seen in the Appendix *, tho' some few of them were before demonstrated in a different manner in the body of the treatise.

* Art. 579.

Upon Art. 530.

500. Line 34. *I judge the breadth of the corona to be equal to the apparent space through which the colours are spread* &c. The sun's apparent diameter must be added to the apparent space here mentioned, as in the Rain-bow; see Art. 502.

501. Line 42. *Now this happens when there is less plenty of such globules*, that is, whose diameters bear equal ratio's to their opake kernels; for a greater or smaller inequality of those ratio's will more or less dilate the breadth of the corona, so as to produce whiteness by a mixture of the generated colours.

A white bow in the place of the rain bow.

502. *Aug.* 26. 1730. at 7 $\frac{1}{2}^{h}$. a. m. travelling between *York* and *Tadcaster*, while the sun shone upon a foggy air, I observed a regular arch in the heavens in the usual situation of the primary rain-bow, or rather, as I judged, contracted into an arch of a smaller circle. But it seemed manifestly broader than the inner rainbow and all over of a strong white, which I took to arise partly from the weakness of the sun's rays in passing through the fogg, but chiefly from the extraordinary breadth of the arch, for a like reason to what I have just mentioned about the corona.

Upon Art. 532.

503. *At the end.* These corona's, if their diameters were small, are accounted for by Sir *Isaac Newton* from principles * quite different from these of *Hugenius*.

* Opt. 8°. pag. 288. Obs. 13.

Upon Chapter 12. *Concerning the apparent shapes of objects seen by reflected or refracted rays.*

Upon Art. 598 to 614.

Appearances through plane glasses.

504. From this theory of the apparent shapes of objects, it follows that a straight object seen through a glass bounded by parallel plane surfaces, should appear a little concave towards the eye, though scarce sensibly, unless the glass be very thick. For, supposing the same notation as before, let the surfaces of the glass be cut perpendicularly by the plane of the figure in the

Fig. 147.

lines Ar, Bs; then we have AO to Af and also Bg to Bf in the given ratio of the sines that determine the refractions *, and alternately $AO:Bg::(Af:Bf::)\ Ar:Bs$. Which shews that the triangles AOr, Bgs are similar, and consequently that the fictitious * ray gsL emerges parallel to Or or OR; and so does the true ray SQ *, which is more refracted both at R and S, as is plain by the position of the aberrations $f\Phi$, $g\gamma$ *. Consequently SQ and sL are parallel; and

* Art. 223.

* Art. 614.

* Art. 36.

* Rem. 485.

and therefore PQ, being always leſs than PL, * Art. 610. ſhould appear always concave towards the eye *; but their difference QL, being always equal to Ss, whatever be the diſtance between the glaſs and object, is ſo ſmall, unleſs the glaſs be very thick, that the apparent curvity of the line $\Pi\varkappa$ may be quite inſenſible, as we find it is in looking through coach-glaſſes, whoſe ſurfaces are truly plane and parallel.

And thro' uneven glaſs.

505. Therefore the diſtortions of objects ſeen through windows made with unwrought glaſs, is owing to its curvity and uneven thickneſs. For the ſmalleſt angular deviations of the refracted rays from their due courſe, thence ariſing, will cauſe them to fall very wide of the eye placed at a good diſtance from the window; ſo that an object cannot be ſeen through it by ſuch rays as being equally refracted in every part of the glaſs, come in due order to the eye, but by others, which by coming through improper places of the glaſs, cauſe a diſtorted image on the retina. And ſince the errors in thoſe places are greatly augmented by the diſtance of the eye from the window, experience has taught us to go very near to it, in order to ſee things in their proper ſhapes.

Fig. 148.

506. It is well known, that a large plane object, as the bottom of a veſſel, lying deep under water or any liquor, appears concave to- * Art. 610. wards the eye; which agrees with our theory *. For ſuppoſing AR to be the ſurface of water, the object PQ is leſs than PL, becauſe the aberration $f\varphi$ tends from the ſurface AR *; and * Rem. 495. when the angle AOR is given, QL and $\varkappa\lambda$ are as the depth AP. In the preſent caſe of a ſingle refraction at a plane ſurface, I find the curve $\Pi\varkappa$ is of the third order.

Apparent ſhapes thro' ſenſes.

507. As a farther confirmation of the truth of this theory, it is obſervable that a large plane object ſeen through a deep concave lens appears always convex towards the eye, agreeably to FIG. 513, and the more concave as the object is more remote and the lens at ſome diſtance from the eye.

508 Secondly, that a large plane object, ſeen inverted through a very thick convex lens or a ſphere (their properties being alike) appears convex towards the eye placed at ſome diſtance from the glaſs; and concave when it is ſeen upright, agreeably to FIG. 514, 515. The parallel hairs of a micrometer are another known inſtance of this laſt caſe, as appearing always concave towards the eye and convex towards each other, when opened ſo wide as to be ſeen thro' the edges of a thick eye-glaſs. For conceiving the ſurface deſcribed by the revolution of the curve $\Pi\varkappa$ about its axis $O\Pi$, to be cut by two Fig. 515. planes parallel to the axis and paſſing through the hairs, the ſections ſeen by the naked eye would appear in the ſhape of the hairs ſeen through the glaſs *. If a ſcruple ſhould ariſe * Art. 599. from the place of the interſection d abovementioned *, it will vaniſh by ſuppoſing the ſurface * Art. 611. AR to be plane inſtead of ſpherical The phænomena of the bars of a ſaſh-window viewed as above through concave and convex glaſſes are explicable in the ſame manner

The ſhape of an object ſeen in an elliptical or hyperbolical ſpeculum. Fig. 149. to 156.

509. Let O and f be the foci of an elliptical or hyperbolical ſpeculum repreſented by AR, and PQ a ſtraight line perpendicular to its axis OPA, in which taking $O\Pi = \frac{AO}{Af} \times fP$ and $\Pi\Omega = \frac{Af}{AO} \times fP$, place them both the ſame way from O as P lyes from f in the ellipſis, and both the contrary way in the hyperbola; then with the vertex Π and foci O, Ω deſcribe another hyperbola $\Pi\varkappa$, and draw any line fQ cutting the object PQ in Q and the ſpeculum AR in R, and let OR (produced) cut the hyperbola in $\varkappa$; I ſay the object PQ will appear in the ſpeculum AR, to the eye at O, of the ſame ſhape and magnitude and at the ſame diſtance, with which the hyperbolick arch $\Pi\varkappa$ would appear to the naked eye at O; and in the ſame ſituation too, provided $\Pi\varkappa$ be inverted, when it falls behind the eye, and removed to an equal diſtance before it.

For, from the known property of an elliptical or hyperbolical ſpeculum AR, *viz.* that the rays which flow from its focus O will all belong to its focus f after reflection, I find, that while the viſual angle AOR is gradually varied, the hyperbolick arch $\Pi\varkappa$ is the geometrick place of the interſection $\varkappa$ of the line OR (produced) and the line $Q\varkappa$ drawn parallel to the axis. The truth of the propoſition is therefore evident by Art. 599.

510. It is well known, that the hyperbolick arch $\Pi\varkappa$ is concave towards the leſſer of the two lines $O\Pi$, $\Pi\Omega$, and that its curvity is greater according to their greater inequality. The poſition and curvity of the apparent object $\Pi\varkappa$ is therefore known by the conſtruction *, which * Rem. 509. gives $O\Pi : \Pi\Omega :: \overline{OA}^2 : \overline{Af}^2$.

And in a parabolick ſpeculum. Fig. 151.

511. Hence, if the eye be at O in the focus of a parabolick ſpeculum AR, draw any line QR parallel to its axis OA, and the object PQ will appear in the ſhape and place of the parabolick arch AR. For, while the vertex A and focus O of the concave hyperbola AR, keep Fig. 150. fixt, let the focus f recede to an infinite diſtance, and the hyperbola's AR, $\Pi\varkappa$ will both be changed into one and the ſame parabola AR, as * Rem. 509. is evident by the conſtruction above *.

And in a ſpherical ſpeculum. Fig. 153.

512. Hence alſo, if the eye be at O in the center of a ſpherical ſpeculum AR, the object PQ will appear in the ſame ſhape and magnitude and at the ſame diſtance as it does to the naked

naked eye at *O*. For by diminiſhing the interval *Of*, of the elliptical foci, to nothing, the ellipſis is changed into a circle and the hyperbola Π*x* into the ſtraight line *PQ*, by the ſaid conſtruction *.

513. Now let *O* and *f* be conjugate foci of rays reflected at the vertex *A* of a ſpherical ſpeculum *Ab*, and the ſtraight object *PQ*, if not too long, will appear to the eye at *O* nearly of the ſame ſhape and magnitude, and at the ſame diſtance, with which the hyperbolick arch Π*x* would appear to the naked eye at *O*. For according as the foci *O*, *f* lye on the ſame or contrary ſides of the circular arch *Ab*, its curvature muſt be the ſame at the vertex *A* with that of an ellipſis or hyperbola that has the ſame points *O*, *f* for its foci: becauſe the circular arch *Ab* reflects the rays, from the common vertex *A*, into the ſame lines as they do, by ſuppoſition: and therefore the apparent curvature of the object *PQ* ſeen in the ſpherical ſpeculum will be nearly the ſame as if it were ſeen in the elliptical or hyperbolical ſpeculum.

514. But if the ſtraight object *PQ* be very long, its apparent ſhape in the ſpherical ſpeculum will come nearer in moſt caſes to that of a parabolical or elliptical arch Π*k*, of the ſame curvature with that of the hyperbola Π*x* at its vertex Π. Firſt, becauſe the hyperbola and ellipſis grow gradually ſtraighter in going from their vertex, and therefore deviate more and more from the invariable curvature of the circle. This being ſo, let the ray *OR* (produced) fall upon the circle in *b*, and, after reflection, upon the object *PQ* in *K*, and let *Kk*, drawn parallel to the axis *AO*, cut the viſual ray *Ob* (produced) in *k*. Then, ſuppoſing the reflected ray *bK* (produced) would paſs through *f*, it is plain by the figures, that the interſection *k* would fall within the hyperbola Π*x*; and the aberration of the ray *bK* from *f* towards φ, will generally carry the interſection *k* within the cavity of the hyperbola Π*x*; and thus the ſhape of the apparent object or curve Π*k*, deſcribed by the interſection *k*, will deviate from the ſhape of the hyperbola Π*x* towards that of a parabola or an ellipſis.

515. The geometrick places deſcribed by the interſection *k* being curves of the third order, whoſe ſhapes are not familiar to us, I choſe to compare them with the known ſhapes of conick ſections.

Let us now proceed to determine the apparent ſhape, magnitude and place of an object ſeen with both eyes in a ſpherical ſpeculum.

516. Other things remaining, let a ſtraight object *QS* be perpendicular to the plane of the figures, and let the curved ſurface, deſcribed by the revolution of the curve Π*k* round its axis *P*Π, be cut in the curve *kt* by a plane paſſing through the lines *QS*, *Qk*; then drawing the line *St* parallel to *Qk*, the object *QS* will appear in the ſpherical ſpeculum *Ab* in the ſame ſhape, magnitude and ſituation to the eye at *O*, as the ſection *kt* would do to the naked eye at *O* *; provided the ſection, when it falls behind the eye, be inverted and carried as far before it *, as in Fig. 160. This being premiſed, thro' the center *E* of the ſpeculum *Ab* draw *QE* and make another angle *QEA* equal to the adjoining angle *QEA*, and in the new radius *EA* take another line *EO* equal to the given *EO*, then the phænomena of the object *QS* viewed with both eyes placed at the points *O*, *O* are reducible to the following caſes. Fig. 157. to 160. * Art. 598. * Art. 139.

517. *Caſ.* 1. If the reflected ray *bO* and the line *QE* be not parallel, produce them, if need be, till they meet in *q*, and if *q* falls behind the eyes, the object *QS* will appear *double*, and the appearance to each eye will be the ſame as above deſcribed when the other was ſhut. We may eaſily adapt the 158th and 159th figures to this caſe, by conceiving the object drawn a little farther from the ſpeculum.

518. *Caſ.* 2. But if the interſection *q* falls before the eyes, and their axes be inclined to one another in an angle equal to *OqO*, the object *QS* will appear *ſingle* in diverſe manners.

519. *Caſ.* 3. For, if the line *Oq* be leſs than *Ok*, the object *QS* will appear to both eyes *nearer and ſmaller* than to either eye alone. Fig. 157. 160.

520. *Caſ.* 4. But if *Oq* be greater than *Ok*, the object *QS* will appear to both eyes *remoter and larger* than to either eye alone. Fig. 158, 159.

521. *Caſ.* 5. Laſtly, having compleated the rectangle *QStu*, let the line *Ou* (produced) meet *Qq* in *x*, or be parallel to it, and if the line *Qx* be longer than *Qq*, as in fig. 157, the object *QS* will appear *convex* towards both eyes, otherwiſe *concave*; and whether *Qx* be longer or ſhorter than *Qq*, will be evident from the following conſiderations.

522. According as the ſurface deſcribed by the revolution of the curve Π*k*, is convex or concave towards *PQ*, which was determined above *, its ſection *kt* will alſo be convex or concave towards the object *QS*, and the line *Qu* will accordingly be longer or ſhorter than *Qk*. Hence, and from the given poſition of the points *Q*, *O* to the ſubtenſes *ku*, *qx* of the angle *kOu* or *qOx*, it will be evident in fig. 157, where *Qu* is longer than *Qk*, that *Qx* is alſo longer than *Qq*; and in fig. 158, 159, where *Qu* is ſhorter than *Qk*, that *Qx* is alſo ſhorter than *Qq*; but in fig. 160, though *Qu* is longer than *Qk* yet *Qx* is ſhorter than *Qq*. * Rem. 513.

523. Let the the ſame conſtruction be compleated for the ſecond eye as we made for the firſt, and from the equality of the angles *QEO*, *QEO* and lines *EO*, *EO*, it is plain that all Demonſtration of the phænomena.

other

other parts of the figures on oppoſite ſides of the line QE are equal and ſimilar to one another.

524. 1. Now if the reflected rays bO, bO, which came originally from Q, tend to meet in q behind the eyes, they muſt needs fall upon points of the two retina's which do not correſpond * to each other; becauſe the axes of the eyes produced forwards cannot diverge as the viſual rays Ob, Ob are ſuppoſed to do. Conſequently the point Q and object QS will appear double *.

* Art. 137.

* Art. 137.

525. 2. But if the point q falls before the eyes and their axes be directed towards it or any collateral point lying nearly at the ſame diſtance from the eyes as q does, their axes will be inclined to one another in the angle OqO or ſome other equal to it, and then the rays bO, bO will fall upon correſponding points of the retina's, and excite a ſingle appearance of the object QS *.

* Art. 137.

A previous Experiment or two explained.

526. In order to explain the reſt of the phænomena we muſt conſider an experiment deſcribed in Art. 977. and add to it another caſe or manner of viewing the compaſſes there mentioned; whoſe legs being opened to any angle and held by the joint in a plane nearly perpendicular to a plane paſſing through the axes of the eyes, with their points in this latter plane, let the axis of the right eye be directed ſteadily towards the point on the left hand, and that of the left eye towards the point on the right hand, then, of the legs that appeared double, the two innermoſt will ſoon unite in one, appearing like a third leg in the middle between the other two, and will tend from the joint towards the interſection of the axis of the eyes; and this middle leg, if viewed attentively while the real legs are gradually opened, will decreaſe in apparent magnitude and approach towards the eyes *; and on the contrary will increaſe and recede from you while the angle of the legs is contracted. The like phænomenon is obſervable in viewing two lighted candles of an equal height and thickneſs at the diſtance of two or three feet from the eyes; but the apparent candle in the middle does not approach quite ſo near the eyes as the correſponding leg of the compaſſes, I mean in proportion to the given diſtance of the original objects from the eyes. In the figure, $a\alpha$ and $b\beta$ are the diameters of the candles, d and e the centers of the pupils, $ae\alpha$, and $bd\beta$ two cones of rays croſſing one another in f, where the candles appear united in one, of a ſize proportional to the thickneſs of the cones at f or a little beyond it. Now if the candles a, b be gradually drawn aſunder, their apparent union near f, while the eyes are fixt upon it, will decreaſe in apparent magnitude and approach towards you. For the ſimilar and equal pictures of the two candles upon correſponding places of the two retina's cauſe the ſame ſenſation as two ſuch pictures of a ſingle candle at f would do; and this ſenſation excites the uſual idea of a ſingle candle*.

* Rem. 301.

* Rem. 301.

Fig. 161.

527. And by the way, if the lines afe, bfd be drawn upon a plane board a foot or two long, and a pin be ſtuck upright at their interſection f, and the pupils of the eyes be placed near the edge of the board at any ſmall height above the points d, e, while the pin is viewed ſteadily the two lines fa, fb will appear united and upright in the place or by the ſide of the pin. For in this caſe they cannot appear in two different places * and therefore muſt appear in the common interſection of two planes paſſing thro' the lines af, bf and the pupils of the eyes.

* Art. 137.

Application of the experiments.

528. 3. 4. By applying what has been ſaid of the points of the compaſſes to any two points k, k or t, t of the curves kt, kt ſuppoſed to be viewed by the naked eyes at O, O directed to the point q, the reaſon of the third and fourth phænomena is very evident.

529. 5. Hence alſo we have the reaſon of the apparent convexity or concavity of the object QS. For the two triangular planes Out, Out (produced) will interſect one another in a line xs perpendicular to the plane of the ſcheme, becauſe the lines tu, tu are perpendicular to it. And the point S, being ſeen by reflected rays coming to the eyes O, O in the ſame directions tO, tO in which they would come from two radiating points t, t, to the naked eyes at O, O, muſt appear in the ſame place in which the radiating points t, t would appear united to the naked eyes. And this place of apparent union will be nearer to, or farther from, the eyes than the place of apparent union of the two other points k, k to the naked eyes, or the apparent place of Q in the ſpeculum, according as the viſual angle OsO or even OxO is greater or leſs than OqO, that is, according as Qx is leſs or greater than Qq.

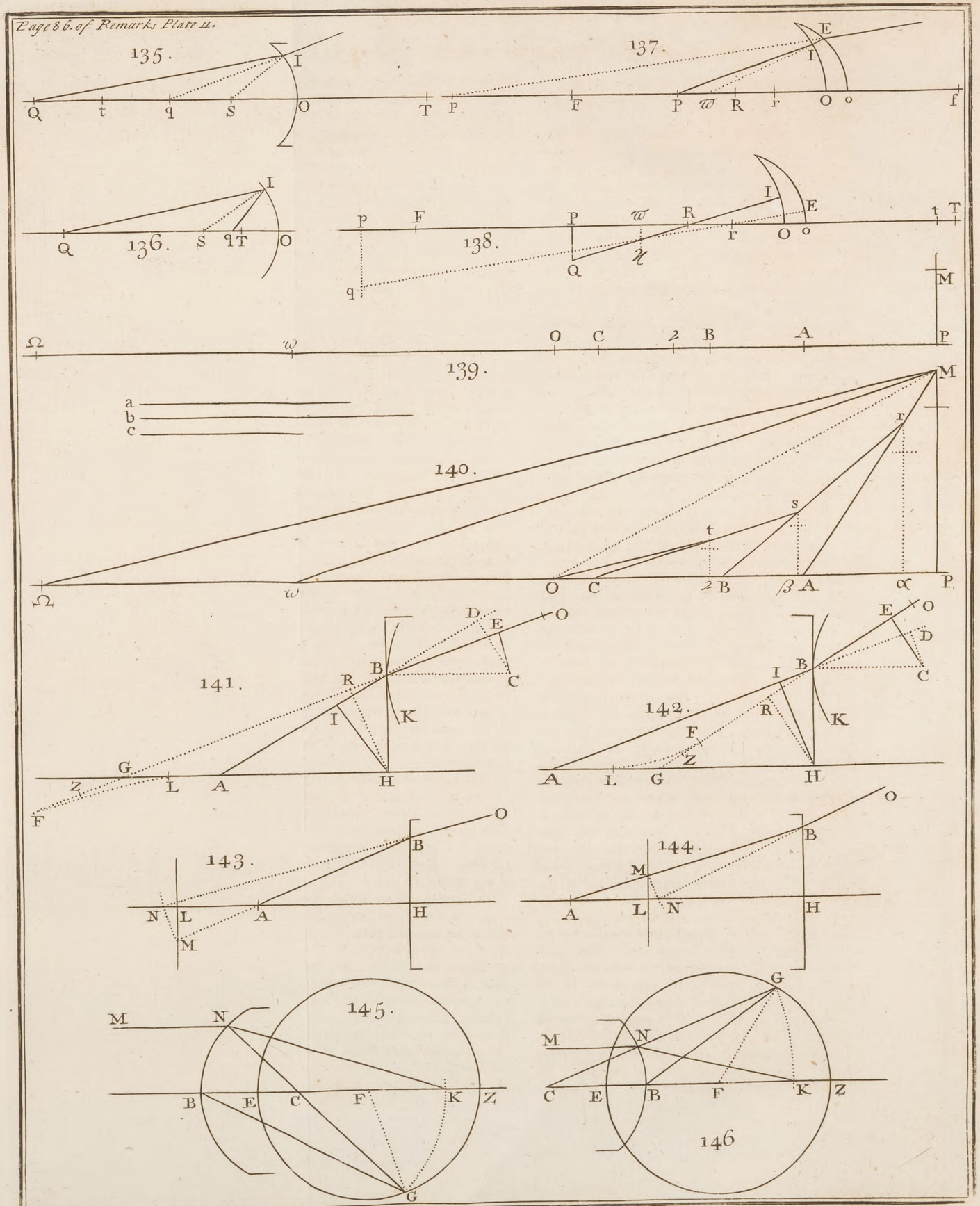
The theory diſcovers an extraordinary caſe.

330. In the caſes of fig. 159, 160, the object QS ought to appear convex to either eye alone and concave to both at once; which change of ſhape, I was very much ſurpriſed with, in viewing a foot ruler held upright between my eyes, with its flat ſide againſt my forehead, placed about a foot farther from the ſpeculum than the center of its concavity. In this caſe the object appears inverted, and accordingly in fig 160, the curves Πk, Πk and their ſections kt, kt are inverted too, and carried along the viſual rays Ob, Ob as far before the eyes as, by conſtruction, they fell behind them.

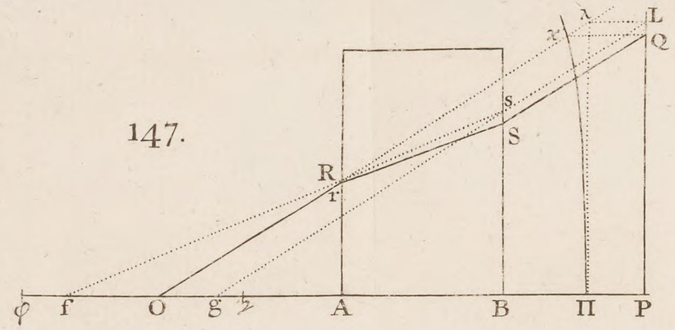
Is farther confirmed by experiments.

531. The whole theory may be farther confirmed by forming any two ſlender equal objects, as two quills ſtript of their feathers, into two equal curves, to repreſent the ſections kt, kt. For by placing them in the ſeveral poſitions

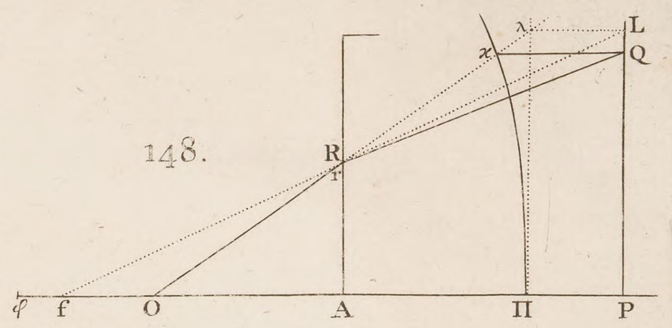
Page 86. of Remarks Plate 11.



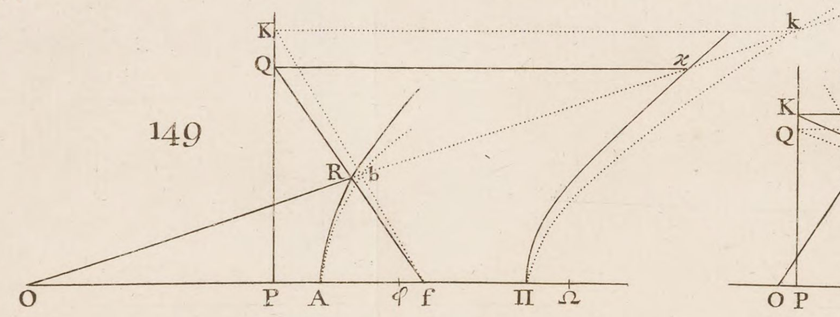
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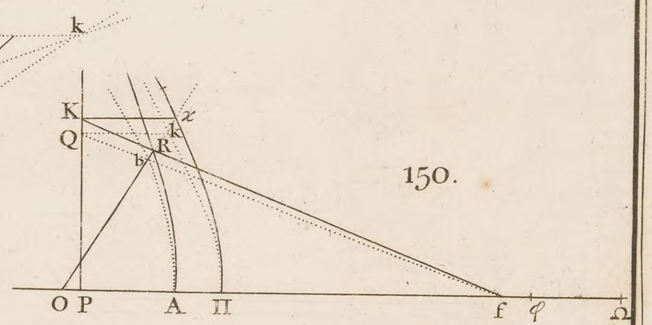
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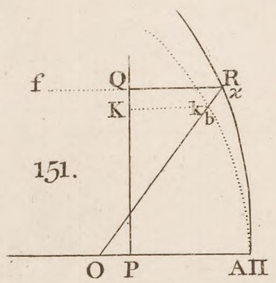
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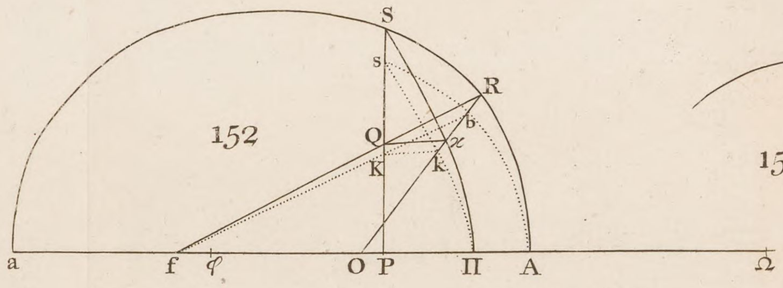
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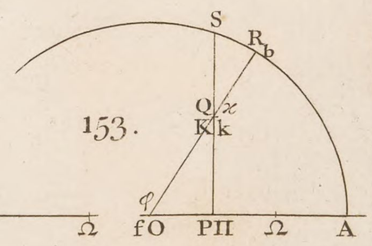
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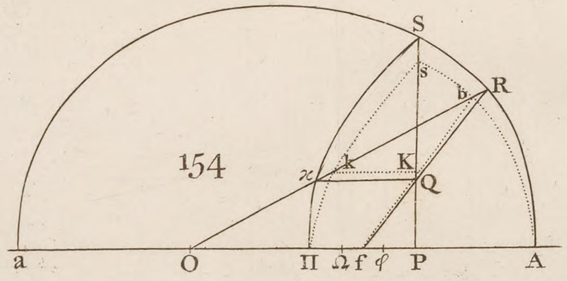
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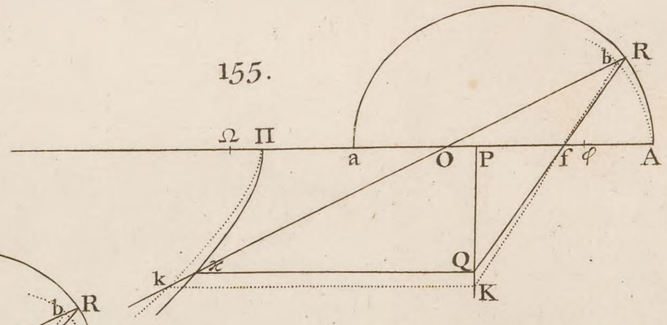
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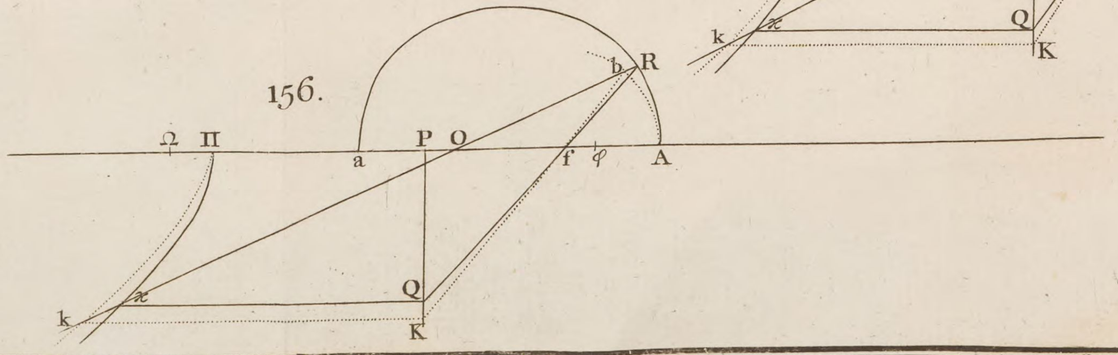
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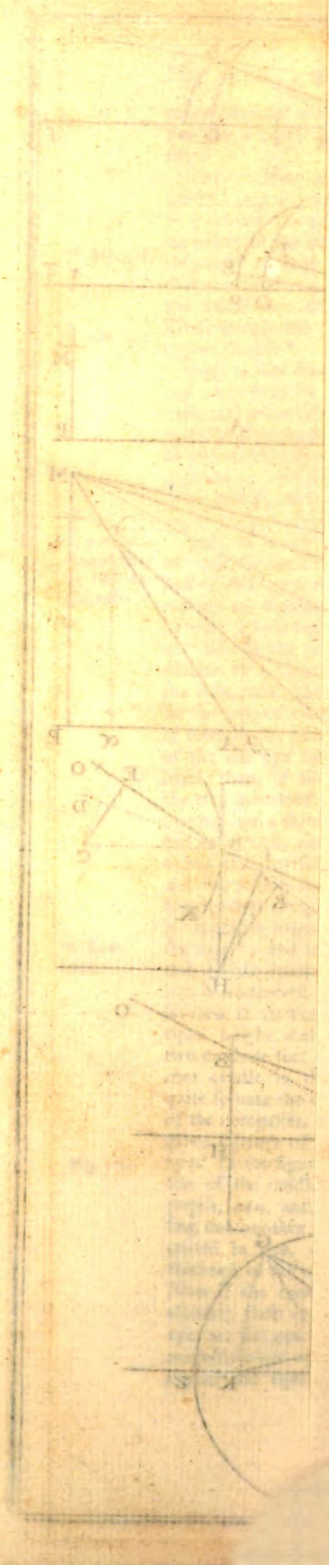


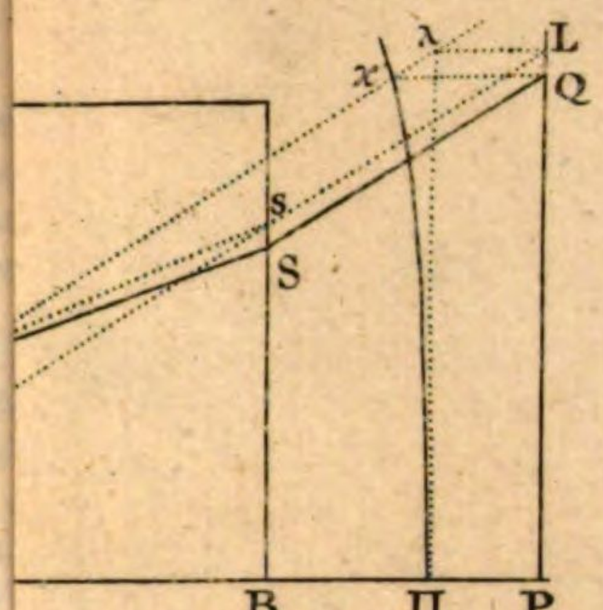
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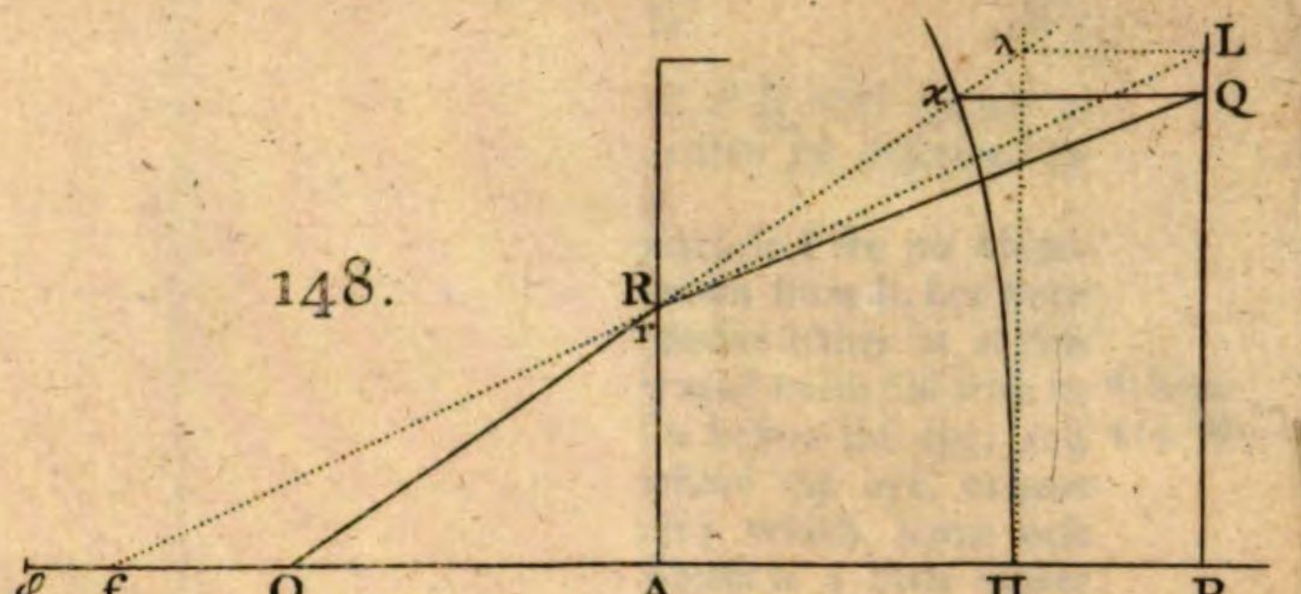
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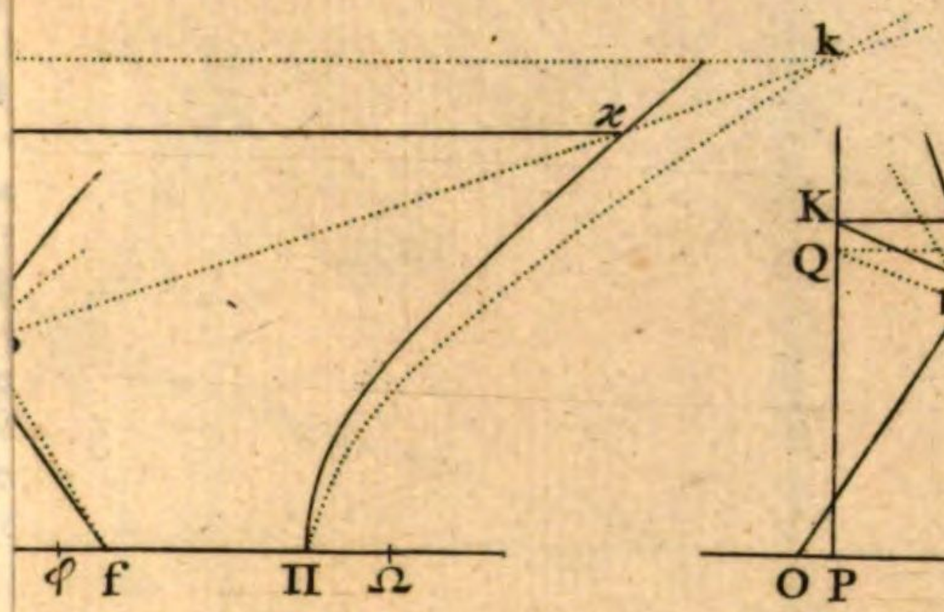


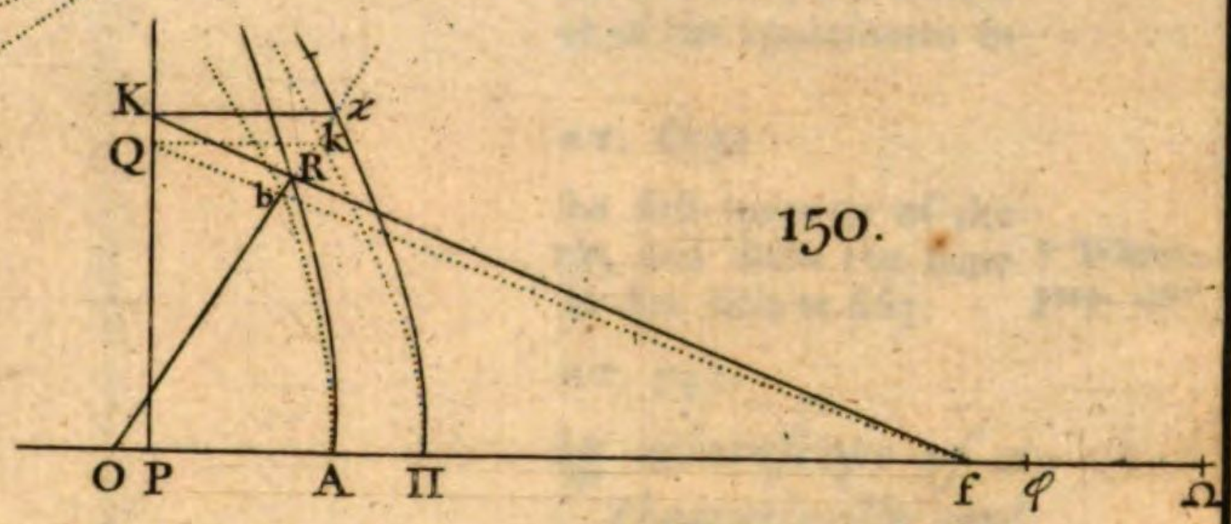




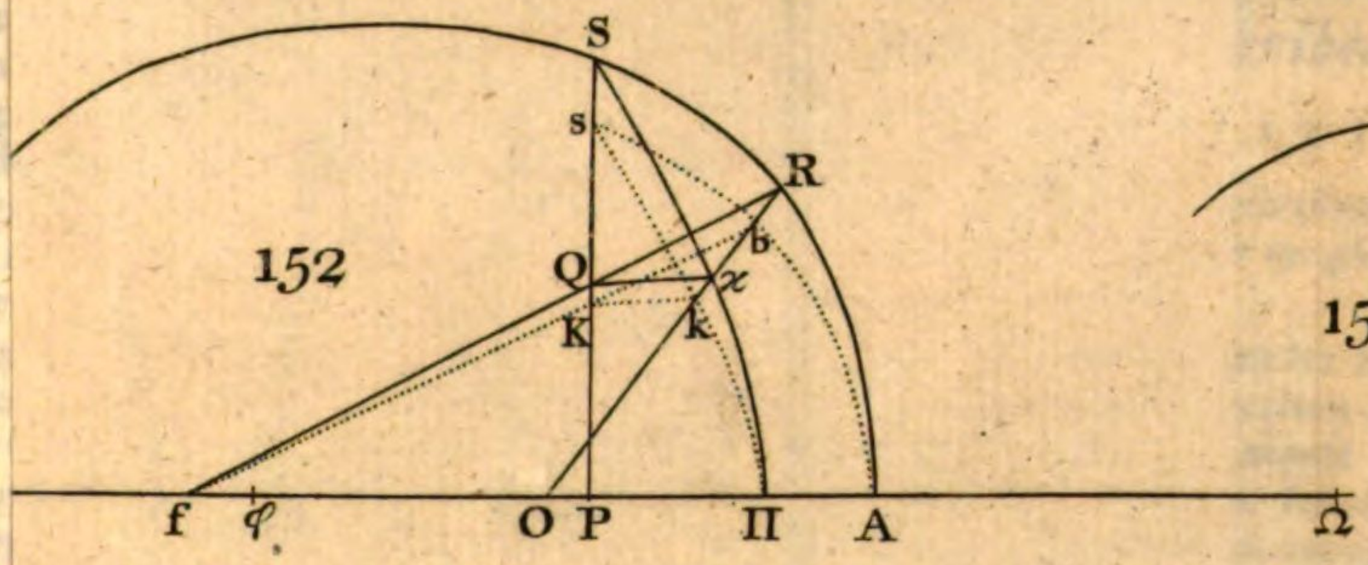
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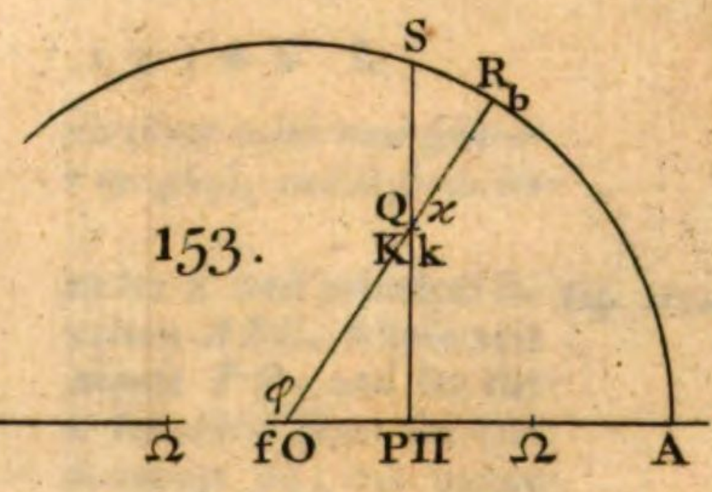




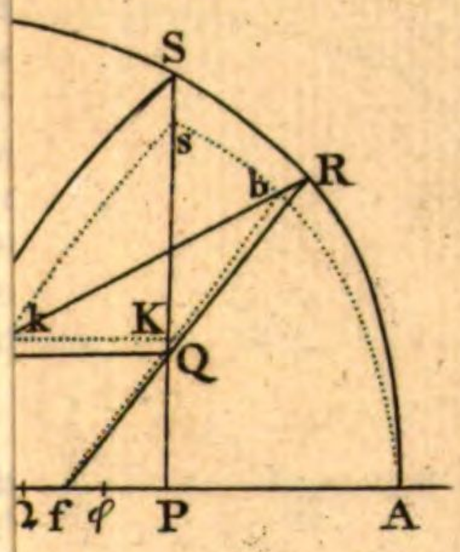
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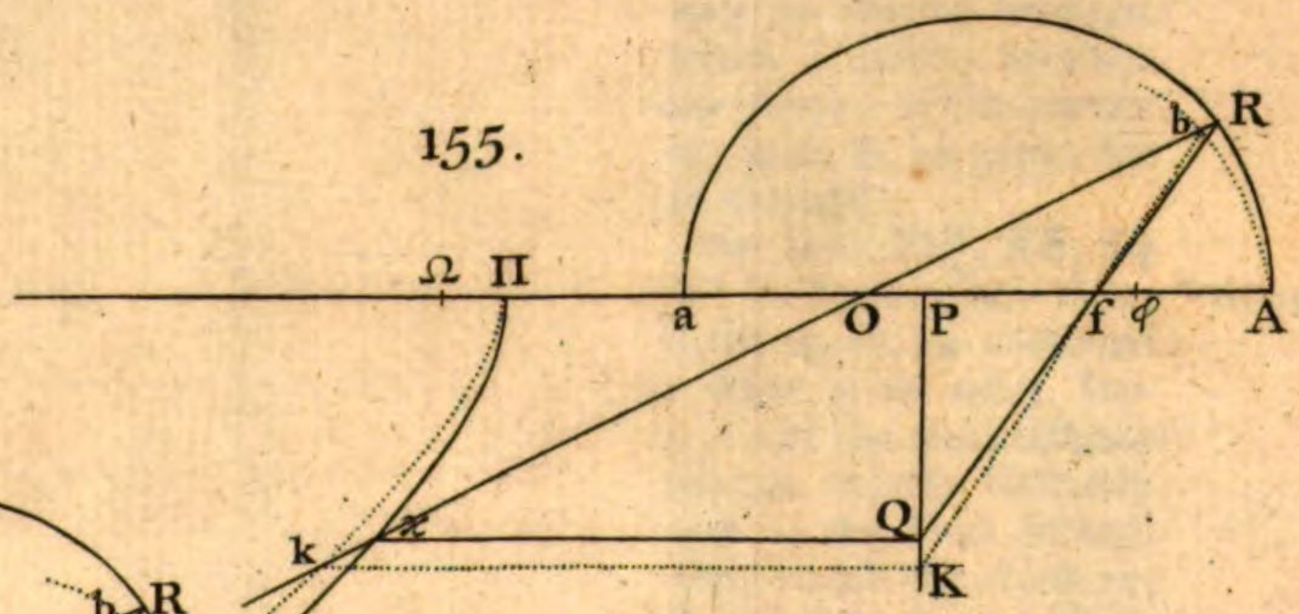
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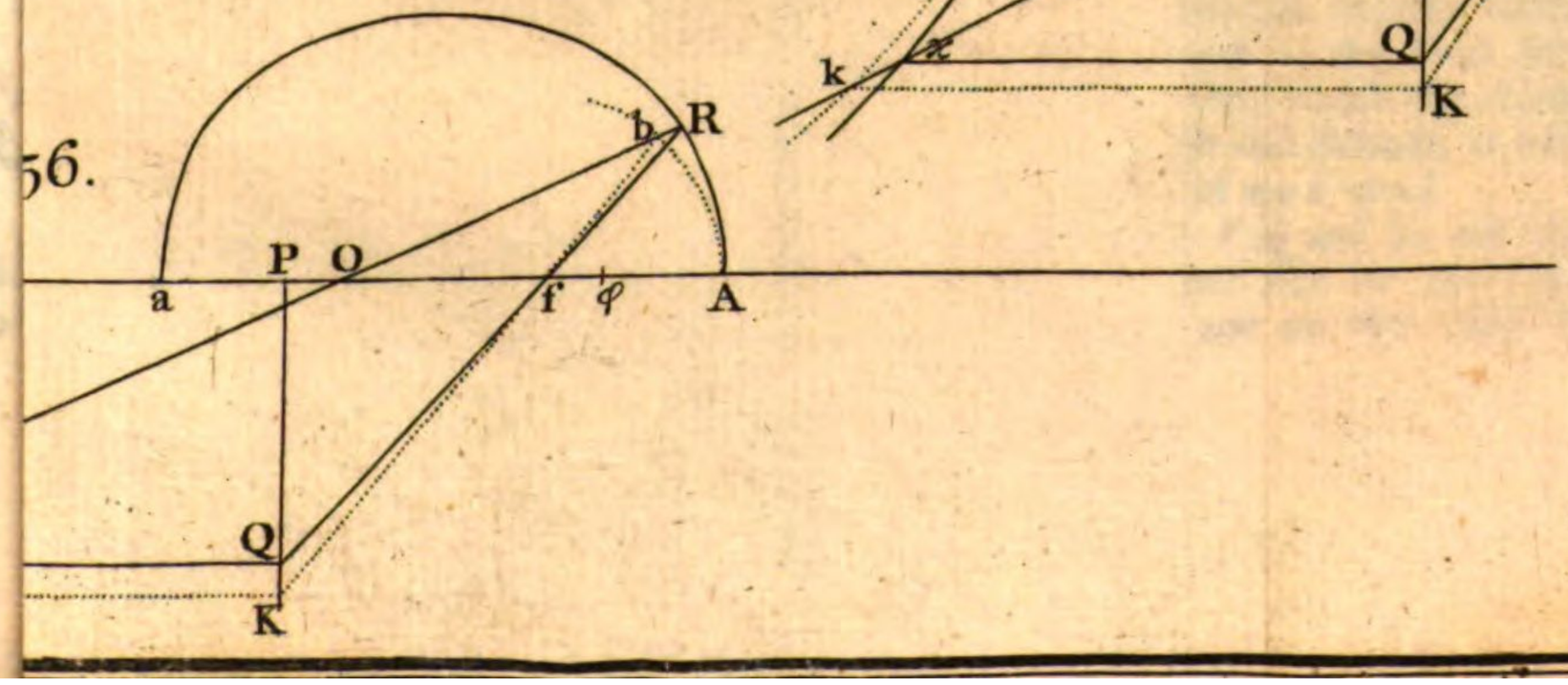
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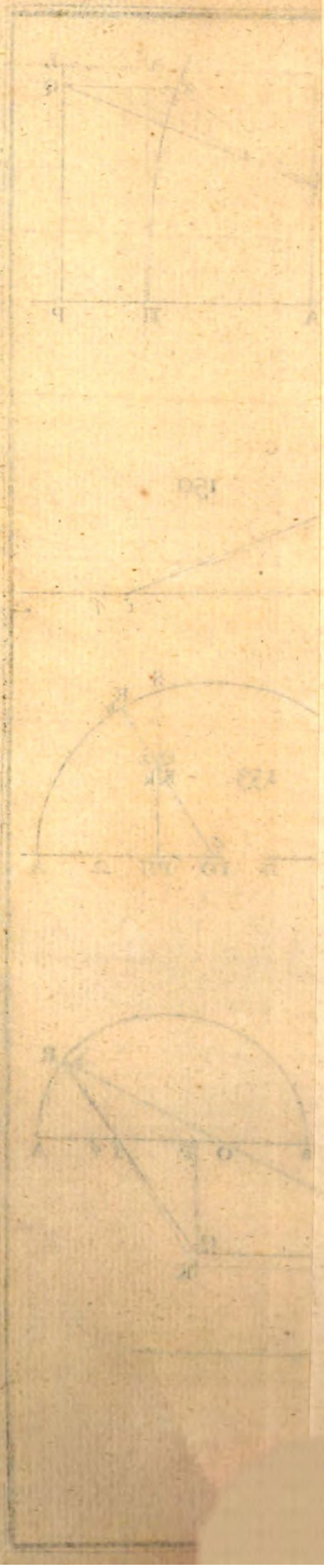


155.



156.





tions represented in the figures and by directing the eyes according to the position of the point x, the curvity of the two apparent inner quills, when apparently united, will have the same appearance, as to convexity or concavity and the degree of either, as the straight object QS will have in the speculum; as I have often tryed.

xplains ome re- narkable hænome- a. Lect. Opt. ect. 15. ppend.

532. The 158th figure gives me an opportunity of explaining a phænomenon which Dr. *Barrow* took notice of as somewhat extraordinary *. Viewing your face in a concave speculum held pretty near it, first with your right eye, while the left is shut, observe its apparent place and size. Then do the same with your left eye, while the right is shut, and your face will appear of the same size as before, only moved a little towards the left hand. But upon looking immediately with both eyes, the two apparent faces will seem united in a face much larger, remoter and more concave than before. The reason of which is very evident from the theory above.

A piece of perspective appears in a concave speculum to great advantage.

533. And for the same reason, a piece of perspective placed a little nearer to a very large concave speculum than its principal focus, as in fig 158, and viewed by a person standing close behind the piece and looking over the top of it, appears in the glass much larger and more projected, and consequeutly to much greater advantage than when it is seen by direct rays.

How Dr. *Barrow* considered this Subject.

534. I do not find that Sir *Isaac Newton* has any where touched upon this subject of apparent shapes, and Mr. *Huygens* has only given us this one observation upon a particular case of it *, "that the apparent curvity of straight "lines often seen about the margin of a lens, "is extremely difficult to be reduced to geome- "try"; which upon tryal I found so true, that I should scarce have pursued the speculation, but through a desire of having the most rigorous test of the validity and extent of the principle I have all along followed. I will conclude this subject with giving an account of Dr. *Barrow's* manner of considering it *. Let E be the center and $APEpO$ the axis of a large concave speculum AB, PQ a straight object perpendicular to it, QEq the axis of a large pencil of incident rays flowing from Q, qk the caustick formed by the reflected rays, q its cusp, O the pupil of the eye, OkB a tangent to the caustick. Joining QB, the ray which describes it, will be reflected into BkO, and the rays adjoining to it, will enter the pupil diverging from k, which therefore, according to Dr. *Barrow's* Principle *, is the apparent place of the point Q.

* Dioptr. prop. 52. p. 189.

* Lect. Opt. sub finem. Fig. 162.

* Rem. 212.

535. While the axis QEq, of the variable caustick qk, is turning about the center E, let a line OkB turning about O, touch the caustick continually in k; and, in consequence of the said principle, the object PQ will appear in the place and shape of a curve pk described by the motion of the point k.

536. Admiting the principle I see no objection to the consequence drawn from it, but since the principle has no foundation either in reason or experience *, the theory must needs fall with it, even when the image pk is before the eye; and much more when it is behind the eye, or part behind and part before it; which latter case must happen when the object is a little nearer to the speculum than its principal focus. These cases create no difficulty in my theory, but are utterly inexplicable by the Doctor's, and besides that, he gives no account of the appearances to both eyes.

* Rem. 213. &c.

Upon Art. 653.

537. *Hugenius* was the first inventor of the theorem * in this Article, and drew the same consequences from it as in Art. 660 to 665.

* Dioptr. prop. 27.

Upon Art. 741.

A double reflecting microscope of a new invention, theoretically and practically described.

Proposition I.

538. *To compose a microscope with two spherical specula and a convex eye-glass, and to shew its magnifying power.*

539. Between the center E and principal focus T of a concave speculum ABC, whose axis is $EQTC$, place an object PQ; and let the rays flowing from it be reflected from the speculum AB towards an image pq; but before they unite in it, let them be received by a convex speculum abc, and thence be reflected, through a hole BC in the vertex of the concave, to a second image ϖx, to be viewed thro' an eye-glass l. Fig. 163.

540. The object may be situated between the specula C, c; or, which is better, between the principal focus t and vertex c of the convex one, a small hole being made in its vertex for the incident rays to pass through.

541. In both cases we have TQ, TE, Tq continual proportionals * in some given ratio, suppose of 1 to n; and also tq, tc, $t\varpi$ continual proportionals in some other given ratio, suppose, of 1 to m. Then if d be the usual distance at which we view minute objects distinctly with the naked eye, and ϖl the focal distance of the least eye-glass, thro' which the object appears sufficiently bright and distinct, it will be magnified in the ratio of mnd to ϖl.

* Art. 207.

542. For the object PQ and its first image pq are terminated on one side by the common axis of the specula, and on the other by a line

line PEp, drawn through the center E of the concave ABC *. Likewise the images pq and $\varpi\varkappa$ are terminated by the common axis and by the line $ep\varpi$, drawn through the center e of the convex abc *. Hence, by the similar triangles $\varpi\varkappa e$, pqe, and also pqE, $P\mathcal{Q}E$, we have $\varpi\varkappa : pq :: \varkappa e : qe :: m : 1$ *, and $pq : P\mathcal{Q} :: qE : \mathcal{Q}E :: n : 1$ *; and consequently $\varpi\varkappa : P\mathcal{Q} :: mn : 1$, whence $\varpi\varkappa = mn \times P\mathcal{Q}$. Now if $l\varkappa$ be the focal distance of the eye-glass l, the points P, $\mathcal{Q}$, of the object, are seen through it by the rays of two pencils emerging parallel to the lines ϖl, $\varkappa l$ respectively; that is, $P\mathcal{Q}$ appears under an angle equal to $\varpi l\varkappa$, which is as $\frac{\varpi\varkappa}{\varkappa l}$ * $= \frac{mnP\mathcal{Q}}{\varkappa l}$; and to the naked eye at the distance d from $P\mathcal{Q}$, it appears under an angle $Po\mathcal{Q}$ which is as $\frac{P\mathcal{Q}}{d}$ *, and therefore is magnified in the ratio of these angles *, that is of mnd to $\varkappa l$.

* Art. 215.
** Euclid. V. 12.
* Art. 60.
* Art. 60.
* Art. 104.

543. *Corol.* 1. Having the numbers m, n, d, to find an eye-glass which shall cause the microscope to magnify M times in diameter, take $\varkappa l = \frac{mnd}{M}$. For the apparent magnitude is to the true, as $M : 1 :: mnd : \varkappa l$.

544. *Corol.* 2. As soon as the propereſt numbers m, n and focal distances $\mathcal{T}$, t can be determined, the following rules will give the several parts of the axis of the microscope.

$t\varkappa = mt$ | $q\mathcal{T} = n\mathcal{T}$

$tq = \frac{1}{m}t$ | $\mathcal{T}\mathcal{Q} = \frac{1}{n}\mathcal{T}$

$qc = tc - tq = \frac{m-1}{m}t$ | $q\mathcal{Q} = \mathcal{T}q - \mathcal{T}\mathcal{Q} = \frac{nn-1}{n}\mathcal{T}$

$c\varkappa = t\varkappa - tc = \overline{m-1,}\, t$ | $qC = q\mathcal{T} + \mathcal{T}C = \overline{n+1,}\, \mathcal{T}$

$q\varkappa = t\varkappa - tq = \frac{mm-1}{m}t.$ | $\mathcal{Q}C = \mathcal{Q}\mathcal{T} + \mathcal{T}C = \frac{n+1}{n}\mathcal{T}$

$Cc = qC - qc = \overline{n+1,}\, \mathcal{T} - \frac{m-1}{m}t$

$C\varkappa = q\varkappa - qC = \frac{mm-1}{m}t - n\overline{+1,}\, \mathcal{T}.$

LEMMA.

545. The focus of incident rays upon a spherical speculum being given, to find the exact aberration of any given ray from the conjugate focus.

Fig. 164.

546. Let E be the center of the speculum AC, $\mathcal{T}$ its principal focus, $\mathcal{Q}$ and q any two conjugate foci of the neareſt rays to the axis $qE\mathcal{Q}\mathcal{T}C$, $\mathcal{Q}A$ any other incident ray, AV perpendicular to the axis. Towards $\mathcal{Q}$ take $EF : EC :: E\mathcal{T} : E\mathcal{Q}$ and $qR : qE :: VC : VF$, placing qR the contrary way from q to that of VC from C; then will AR be the reflected ray and qR its exact aberration from the focus q.

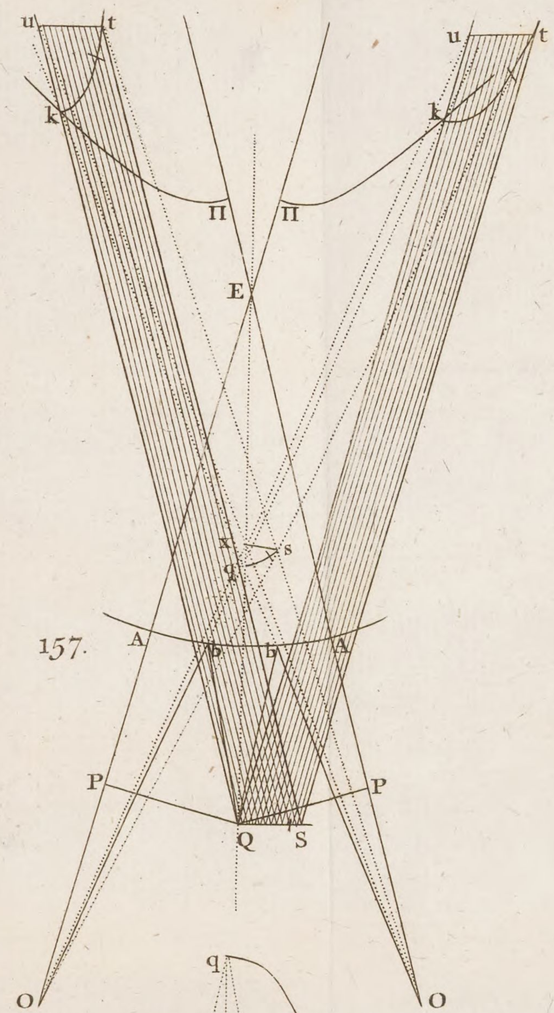
547. For let the incident rays $\mathcal{Q}A$, $\mathcal{Q}C$, produced, cut the sphere of the speculum again in B and D, and joining EA, EB, let RS drawn parallel to EA, meet $\mathcal{Q}A$ produced in S. Then we have ER to $R\mathcal{Q}$ in a ratio compounded of ER to $E\mathcal{Q}$ and $E\mathcal{Q}$ to $\mathcal{Q}R$, that is, of AS to $A\mathcal{Q}$ * and EA to RS, or, because the compound ratio will be the same, of EA to $A\mathcal{Q}$ and AS to RS, or EA to AB *. We have then $ER : R\mathcal{Q} :: \overline{EA}^2 : (A\mathcal{Q} \times AB = A\mathcal{Q} \times \overline{A\mathcal{Q} + \mathcal{Q}B} = A\overline{\mathcal{Q}}^2 + A\mathcal{Q} \times \mathcal{Q}B = A\overline{\mathcal{Q}}^2 + C\mathcal{Q} \times \mathcal{Q}D$ *$) = \overline{A\mathcal{Q}}^2 + \overline{EC}^2 - \overline{E\mathcal{Q}}^2$ *; and disjointly $ER : E\mathcal{Q} :: \overline{EA}^2 : (\overline{A\mathcal{Q}}^2 - \overline{E\mathcal{Q}}^2$ or $) \overline{E\mathcal{Q}}^2 + \overline{EA}^2 - 2E\mathcal{Q} \times EV$ *$- \overline{E\mathcal{Q}}^2 :: \frac{\overline{EA}^2}{2E\mathcal{Q}}$ * $: \frac{\overline{EA}^2}{2E\mathcal{Q}} - EV :: \frac{CE \times E\mathcal{T}}{E\mathcal{Q}} : \frac{CE \times E\mathcal{T}}{E\mathcal{Q}} - EV$; that is, by the construction, we have $ER : E\mathcal{Q} :: EF : FV$. Which proportion, when the point $\mathcal{Q}$ is fixt and A, V are coinciding with C and consequently R with q, becomes $Eq : E\mathcal{Q} :: EF : FC$; and the rectangles under the means being the same in both proportions, we have $ER : Eq :: FC : FV$, and disjointly $qR : qE :: VC : VF$; and by the proportion next before this, it appears that ER is less than Eq, and consequently that qR and CV lye contrary ways from q and C, which will easily be found so in all other cases.

* Euclid. VI. 2.
* Euclid. VI. 4.
* Euclid. I. 35.
* Euclid. II. 5.
* Euclid. II. 13.

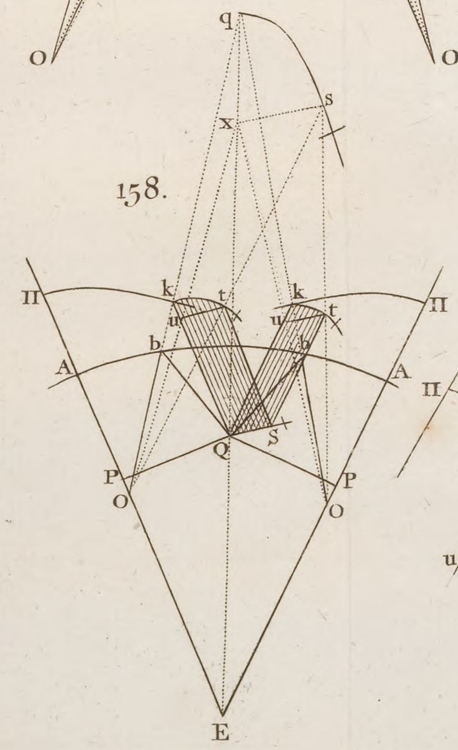
548. *Corol.* 1. Putting $\mathcal{T}\mathcal{Q} : \mathcal{T}E : \mathcal{T}q \therefore 1 : n$, as in prop. 1, $\mathcal{T}E$ or $\mathcal{T}C = \mathcal{T}$, and $CV = V$, in the concave speculum we have the exact aberration $qR = \frac{\overline{n-1}^2, \mathcal{T}, V}{2\mathcal{T} + n\overline{-1}, V}$. For we had $EF : EC :: E\mathcal{T} : E\mathcal{Q}$ *, and disjointly $CF : CE :: \mathcal{T}\mathcal{Q} : \mathcal{Q}E :: 1 : n-1$; whence $CF = \frac{2\mathcal{T}}{n-1}$, VF

* Rem. 546.

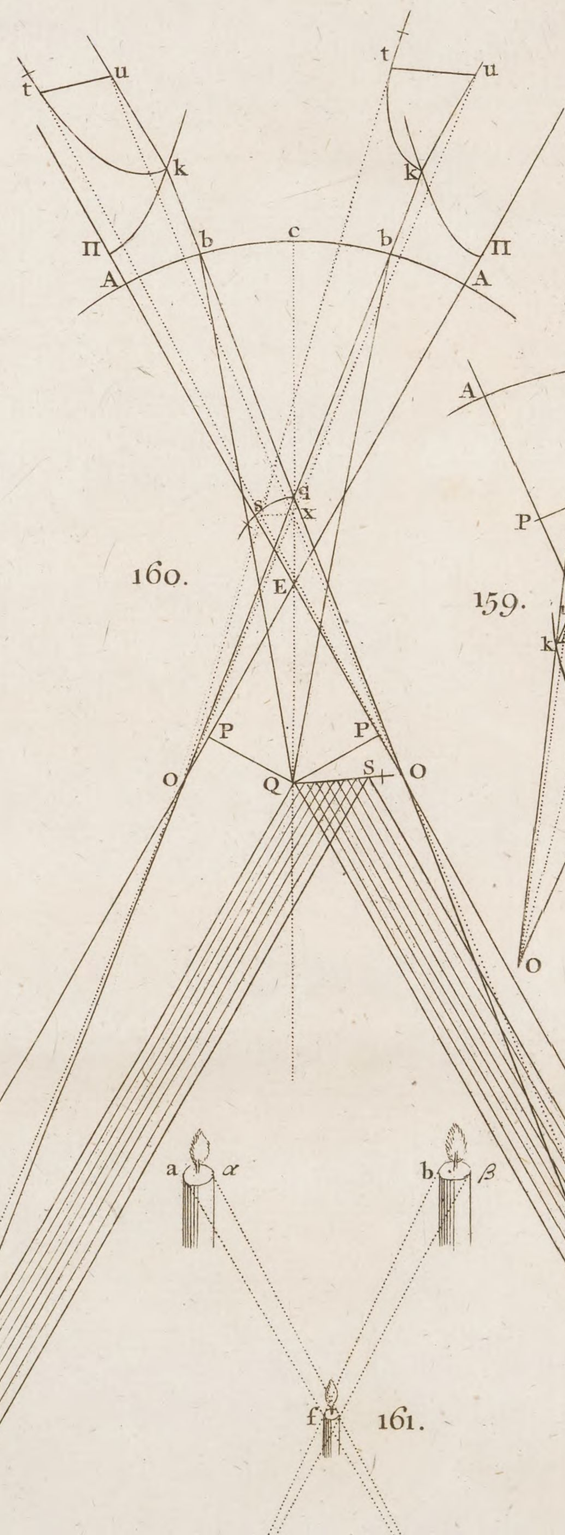
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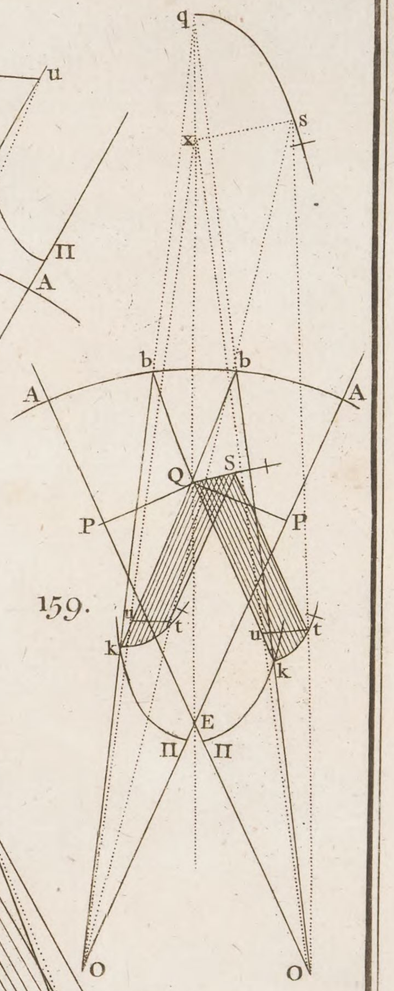
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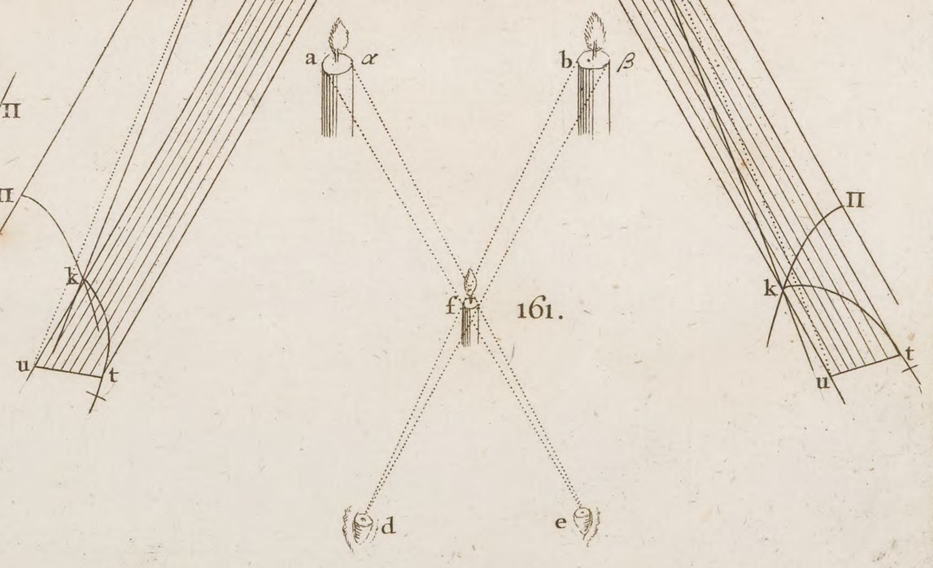
160.


159.



161.





$VF = \frac{2T}{n-1} + V$ and $qR = \frac{VC}{VF} \times qE = \frac{n-1, T, V}{\frac{2T}{n-1} + V}$.

Fig. 165.

549. *Corol.* 2. Likewiſe ſuppoſing za an incident ray on the convex ſpeculum ac, zar the reflected ray produced, and putting $tz : te : tq \div m : 1$, te or $tc = t$ and $cv = v$, we have the exact aberration $qr = \frac{\overline{\frac{m+1}{m}}^2, t, v}{2t - \frac{m+1}{m}v}$.

For we had $ef : ec :: et : ex$ and disjointly $cf : ce :: tz : ze :: m : m+1$; whence $cf = \frac{2tm}{m+1}$ and $vf = \frac{2tm}{m+1} - v$ and $qr = \frac{vc}{vf} \times qe = \frac{\frac{m+1}{m}t, v}{\frac{2tm}{m+1} - v}$.

550. *Corol.* 3. Theſe aberrations, by dividing an unite by the denominator of each fraction and multiplying the quotients by the numerator of each, may be turned into theſe ſeries. $qR = \overline{n-1}^2, \frac{V}{2} - \overline{n-1}^3, \frac{VV}{4T} + \overline{n-1}^4 \frac{V^3}{8TT}$ — &c. $qr = \overline{\frac{m+1}{m}}^2, \frac{v}{2} + \overline{\frac{m+1}{m}}^3, \frac{vv}{4T} + \overline{\frac{m+1}{m}}^4, \frac{v^3}{8tt}$ + &c.

Fig. 164.

551. *Corol.* 4. Compleat the rectangle $AVRY$, and while the point A deſcribes the quadrantal arch CAA, the point Y will deſcribe a curve qY, continually convex towards its tangent qX drawn perpendicular to the axis qC. For, we had $qR = \frac{VC}{VF} \times qE$ *, in which qE is given, and the ratio $\frac{VC}{VF}$ increaſes continually.

* Rem. 546.

Fig. 165.

552. *Corol.* 5. Likewiſe if z be the focus of incident rays, as za, on the part of the ſpeculum $zacz$ which is convex towards z, the curve qy, deſcribed by the angular point y of the variable rectangle $avry$, will be continually convex towards its tangent qx, drawn perpendicular to the axis qc. For we had $qr = \frac{vc}{vf} \times qe$ *, which increaſes continually ad infinitum till the point v arrives at f, and then decreaſes. Theſe two curves belong to the third order.

* Rem. 549.

553. *Corol.* 6. When V is very ſmall we have $qR = \overline{n-1}^2 \times \frac{1}{2}V$ very nearly by cor. 3; and conſequently the aberration from the principal focus T is $\frac{1}{2}V$ very nearly; and $qR : \frac{1}{2}V :: \overline{n-1}^2 : 1$; that is, if the point of incidence be given and the poſition of the ray be varied, its aberrations are as the ſquares of their diſtances from the center of the ſpeculum very nearly.

Proposition II.

Things remaining as in prop. 1, *if the relation among the focal diſtances* T, t *and the numbers* m, n *be expreſſed by this equation* $n\overline{n-1}^2$, T $= \overline{\frac{m\overline{m-1}^2}{mm}}$, t, *the laſt image of the object will be more diſtinct than if thoſe quantities* T, t, m, n *were ſome other ways related.*

554. Any incident ray QA being reflected into the lines Aa, aS, ſuppoſe another ray za flowing from z, to be reflected into the line $a\alpha$, and let aA and $a\alpha$ be produced backwards till they meet the axis of the ſpecula in R and r; then ſince the aberrations qR, qr tend the ſame way from q *, if they were equal to one another, the rays Aa, $a\alpha$, and alſo aS, az, would coincide; and ſo the ray QA would be reflected exactly to z, the focus of the neareſt rays to the axis.

Fig 166.

* Rem. 546.

555. Now the firſt ratio of the aberrations qR, qr, is expreſſed by the firſt terms of the two ſeries found above *, and by ſuppoſing thoſe terms equal, we have $n\overline{n-1}^2 : \overline{\frac{m+1}{m}}^2 :: v : V :: \frac{\overline{av}^2}{4t} : \frac{\overline{AV}^2}{4T}$ *$:: \frac{\overline{cq}^2}{t} : \frac{\overline{Cq}^2}{T}$ * $:: \overline{\frac{m-1}{m}}^2 t : \overline{n+1}^2, T$ *. Whence $n\overline{n-1}^2, T = \overline{\frac{m\overline{m-1}^2}{mm}} t$. Therefore when the relation of m, n, T, t is expreſſed by that equation, the exact aberrations qR, qr, which ſet out from q with equal velocities of increaſe, will continue more nearly equal, during a ſmall increaſe, than if they had ſet out with unequal velocities; and ſince the diminution of Rr (by reaſon of the equal angles raR, αaA, zaS,) diminiſhes the aberration zS, the laſt image ϖz will be more diſtinct than if the quantities T, t, m, n were ſome other ways related.

* Rem. 550.

** Art. 204.

* Rem. 554.

556. *Corol.* 1. Hence if either of the aberrations qR, qr, belonging to any given ſemi-aperture AV, be made ſmaller, their difference Rr will alſo become ſmaller. For ſince the firſt ratio of the ſubtenſes, XY, Xy, of the angles of contact at q, was made the ratio of equality,

the curves qY, qy have the ſame curvature at q; which, if diminiſhed, will diminiſh any given ſubtenſes XY, Xy, and alſo their difference Yy or Rr; as is eaſily underſtood without farther proof.

557. *Corol.* 2. Conſequently, *cæteris paribus*, the appearance of the object will become diſtincter by diminiſhing n. For it will become diſtincter, *cæteris paribus*, by diminiſhing the aberration $\varkappa S$, which decreaſes with Rr, which decreaſes with qR *, which decreaſes with n, as appears by this expreſſion of $qR = \frac{T, V}{\frac{2T}{n-1}^2 + \frac{V}{n-1}}$ *.

* Rem. 556.

* Rem. 548.

PROPOSITION III.

Things remaining as in prop. 2, if the ſpecula have equal focal diſtances, the uſe of their innermoſt parts, which reflect truer than the reſt, will be preſerved in both.*

* Rem. 546.

Fig. 167.

558. The lines $QA\,aB\,\varkappa$ being the courſe of the outermoſt ray, let QA produced cut the tangents $c\,d\,\alpha$, $C\,\beta\,D$ in d and D; join $D\,q$ cutting the former tangent in α, and $\alpha\,\varkappa$ cutting the latter in β, and $\beta\,q$ cutting the former in γ.

559. By inſpection of the figures it appears that $c\alpha$ and $C\beta$ are ſomewhat longer than ca and CB, and conſequently that a hole at C whoſe ſemidiameter is equal to $C\beta$, may be wide enough for collateral pencils to paſs thro' to the exterior parts of the image at $\varkappa$.

560. The ſemidiameter of the hole in the convex ſpeculum muſt be equal to cd, for cd will differ inſenſibly from the arch cb cut off by QA, when cQ is ſmall; and ſince the innermoſt ray that deſcribes the line $Q\beta$ will be very nearly reflected into $\beta\gamma$, if $c\gamma$ happens to be ſhorter than cd, this ray will be loſt in the hole at c; and ſo will all thoſe that fell on a zone of the concave, whoſe breadth $\beta\delta$ is deſcribed by the angular motion of the line $\beta\gamma q$, about the point q, till γ coincides with d.

561. On the other hand, if $c\gamma$ happens to be longer than cd, a zone of the convex ſpeculum, whoſe breadth is γd, will become uſeleſs.

562. Therefore to preſerve the uſe of the innermoſt parts of the ſpecula, which reflect truer than the reſt, we muſt have $c\gamma = cd$; as it will always be if the focal diſtances of the ſpecula be equal to one another, and conſequently $nn - 1 = \frac{mm-1}{mm}$ *.

* Prop. 2.

563. For the ratio of cd to $c\gamma$, being compounded of cd to CD, CD to $c\alpha$, $c\alpha$ to $C\beta$, $C\beta$ to $c\gamma$, that is, of cQ to CQ, Cq to cq, $c\varkappa$ to $C\varkappa$, Cq to cq, will come out a ratio of equality by putting $T = t = 1$, in the values of theſe terms *, and $nn - 1 = \frac{mm-1}{mm}$, where

* Rem. 544.

convenient for the reduction of the compound ratio.

564. *Corol.* 1. When the ſpecula have equal focal diſtances, the quantity of incident light, is to the part loſt in the holes, as 1 to $n - 1 - \frac{1}{m}$ nearly. For it is in a duplicate ratio of CD to $C\beta$ nearly, or of $CD \times c\alpha$ to $c\alpha \times C\beta$, that is of $Cq \times c\varkappa$ to $cq \times C\varkappa$, or of $n+1 \times \frac{m-1}{m}$ to $\frac{m-1}{m} \times \frac{mm-1}{m} - n + 1$ *, or of 1 to $\frac{mm-1}{mm, n+1} - \frac{1}{m}$ or $\frac{nn-1}{n+1}$ * $- \frac{1}{m}$ or $n - 1 - \frac{1}{m}$.

* Rem. 544.

* Prop. 1.

565. *Corol.* 2. Therefore if the angle CQA and conſequently the quantity of incident light be given, the part loſt in the holes is as $n - 1 - \frac{1}{m}$ nearly, and therefore is diminiſhed by diminiſhing n or m; for they both decreaſe together; becauſe $nn - 1 = \frac{mm-1}{mm} = 1 - \frac{1}{mm}$; whence $n = \sqrt{2 - \frac{1}{mm}}$.

566. *Corol.* 3. Therefore to ſave light and improve the apparent diſtinctneſs of the object, I have aſſumed the ſmall numbers 5 and 4 ſucceſſively for m, and taking $n = \sqrt{2 - \frac{1}{mm}}$, have computed the dimenſions of the Iſt and IId microſcopes in the Table, from the rules in cor. 2. prop. 1, taken in the following order: where, putting $T = t = 1$, we have $Cq = n + 1$, $qc = \frac{m-1}{m}$, $cC = Cq - qc$; $CQ = \frac{1}{n} Cq$, $cQ = CQ - Cc$, $c\varkappa = m - 1$, $C\varkappa = c\varkappa - cC$; $lo = \frac{c\varkappa + \varkappa l}{c\varkappa} \times \varkappa l$, and the diameter of a hole in a plate next the eye at $o = \frac{\varkappa l}{c\varkappa} 2c\alpha$.

567. *Corol.* 4. To ſettle the ſize of the holes in the metals, it is neceſſary to aſſume the ratio of CD to CQ, for which I put 1 to 3 in all the microſcopes, judging the reſulting angle, CQD, of 18°. 26'. 06", to afford light enough to magnify the object about 300 times, or more, ſuppoſing, in prop. 1, the line $d = 8$ inches. Hence by ſimilar triangles we have $c\alpha = \frac{cq}{Cq} \times CD$, $C\beta = \frac{C\varkappa}{c\varkappa} \times c\alpha$, $c\gamma = \frac{cq}{Cq} \times C\beta$ and $cd = \frac{cQ}{CQ} \times CD$; the two laſt, by coming out equal, as they ought *, will verify the calculation.

* Rem. 562.

Reflecting

Reflecting compound microscopes.

Fig. 163. 166. 167.

	I.	II.	III.	IV.
m	5.	4.	3. 7895	5.
n	1. 4	1. 39194	1. 39194	1. 39464
CT	1.	1.	1.	1. 0320
ct	1.	1.	1.	1.
$c\mathcal{Q}$	0. 1143	0. 0765	0. 0626	0. 1007
cC	1. 6	1. 6419	1. 6558	1. 6712
Cx	2. 4	1. 3581	1. 1337	2. 3288
CD	0. 5714	0. 5725	0. 5728	0. 5906
$c\alpha$	0. 1905	0. 1795	0. 1763	0. 1912
$C\beta$	0. 1143	0. 0813	0. 0716	0. 1113
$c\gamma$	0. 0381	0. 0225	0. 0220	0. 0360
cd	0. 0381	0. 0225	0. 0209	0. 0333
$\mathcal{Q}g$	0. 5143	0. 5763	0. 4545	0. 5104
gh	0. 0343	0. 0222	0. 0189	0. 0323
Mag. power	300 times	300 times	300 times	300 times
xl	0. 1867	0. 1485	0. 1407	0. 1859
lo	0. 1956	0. 1558	0. 1479	0. 1950
diam. hole o	0. 0186	0. 0187	0. 0190	0. 0185
ang. $C\mathcal{Q}D$	18°. 26′. 06″	18°. 26′. 06″	18°. 26′. 06″	18°. 26′. 06″
ang. $C\mathcal{Q}\beta$	3. 48. 53	2. 42. 37	2. 23. 15	2. 35. 40
Incid. light	0. 0513. 180	0. 0513. 180	0. 0513. 180	0. 0513. 180
loft light	22. 155	11. 186	8. 681	19. 671
Refidue	0. 0491. 025	0. 0501. 994	0. 0504. 499	0. 0493. 509
ang. AEC	15°. 44′. 41″	15°. 47′. 00″	15°. 47′. 00″	15°. 46′. 13″
ang. aec	5. 13. 57	4. 56. 16	4. 50. 49	5. 15. 12
CV	0. 0750. 396	0. 0754. 058	0. 0754. 058	0. 0752. 819
cv	0. 0083. 348	0. 0074. 226	0. 0071. 526	0. 0084. 008
qR	0. 0059. 144	0. 0057. 076	0. 0057. 076	0. 0057. 791
qr	0. 0060. 312	0. 0058. 259	0. 0057. 388	0. 0060. 792
Rr	0. 0001. 168	0. 0001. 183	0. 0000. 312	0. 0003. 001
$\frac{Rr}{nn}$	0. 0000. 596	0. 0000. 611	0. 0000. 161	0. 0001. 543
ang. $xl\lambda$	00°. 02′. 26″	00°. 02′. 29″	00°. 00′. 39″	00°. 06′. 15″

568. *Corol.* 5. The quantity of incident light, the part loft in the holes and the refidue, are to one another exactly as the verfed fines of the angles $C\mathcal{Q}D$, $C\mathcal{Q}\beta$ and the difference of thefe fines. For the arches of any given circle, that fubtend thefe angles, being turned about $C\mathcal{Q}$, will defcribe two fegments of a fpherical furface, proportional to the incident light and the part loft; and fo the refidue of light will be as the difference of the fegments, or of the verfed fines of the generating angles, by *Archimedes*'s Theorems on the Sphere and Cylinder.

PROPOSITION IV.

Having all the dimenfions of a reflecting double microfcope, to find the angle of aberration at the eye-glafs.

569. Let the outermoft ray $\mathcal{Q}A$, or any other that makes a given angle with the axis, be reflected into Aa and aS, to which laft let $x\lambda$ be perpendicular; then joining λl, I call $xl\lambda$ the angle of aberration at the eye-glafs, whofe fine, to the radius 1, will be $\frac{x\lambda}{xl} = \frac{m}{n} \times \frac{AV}{A\mathcal{Q}} \times \frac{Rr}{xl}$ very nearly. Fig. 166.

570. For let ro be perpendicular to RaA, and by the similar triangles $x\lambda a$, roa, and also roR, AVR, we have $x\lambda : ro :: ax : ar :: ex : er$ † $:: ex : eq^*$ $:: m : 1$; and the same $ro : rR :: AV : AR$, or in a compound ratio of $AV : AQ$, and $AQ : AR$ or $EQ : ER^*$, or $EQ\,Eq^*$, or $1 : n$. Therefore $x\lambda : rR :: m \times AV : n \times AQ$; whence $x\lambda = \frac{m}{n} \times \frac{AV}{AQ} \times Rr$, and $\frac{x\lambda}{xl} = \frac{m}{n} \times \frac{AV}{AQ} \times \frac{Rr}{xl}$ very nearly.

† See below.
* Art. 204.
* Eucl. VI. 3.
* Art. 204.

571. *Corol.* 1. In different microscopes the angles of aberration will be very nearly as $\frac{Rr}{nn}$, if the quantities of light or the angles AQC, and the magnifying powers be the same in all. For then the ratio's of AV to AQ, and of xl to dnm* and to mn are the same in all.

* Rem. 543.

572. *Corol.* 2. Hence if the residues of light in these microscopes were equal, that would be the best of them in which $\frac{Rr}{nn}$ is the least. For the apparent brightness of the object, being as the residue of light directly and the magnifying power inversely, would be given; and therefore that microscope would be the best which shews the object most distinctly, or in which $\frac{Rr}{nn}$, and consequently the angle of aberration at the eye-glass, is the least; see Art. 344 and 503.

573. *Corol.* 3. Now Rr, being the difference of the aberrations qR, qr, may be thus computed.

Fig. 166.

574. 1. In the triangle QAR, sin. ang. $R = \frac{1}{n}$ sin. ang. Q very nearly, and ang. $\frac{Q+R}{2}$ = ang. E, and CV or $V = \frac{\text{ver. sin. ang. } E}{\text{Rad.}} \times 2T$, which gives $qR = \frac{\overline{n-1}^2, T, V}{2T + \overline{n-1}, V}$*.

* Rem. 548.

575. 2. In the triangle RaS, sin. ang. $S = \frac{1}{m}$ sin. ang. R very nearly, and ang. $\frac{R-S}{2}$ = ang. e, and cv or $v = \frac{\text{ver. sin. ang } e}{\text{Rad.}} \times 2t$, which gives $qr = \frac{\left|\frac{m+1}{m}\right|^2 t, v}{2t - \frac{m+1}{m} v}$*.

* Rem. 549.

576. 1. For in the triangle QAR, sin. ang. R : sin. ang. $Q :: QA : AR^*$ $:: QE : ER^*$ $:: QE : Eq^*$ $:: 1 : n$.

* Art. 221.
* Eucl. VI. 3.

577. 2. In the triangle RaS, sin. ang. S : sin. ang. $R^* :: aR : aS^* :: eR : eS^* :: eq : ex^* :: 1 : m$, and the ang. Rae = ang. $\frac{R+S}{2}$, and $aeR = (aRe - Rae = R - \frac{R+S}{2} =) \frac{R-S}{2}$.

* Art. 204.
* Art. 221.
* Eucl. VI. 3.
* Art. 204.

578. *Corol.* 4. If greater exactness be judged necessary, we may compute Rr by these rules.

579. 1. Put $a = \frac{n-1}{n}$, and θ to 1 for the given ratio of the radius to the tangent of the angle ACQ, then taking $EV = \frac{aT - 2\theta T \sqrt{1 + \theta\theta - \frac{1}{4}aa}}{1 + \theta\theta}$ we have $CE - EV = CV$ or V, which gives qR as above.

580. 2. Put $b = qR$ and $q = \frac{\overline{n+1}, T-b-V}{V, 4\overline{T-V}}\Big|^2$ and $r = \frac{m-1}{m} t - b$, then we have cv or $v = \frac{2qt + r - 2\sqrt{q, \overline{qtt + rt - \frac{1}{4}rr}}}{1+q}$, which gives qr as above.

581. For 1. $TE : EQ :: n : n-1$, whence $EQ = \frac{n-1}{n} T = aT$; put $z = EV$ then $QV = z - aT : VA :: \theta : 1$, whence $VA = \frac{z-aT}{\theta}$. But $\overline{EV}^2 + \overline{VA}^2 = \overline{AE}^2$, that is $\frac{zz - 2aTz + aaTT}{\theta\theta} = 4TT$, which by reduction gives z or EV as above.

582. 2. $RV = qC - qR - CV = \overline{n+1}, T - b - V$, and $Rv = qc - qR - cv = \frac{m-1}{m} t - b - v = r - v$. Again $\overline{AV}^2 = V, \overline{4T - V}$ and $\overline{av}^2 = v, \overline{4t - v}$, and by similar triangles $\frac{\overline{RV}^2}{\overline{AV}^2} = \frac{\overline{Rv}^2}{\overline{av}^2}$, that is, $q = \frac{\overline{r-v}^2}{4v - vv}$, which by reduction gives v as above.

583. *Corol.* 5. Sin. ang. S : sin. ang. $Q :: 1 : mn$, very nearly by cor. 3.

584. *Corol.* 6 Therefore in the IId microscope the semiangle of the pencil at S is but 3°. 15′. 20″, and consequently the eye-glass needs so small an aperture that it cannot sensibly in-

† See Euclid VI. 3. For, by a like demonstration, a line ae, which bisects the supplement of the vertical angle xar, and cuts the base xr produced, gives two segments ex, er in the same ratio as the sides ax, ar.

crease

creaſe the aberrations cauſed by the metals; whereas the aperture of the object-glaſs of a ſingle microſcope, to give the ſame light as ours does, muſt be ſufficient for receiving a radiating angle, of 18°. 26'. 06", equal to Q, which is 5, 567 times greater than the angle S. So that here is a great advantage over a ſingle microſcope, beſides that our image is 5. 567 times greater in diameter than the object and almoſt 31 times greater in ſurface.

PROPOSITION V.

To improve theſe microſcopes by diminiſhing the angle of aberration at the eye-glaſs.

Fig. 166. 585. The curves $q\Upsilon$, qy, according to prop. 2, have the ſame curvature at q; conſequently while CA increaſes uniformly, tho' Υy or Rr began from q to increaſe with the leaſt poſſible velocity, yet ſince it increaſes continually with qR, it will become pretty large, and ſo will the angle of aberration $\varkappa l\lambda$, which is as Rr, when the light, the magnifying power, and the number n, are given *.

* Rem. 71.

586. But if we retain the curve $q\Upsilon$, as given, and alter qy ſo as to croſs $q\Upsilon$ in any intermediate point z, as repreſented in Fig. 168, then though Υy will begin at η to increaſe faſter than before, yet it will ſoon arrive at its *maximum* and then decreaſe till it vaniſhes at z, and after that will increaſe again, but not to ſo great a magnitude as before, when it increaſed continually, ſuppoſing the ſemi-aperture AC to be the ſame in both caſes.

Fig. 166. 168. 587. In like manner the angle of aberration $\varkappa l\lambda$, which here alſo is as Rr or Υy, will firſt increaſe and then decreaſe to nothing, and then increaſe again on the contrary ſide of the axis $\varkappa l$; and conſequently cannot be ſo great as when it increaſed continually on the ſame ſide of the axis: all which is ſufficiently evident from the idea of the curves $q\Upsilon$, qy.

588. Let the line $z\alpha$ drawn parallel to the axis, cut the ſemiaperture AC in α, and compleating the rectangle $z\alpha UR$, join αR, αE, αQ; then taking the values of n, T and t from the IId microſcope as having a little more light in it than the firſt, and aſſuming at diſcretion the value of U or CU, the verſed ſine of the arch $C\alpha$, we have the correſponding aberration $qR = \frac{\overline{n-1}^2, T, U}{2T+\overline{n-1}, U}$ *.

Rem. 8. lution. 589. Now to determine a new value of m, which when poſſible ſhall make $q\varkappa = qR$, put the given $qR = b$, and the given quantity $\frac{\overline{\overline{\varkappa+1}, T-b-U}|^2}{U, \overline{4T-U}} = q$; and having found a proper root x of this equation

$q = \frac{\overline{\overline{x, b+xt, 2t-b-xt-2bt}|^2}}{8bxtt, \overline{b+xt}-4bbtt}$, take $m = \frac{1}{x-1}$ and the buſineſs is done.

590. For ſince $m = \frac{1}{x-1}$ we have $x = \frac{m+1}{m}$ and $q\varkappa = \frac{\overline{\frac{m+1}{m}}|^2, t, v}{2t - \frac{m+1}{m}v}$ * $= \frac{xxtv}{2t-xv}$ and by putting $q\varkappa = qR$, that is, $\frac{xxtv}{2t-xv} = b$, we have $v = \frac{2bt}{bx+xxt}$. But $\overline{aU}^2 = 4TU - UU$ and $\overline{av}^2 = 4tv - vv = \frac{8btt}{bx+xxt} - \frac{4bbtt}{\overline{bx+xxt}|^2} = \frac{8btt, \overline{bx+xxt}-4bbtt}{\overline{bx+xxt}|^2}$; and $RU = qC - qR - CU = n+1, T-b-U$, and alſo $Rv = qc - qR - cv = \frac{m-1}{m}t - b - v = 2t - xt - b - \frac{2bt}{bx+xxt} = \frac{\overline{bx+xxt, 2t-b-xt-2bt}}{bx+xxt}$. Then by ſimilar triangles we have $\frac{\overline{RU}^2}{\overline{aU}^2} = \frac{\overline{Rv}^2}{\overline{av}^2}$, that is $q = \frac{\overline{\overline{x, b+xt, 2t-b-xt-2bt}|^2}}{8bxtt, \overline{b+xt}-4bbtt}$. Q. E. I.

* Rem. 568.

* Art. 222.

591. *Corol.* 1. When $T=t=1$, the equations are ſomewhat ſimpler; $qR = \frac{\overline{n-1}^2, U}{2+n-1, U}$. $\frac{\overline{\overline{n+1-b-U}|^2}}{U, \overline{4-U}} = q = \frac{\overline{\overline{x, b+x, 2-b-x-2b}|^2}}{8bx, \overline{b+x}-4bb}$. $m = \frac{1}{x-1}$.

592. *Example.* If it be deſired that the halves of the light of a ſolid pencil ſhall fall on oppoſite ſides of the point $\varkappa$, which probably will produce a very ſmall angle of aberration in the outermoſt rays; the verſed ſine of the angle $CQ\alpha$ muſt be half the verſed ſine of the generating angle CQA of the given pencil, by *Archimedes*'s Theorems abovementioned*. Therefore taking this ang. $CQA = 18°.\ 26'.\ 06''$, as before, by a table of verſed ſines we have the ang. $CQ\alpha = 13°.\ 00'.\ 26''$, and thence $\frac{1}{n}$ ſin. ang. $CQ\alpha$ = ſin. ang. $CR\alpha$ *, which, when $n = 1.\ 39194$, gives $CR\alpha = 9°,\ 18'.\ 21''$, and

Fig. 168.

* Rem. 568.

* Art. 222.

the

the ang. $\frac{\mathcal{Q}+R}{2}$ = ang. $CE\alpha = 11°.09'.23''\frac{1}{2}$, whose verſed ſine, to the radius $CE = 2$, is 0.0377948 $= U$. Hence $b = 0.0028811$ and $q = 29.340$.

593. Now becauſe the difference Rr, of the aberrations belonging to the outermoſt ray $\mathcal{Q}A$, is very ſmall, and that which belongs to the ray $\mathcal{Q}\alpha$ is ſmaller; in order to find ſuch a value of x as ſhall cauſe the ray $\mathcal{Q}\alpha$ to have no aberration from α, at firſt we may take $m = 4$ as in the given microſcope, and conſequently $x = \frac{m+1}{m} = 1.25$ and thence approach towards the required value of x by correcting the aſſumed one as follows.

594. Putting $y = x, \sqrt{b+x}, 2 - b - x - 2b$ and $z = 8bx, \sqrt{b+x} - 4bb$ the equation to be reſolved has this form $q = \frac{yy}{z}$ * or $yy - qz = 0$. Suppoſe the computed $yy - qz = p$, and if p comes out very ſmall, the value of x was nearly right; if not, compute $\dot{y} = 2\sqrt{-b-x}, \sqrt{b+2x} - x, \sqrt{b+x}$ and $\dot{z} = 8b, \sqrt{b+2x}$, and $2yy - q\dot{z} = p$; then $x - \frac{p}{\dot{p}}$ will be a correcter value of x, to be uſed in repeating the work over again till p comes out as ſmall as you pleaſe. This method of reſolving the equation I take for granted, and thus I found $x - \frac{p}{\dot{p}} = 1.56444 =$ the new x', and therefore $m' = \frac{1}{0.56444} = 1.7717$; but this value of m being ſo ſmall as to produce a negative value of $c\mathcal{Q} = qc - q\mathcal{Q} = 1 - \frac{1}{m} - n - \frac{1}{n}$ * $= -0.098$, I therefore reaſſumed at diſcretion a ſmaller value of $U = 0.03230$ and by repeating the ſame work I found another new $x'' = 1.263891$ and $m'' = 3.7895$, from which I computed the dimenſions of the IIId microſcope.

595. Now this latter value of $U = 0,03230$ is the verſed ſine 10°. 19′ to the radius $CE = 2$; and ſo the conical periphery of incident rays (deſcribed by the ray $\mathcal{Q}\alpha$ turned about the axis $\mathcal{Q}C$,) that will be collected accurately to α, lyes nearer to the axis than the periphery which biſects the whole pencil by 50′. $23''\frac{1}{2}$ *. In this I fully ſatisfied my ſelf by calculating, from the 2d theorem in cor. 4. prop. 4, the correſponding values of $v' = 0,0030.805$, $qR' = 0,0024.6531$ and $qr = 0,0024.6524$, whoſe difference Rr', $= 0,0000.0007$, is quite inconſiderable.

596. The ſemi-angle of the radiating pencil, $C\mathcal{Q}A = 18°.26'.06''$, will afford light enough for magnifying the object about 300 times in diameter, as I collected from an eſtimate taken from the beſt dioptrick microſcopes, and was afterwards confirmed in it by an actual eſſay of the IVth double reflecting microſcope, hereafter deſcribed. An eye-glaſs whoſe focal diſtance αl is $= 0.1407$ of an inch, will give the IIId microſcope that magnifying power *; and hence the angle of aberration $\alpha l\lambda = 39''$, which is far too ſmall to cauſe a perceptible indiſtinctneſs, eſpecially in that degree of brightneſs with which the object will appear. For in broad daylight, ſcarce one perſon in a hundred can diſcern an object, with the naked eye, that ſubtends a ſmaller angle than a minute *. Beſides, it is well known that the beſt dioptrick microſcopes do uſually bear an angle of aberration at the eyeglaſs of 15 or 20 minutes without much inconvenience.

597. Therefore if the IIId microſcope be well executed, we have great reaſon to expect it will bear a much larger angle of aberration than 39″ without inconvenience, and conſequently a larger aperture or angle of the radiating pencil, in order to give light enough for magnifying much more than 300 times: whereas I find that the beſt dioptrick double microſcopes now in uſe, do not magnify much above 200 times with ſufficient light and diſtinctneſs.

PROPOSITION VI.

The place of the object and the length of the microſcope being given, to find its other dimenſions agreeably to prop. 2.

598. What I call the length of the microſcope is $cz = m - 1 \times t$, which, by aſſuming t, gives m. The given place of the object gives the ratio of $t\mathcal{Q}$ to tc, for which putting r to 1 we have the number r. Then putting s for $\frac{mm-1}{mm}\Big|^2$ find the root n of this equation $n^3 - n - \frac{s}{r - \frac{1}{m}} = 0$; and taking $\mathcal{T} = \frac{s \times t}{nn-1}\Big|^2$ compute all the other dimenſions by the rules above *.

599. For by prop. 2, we have $\mathcal{T} = \frac{s \times t}{nn-1}\Big|^2$ and ſince $t\mathcal{Q} = tq + q\mathcal{Q}$, that is, $r \times t = \frac{1}{m}t + \frac{nn-1}{n}\mathcal{T}$ *, by ſubſtituting for $\mathcal{T}$ its value

* Rem. 591.

* Rem. 544.

* Rem. 592.

* Rem. 544.

* Art. 9

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* Rem. 544.

* Rem. 544.

and

and by reduction, we have the equation $n^3 - n - \frac{s}{r - \frac{1}{m}} = o$.

The reflecting microscopes preferable to refracting ones.

600. I computed the dimensions of the IVth microscope, by the rule in this proposition, some years before I had thoroughly considered the foregoing method by equal speculums; and having caused an essay to be made of it, which I have now by me, I found it performed very nearly as well in all respects as the very best refracting microscopes, and do not doubt but it might have excelled them had it been more exactly executed according to the proposed dimensions in the IVth column; where the angle of aberration, being but 6′. 15″, is above three times less than the like angle in the best refracting microscopes *: and since the angle of aberration of 39″ in the IIId column is near ten times less than that of the IV, we have reason to expect that a microscope exactly executed according to those dimensions, will far excel any other yet invented; for the essay-microscope wants distinctness rather than light. I will therefore conclude with some practical directions to the workman.

* *Hugenii* Dioptr. prop. 64.

Some directions for making and managing this microscope. Fig. 169.

601. Supposing the construction described in prop. 1, let the object be included between two little round plates of moscovy-glass, fixt as usual in a hole of an oblong brass plate mn, intended to slide close to the backside of the convex speculum; which therefore must be ground flat on that side, and so thin that the object may come precisely to its computed distance from the vertex of the speculum. The exactest way to determine this small distance is, first to fix the speculums and the eye-glass precisely at their computed distances from one another; and then having made the sliding plate full thick enough at first, to file it gradually thinner and thinner, till, upon its application to the metal, you find the object appears perfectly distinct. The slider must be kept tight to the back of the metal by a gentle spring. The distance of the object being thus determined once for all, at other times distinct vision to different sorts of eyes, and thro' different eye-glasses, must be procured by a gentle motion of the little tubes that contain these glasses. These tubes must be made in the usual form of those that belong to Sir *Isaac Newton*'s reflecting telescope, having a small hole in the middle of each plate, at the ends of the tube, situated exactly in each focus of the glass; the use of these holes and plates is to limit the visible area and hinder any straggling rays from entering the eye.

602. One way of procuring distinct vision by a skrew-motion of the eye-glass, while the object is viewed, may be easily understood by the figure. The tube of the eye-glass is skrewed into a ring or collar pq, having a handle qr, in which there is a hole at r; the end of the rod rs turns round in this hole without sliding endways, and its other end turns and slides endways in a hole s, while the middle part t works in a hollow skrew t, made in the handle tu of another collar ux fixt to the tube of the microscope. The knob for turning the rod is at y. Different eye-glasses must have different tubes.

603. The rays which flow from the object directly through the hole in the concave speculum and through the eye-glass, by mixing with the reflected rays, would dilute the image upon the retina, and therefore must be intercepted as follows. Let the little hole in the convex speculum be ground conical, with a conical tool or bit whose semi-angle is rather less than the angle $C\,Q\,D$, till the diameter of the larger orifice, on the polished side, be exactly equal to its computed measure $2cd$, and then the narrower orifice, on the backside will be rather wider than a section of the principal pencil, made by the plane of the backside; an exceeding small excess above that section will give room enough for the passage of collateral pencils equal to the principal one. Now let the first and last parts $Q\beta$, γx of the innermost ray $Q\beta\gamma x$, cross each other in b, and let bg, drawn perpendicular to the axis, be the semidiameter of the base of a conical solid; this base being larger than the orifice in the back of the convex speculum, will intercept all direct rays from the eye-glass. All the tubes must be strongly blacked on their insides, and so must the conical solid to hinder its reflection of rays upon the convex speculum; its little base may be made concave, having the object Q for the center of its cavity, that whatever light it may still reflect, may be thrown back upon the object, and its backside being conical and blacked all over, will either absorbe or laterally disperse any straggling rays which the concave speculum may scatter upon it, and so prevent their coming to the eye-glass. This conical solid may be held in its proper place by a very thin arm, like a narrow blade of a knife, whose edge is turned towards the object. Fig. 167, 169.

604. Notwithstanding the interposition of this conical solid, yet when the eye glass is taken out, you may see distant objects distinctly through the microscope, by rays reflected from the metals and diverging upon the eye from an image behind the convex speculum. But this mixture of foreign rays with those of the object is common to all kinds of microscopes, in viewing transparent objects, and is usually prevented by placing before the object a thick double convex lens, to collect the sky-light, exactly upon the object. This lens should be just so broad as to subtend the opposite angle to that which Fig. 168.

the

which the concave ſpeculum ſubtends at the object. The annular frame of the lens muſt be very narrow, and connected to the microſcope by two or three ſlender wires or blades whoſe planes produced may paſs through the object, and intercept from it as little ſky-light as poſſible.

605. A whitiſh grey ſky-light is the beſt for illuminating microſcopical objects, and the due quantity of it may be found by holding the microſcope at different diſtances from the window, or if this light be too faint, by going without doors and giving the microſcope different elevations, that more or leſs of the ſky may be expoſed to the object.

Fig. 167. 606. The rule for computing the place and breadth of the conical ſolid is this. $\mathfrak{Q}g = \frac{\mathfrak{Q}\varkappa}{1+mn}$, $gh = \frac{\text{ſin. ang. } C\mathfrak{Q}\beta}{\text{Rad.}} \times \mathfrak{Q}g$. For we had $1 : mn ::$ ſin. ang. $\varkappa :$ ſin. ang. $\mathfrak{Q}$ * $:: \mathfrak{Q}h : h\varkappa$ *, and conjointly $1+mn : 1 :: \mathfrak{Q}\varkappa : \mathfrak{Q}g$ when the angles are vaniſhing. Then in the triangle $\mathfrak{Q}gh$, we have $gh : g\mathfrak{Q} ::$ tang. ang. $g\mathfrak{Q}h$: Radius. The light ſtopt by this ſolid is an inconſiderable part of the whole pencil. For if it could poſſibly be conveyed to the eye, in the IVth microſcope it would only ſuffice to increaſe the apparent diameter of the object in the ratio of 51 to 52. In the third microſcope the light loſt is not half ſo much as in the IV, and therefore would not ſerve to increaſe the apparent magnitude in the ratio of 100 to 101 with the ſame degree of brightneſs. And Mr. *Huygens*, ſpeaking of Sir *Iſaac Newton*'s teleſcope, ſays *, "that by the meer reflection of the metalline ſpeculum, there are not ſo many rays loſt as in glaſſes; which reflect a conſiderable quantity by each of their ſurfaces, and beſides intercept many of them by the obſcurity of the matter."

* Rem. 583.
* Art. 221.
Phil. Tranſ. Abr. Vol. I. p. 199.

Rule for the meaſures upon the axis

607. Extraordinary care muſt be taken to get two ſpeculums exactly ſpherical and exactly of the ſame ſphere. The leaſt deflection of their figures, even towards thoſe of the conick ſections, though advantageous in teleſcopes, would here have a contrary effect, as being contrary to the deſign of the preſent theory. That the angle of aberration may be the ſame as in the Table, the focal diſtance of each ſpeculum ſhould be an inch, but ſince it is difficult to figure the ſpecula ſo exactly to an inch focal diſtance, as to ſuit preciſely with the other meaſures calculated from it; when ſeveral concave and convex ſpecula are finiſhed by the ſame tools, firſt examine their ſeveral focal diſtances, by analogy of the methods deſcribed in Art 63 and 64, if no better occurs, and meaſure them by a diagonal ſcale of inches and decimals, and couple each concave and convex whoſe focal diſtances come the neareſt to equality: then, by an arithmetical mean between the two focal diſtances multiply the meaſures in the axis of the IIId microſcope, *viz.* 0.0626, 1.6585 1.1337, 0.4545 for obtaining the meaſures of $c\mathfrak{Q}$, cC, $C\varkappa$, $\mathfrak{Q}g$ ſuitable to the preſent ſpecula. The rule is eivdent by cor. 2. prop 1.

Rule for the apertures and holes.

608. The ſum of theſe new meaſures of $c\mathfrak{Q}$ and cC gives the new $C\mathfrak{Q}$, and $\frac{1}{3}$ of it the new CD, which being divided by the given $CD = 0.5728$, gives a quotient $\mathfrak{Q}$, which being multiplied into the given meaſures of $c\alpha$, $C\beta$, $c\gamma$, cd, gh, in the IIId column, gives the new ones: So that the new microſcope, being ſimilar to the old one in all its parts, will have exactly the ſame magnifying power * and light, and nearly the ſame diſtinctneſs * as before, when uſed with the ſame eye-glaſs.

* Rem. 541.
* Rem. 571.

609. The concave ſpeculum muſt have an annular aperture of thin braſs blacked all over; the ſemidiameter AV, of the hole, may be found by numb. 1. cor. 3. prop. 3; or with leſs trouble by drawing the figure by a ſcale.

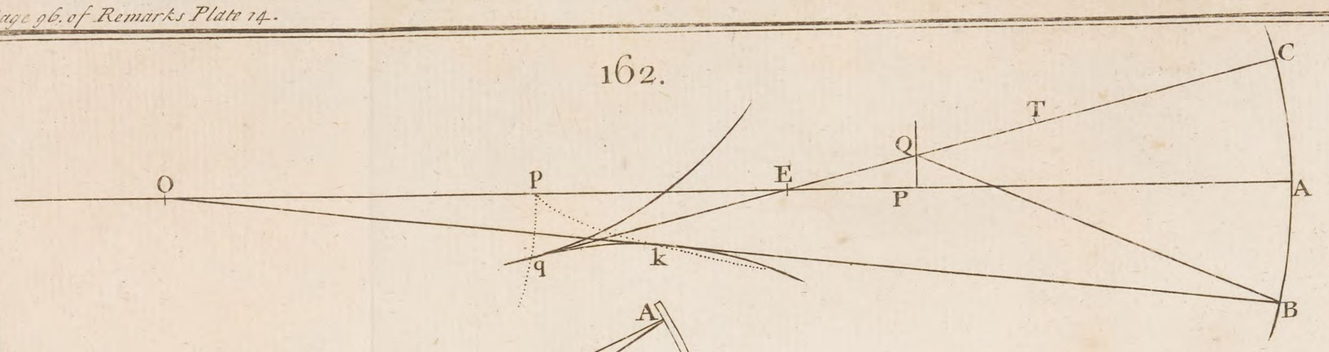
610. If in magnifying much the object ſhould appear very diſtinct but not bright enough, multiply all the the new tranſverſe meaſures CD, $c\alpha$, $C\beta$, $c\gamma$, cd, gh, by ſome ſmall number as 1.05 or 1.1, and enlarge the apertures, the baſe of the conical ſolid and the holes in the metals according to the reſulting numbers. And if upon tryal the microſcope will bear a greater enlargement of thoſe meaſures, repeat the ſame operation. Becauſe, if the holes in the metals be made too large at firſt, ſome of the innermoſt and beſt rays will be loſt in them. It may not be improper to have an aperture upon the convex ſpeculum, leſt the uncovered annulus ſhould reflect any ſcattered rays to the eye,

Sir *Iſaac Newton*'s thoughts upon teleſcopes and microſcopes.

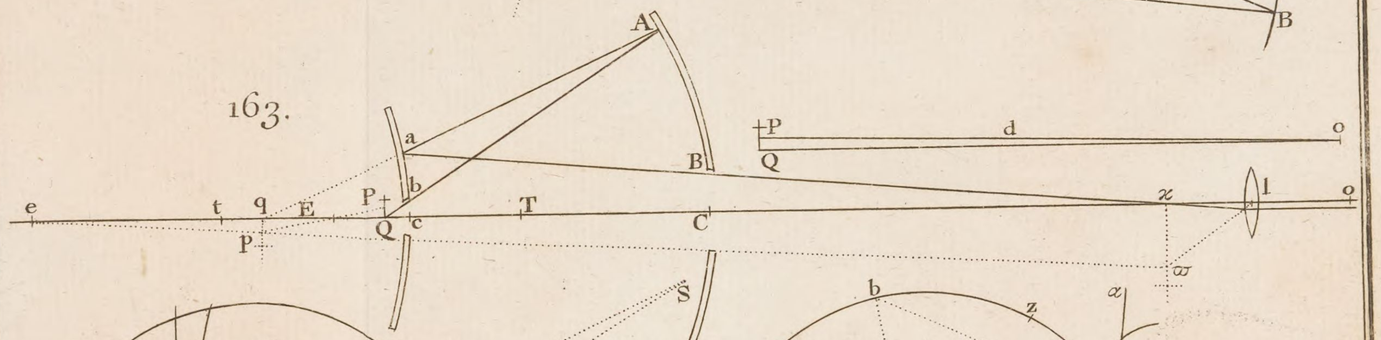
611. In looking over Sir *Iſaac Newton*'s and Mr. *Ja. Gregorie*'s letters, publiſhed * ſoon after I had invented the foregoing microſcope, I had the pleaſure to find its conſtruction anſwered more perfectly to Sir *Iſaac*'s thoughts upon the improvement of microſcopick inſtruments than any other conſtruction yet extant. Mr *Gregorie*, in arguing for the preference of his own to Sir *Iſaac Newton*'s reflecting teleſcope; propoſes the following queſtion *. "But above "all things I deſire to know this; that ſeeing "the image made by the great ſpeculum may "be eſteemed a ſmall viſible; and ſeeing Mr. "*Newton*, in the *Tranſactions* pag. 3080, think-"eth it fitter to mak an microſcope, or tube "to hold an ſmal viſible, of one concave ſpe-"culum and one eye-glaſſe, rather than with "one ſingle eye-glaſſe, and much rather than "with one plane ſpeculum and with one eye-"glaſſe: wherefor alſo to look to this ſmal vi-"ſible the firſt alſo ſhould not be preferred to "the laſt." To this Sir *Iſaac* anſwered as follows.

* Append. to *Gregorie*'s Elem. of opticks.

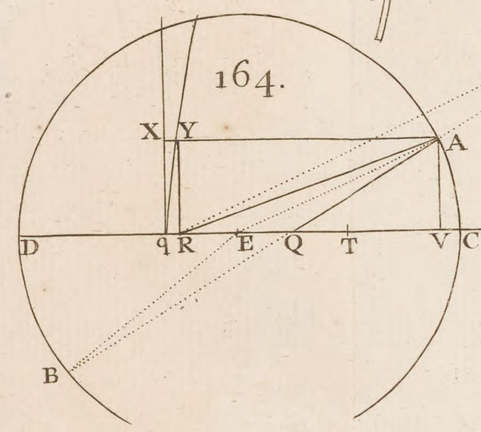
162.



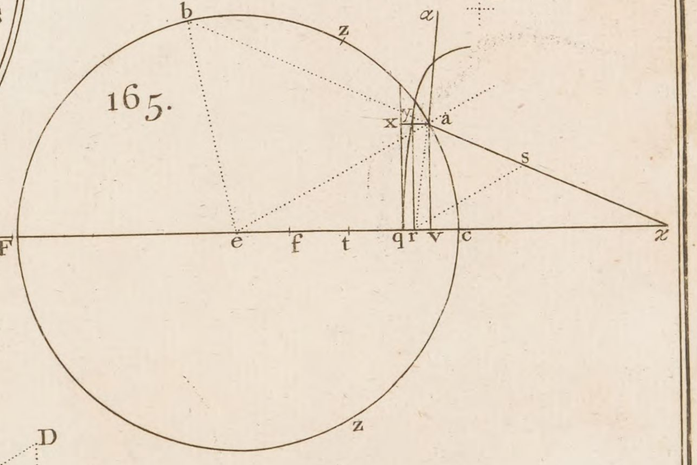
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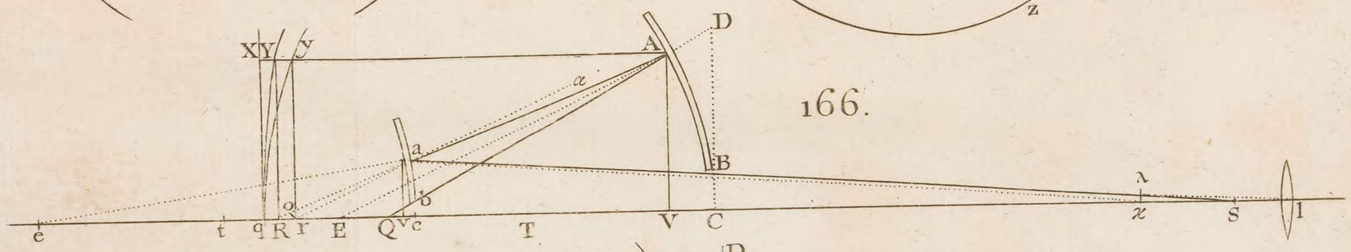
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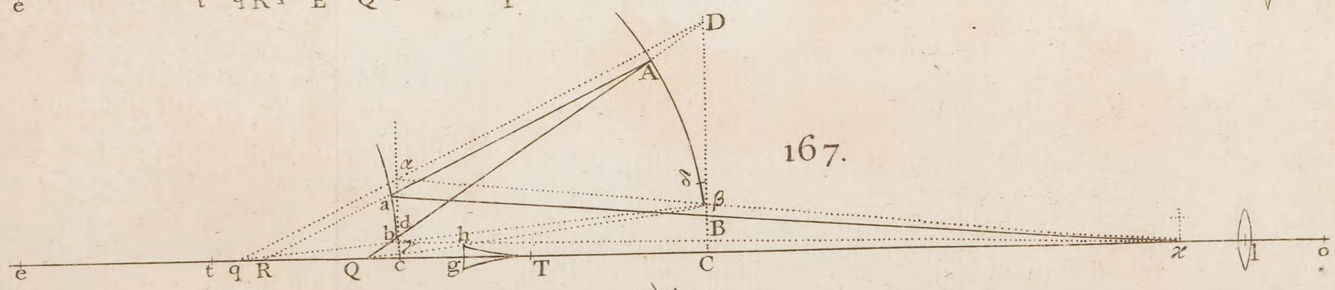
165.



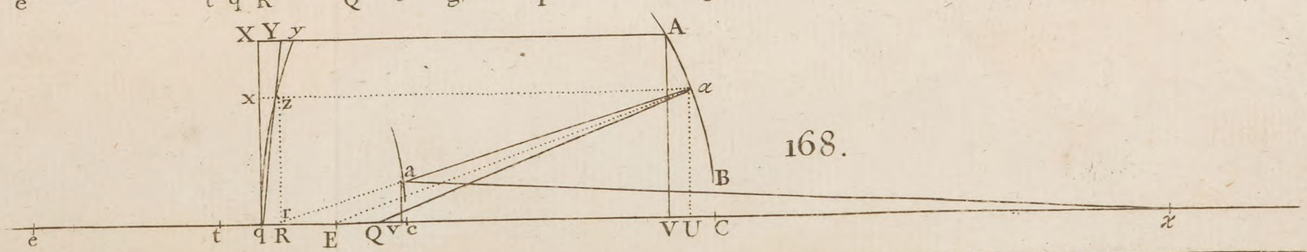
166.



167.



168.





Ibid. pag. 77. lows *. "Why I aſſign a ſmall concave with "an eye-glaſs to magnifie ſmall objects (in *Tran-* "*ſactions* pag. 3080.) and yet an eye-glaſs with- "out ſuch a concave to magnifie the image of "the great concave, is, becauſe that image doth "not require to be magnified ſo much as an "object by a microſcope; and further becauſe "the angle of the penicil of rays which flow from "any point of the ſmall object, that the object "may appear ſufficiently luminous, ought to "be as great as poſſible; and a concave will, "with equal diſtinctneſs, reflect the rays with "a greater angle of the penicil than a lens; but "in the teleſcope the angles of thoſe penicils "are not ſo great as to tranſcend the limits at "which an eye-glaſs may with ſufficient di- "ſtinctneſs refract them: and therefore in theſe "inſtrumenss I chuſe to lay all the ſtreſs of "magnifying upon the eye-glaſſes. In micro- "ſcopes alſo I would lay as much ſtreſs of mag- "nifying upon the eye-glaſs as it is well capa- "ble of, and the exceſs only upon the concave." Since the deſign of this double microſcope is, to magnify the minuteſt objects more than uſual, it anſwers the end though it ſerves only for tranſparent objects; becauſe the minute parts of all kinds of objects are tranſparent, as Sir *Iſaac Newton* and others have obſerved.

Application to Mr. *Caſſegrain*'s teleſcope. Fig. 170.

612. Imagine TQ to increaſe till it vaſtly exceeds TE, and our microſcope will be changed into Mr. *Caſſegrain*'s teleſcope, whoſe magnifying power will be as $m \times TC$ to $\varkappa l$.

613. For the apparent magnitude is to the true as the angle $\varpi l \varkappa$ to PEQ or pEq, or in a ratio compounded of $\varpi l \varkappa$ to $\varpi e \varkappa$ and of $\varpi e \varkappa$ or peq to pEq, that is, of $e\varkappa$ to $\varkappa l$ * and Eq to eq, or becauſe the rectangles will be the ſame, of Eq to $\varkappa l$ and $e\varkappa$ to eq or $t\varkappa$ to tc or m to 1; which make the ratio of $m \times Eq$ to $\varkappa l$, or, when the object is very remote, of $m \times ET$ to $\varkappa l$. Art. 60.

614. Here the naſcent aberrations from the intermediate focus q, will become equal to one another by putting $ct : CT :: m^4 : \overline{mm-1}^2$ *. For the number n, being infinitely diminiſhed, gives $\overline{nn-1}^2 = 1$. Hence I collect that the ſpeculum ac will intercept the leaſt light from AC, when $m = 3$ and conſequently when the ſemiaperture ac is to AC, that is, qc to qC or TC as 27 to 32 *; the duplicate of which being 729 to 1024, ſhews that almoſt three quarters of the incident light will be intercepted; which renders the deſign of a perfect image at $\varkappa$ impracticable. Nevertheleſs though we cannot apply a convex ac of ſo large a ſphere as is neceſſary for that purpoſe (for $ct = \frac{8}{6}\frac{1}{4} CT$) yet any ſmaller convex, *cæteris paribus*, is preferable to a concave of the ſame ſize; becauſe the imperfection of the image $\varkappa$ will reſult only from the difference of the aberrations from q in the former caſe and from their ſum in the latter. Rem. prop. 2. Rem. 44.

615. Hence by making the convex ac exactly ſpherical, and by inclining the concave AC towards a parabolick figure *, the aberrations in the image $\varpi\varkappa$ will be diminiſhed. For when both ſpeculums are ſpherical, it is eaſy to collect * that the aberration qR is to qr as CT to $\left|\frac{\overline{mm-1}^2}{mm}\right| \times ct$ nearly, and ſince to gain light we are obliged to make ct ſmaller than is requiſite for producing qR equal to qr, it follows that qR is bigger than qr, and therefore ought to be diminiſhed by the method propoſed. * Art. 304. * Rem. prop. 2.

616. The late admirable improvements in the performance of Mr. *Gregorie*'s teleſcope, even beyond what its theory ſeemed to promiſe *, has encouraged me to conſider it more minutely than before *; and to calculate tables of its dimenſions, and alſo of Mr. *Caſſegrain*'s, in order to farther improvements of both. *Gregorie*'s and *Caſſegrain*'s teleſcope farther conſidered. * Phil. Tranſ. Abr. Vol. 1. p. 204. * Rem. 137.

LEMMA I.

617. The aperture of the larger ſpeculum being given, if the area of the leſſer be juſt ſufficient to receive all the rays of the principal pencil, and the hole in the larger be not greater than this area, the brightneſs of the laſt image will be as great as poſſible at its center; but will go decreaſing outwards though very ſlowly and perhaps imperceptibly in teleſcopes that magnify much.

618. Let T be the focus and TC the focal diſtance of the larger ſpeculum $ABCBA$, CA its ſemiaperture, CB the ſemidiameter of the hole in its vertex, through which the laſt image $\varpi\varkappa$ of a remote object PQ, is projected from the leſſer ſpeculum aca. And let the outermoſt rays QA, QA, which come parallel to the axis cC and after the firſt reflection belong to the focus T, fall upon the leſſer ſpeculum at a and a. Then the area whoſe breadth is aca will be juſt ſufficient to receive all the rays of the principal pencil and to reflect them to $\varkappa$, the center of the laſt image. Fig. 171. 172.

619. Now if the leſſer ſpeculum were narrower than aa, ſome rays of this pencil, after their firſt reflection would paſs by it and be loſt; and if it were broader than aa, it would intercept more of the incident rays, which would alſo be loſt.

620. As to the breadth BB of the hole in the larger ſpeculum, if it were greater than aa, ſome of the innermoſt incident rays of the principal pencil would be loſt in it; and if it were leſs than aa, whoſe ſhadow is rather larger than it ſelf, no more of the principal pencil could fall on the ambient annulus, whoſe breadth is AB, than if BB were as large as aa. There-

fore the point $\varkappa$, to which these rays are reflected, is as bright as possible when the breadth aa is just sufficient to receive the principal pencil, and BB not greater than aa, which was the first thing to be proved.

621. The half breadth ST of the first image of the half object PQ, is terminated by the line PES drawn through the center E of the larger speculum AC*; and likewise the half breadth $\varpi\varkappa$ of the last image, is terminated by the line $Se\varpi$ drawn through the center e of the lesser speculum ac*.

* Art. 215.

* Art. 215.

622. Now to determine the ratio of the quantities of light at ϖ and $\varkappa$, let the circle $AGAG$ be the section of the sphere of the larger speculum cut by a plane passing through the circumference of its aperture AA, and $BgBg$ another section of it through the circumference of the hole BB, and K the common center of the sections. In the radius KA take $KF : ST :: Cc : cT$ and with the center F and semidiameter FD, equal to KA, describe the circle $DGHG$. Again upon KF take $Kf : ST :: Cc : CT$, and with the center f and semidiameter fh equal to KB, describe the circle $hgig$. Then, I say, the brightness of the point $\varkappa$ will be to that of ϖ, as the annular difference of the circles $AGAG$, $BgBg$, to the same annulus diminished by the lunar spaces $AGDGA$, $BghgB$ whose greatest breadths are AD, Bh.

623. For, drawing the lines aS, aS and producing them, the one will fall at D within the aperture AA and the other at H as much without it very nearly, because the angles SaT, SaT are very nearly equal. Consequently if the speculum AA were inlarged, and a circular section of it of the breadth DH nearly equal to AA, were filled with rays that come from P, parallel to PES, these rays after their first reflection would just fill the whole area aca of the lesser speculum, as before, and be thence reflected to ϖ, which would now be as bright as $\varkappa$. But the rays which fall on the lunar part $AGHGA$, without the given aperture AA, are wanting at ϖ while $\varkappa$ has those of the principal pencil which fall on the equal lunula $AGDGA$. Now by the similar* figures ADa, STa, we have $AD : ST :: (Aa : aT ::) Cc : cT$; therefore KF by construction is equal to AD, and consequently the center F of the circle $DGHG$ was rightly determined.

* Art. 204.

624. Again the smaller circle $hgig$, answers to the shadow of the lesser speculum aca, cast upon the section of the larger by the rays that come from P parallel, as it were, to ES. For the oblique ray ah, being parallel to ES, and the line aB to ET, the angle Bah is equal to SET. Whence, by the similar triangles Bah, SET, we have $Bh : ST :: (Ba : TE ::) Cc : CT$; therefore by construction Kf is equal to Bh, and consequently the center f of the said shadow was rightly determined. Now $\varkappa$ has all the rays which fall upon the lunar part $BgbgB$, and ϖ wants an equal quantity intercepted from that part. Hence the proportion of their brightness is very plain.

625. Lastly by diminishing the angle PEQ, or SET, the images ST, $\varpi\varkappa$ and the breadths AD, Bh of those lunar areas are diminished, so that the brightness of ϖ increases towards $\varkappa$, and differs less from that of $\varkappa$ as the angle PEQ is smaller, that is as the telescope magnifies more.

626. For the magnifying power of the telescope is measured by the ratio of the angles $\varpi l\varkappa$, PEQ, the former of which, must be nearly the same in every telescope as being limited by the breadth of the eye-glass, which breadth must bear a certain given ratio to its focal distance. For then the refracting angle at its edge will be of a given magnitude, and consequently the margin of the visible area will appear with a given degree of distinctness. And this is the reason, by the by, that the more a telescope magnifies, the less it takes in at one view.

627. *Corol.* 1. Hence the lunar space $AGDGA$ is equal to the rectangle $AD \times AA$ very nearly when AD is small. Perpendicular to AKA draw the semidiameter KL, cutting the arch DG in M, and FN cutting the arch AGA in O; then from the quadrants AKL, DFN, taking away the common area DKM, we have the space $ADML$ equal to $MNFK$, and by adding the space LMG to both, we have the semilunar space $ADGA$ equal to $KLGNF$ equal to $KL \times KF$ or $KA \times AD$ very nearly when AD is small. The same is true of the lunar space adjoining to the hole $BgBg$.

628. *Corol.* 2. Hence it is easy to collect, that the circle $AGAG$ is to the lunar part $AGDGA$, as the quadrantal arch AL to AD very nearly.

629. *Corol.* 3. If the lesser speculum be convex and the hole in the larger be equal to it, the last image may also be equal to the hole, but not exceed it without suffering a farther diminution of its brightness towards the margin. For, the outermost ray PD will be reflected into the lines Da, $a\varpi$; because all the rays that come from P belong to the points S, ϖ; and if $ca = CB = \varpi\varkappa$, the line $aB\varpi$ is parallel to the axis cC. But if $\varpi\varkappa$ exceeds CB, the outermost ray as $a\varpi$ will be stopt by the speculum at B.

630. *Corol.* 4. If the lesser speculum be concave and the hole in the larger be equal to it, the last image may be bigger than the hole, but not much, unless it be projected far beyond the hole, which would increase the length of the telescope.

teleſcope. For, the outermoſt ray *PA* will be reflected into the lines *ASb*, *b*ϖ; and *cb* being leſs than *ca* or *CB*, the ray *b*ϖ has room to diverge a little from the axis *cC*.

LEMMA II.

631. The hole in the larger ſpeculum being equal to the leſſer ſpeculum as before, if this ſpeculum be increaſed by a ſmall zone whoſe breadth is to half the breadth of the firſt image, as the diſtance between the ſpeculum's is to the focal diſtance of the larger, the laſt image will become uniformly bright; but ſomewhat leſs bright than its center was before, by the loſs of as much light as this zone intercepts.

632. For, drawing the lines *AS*, *AS*, the arch *aca* will cut the one in *b*, and if produced will touch the other in *d*; and then the rays flowing from *P* upon the arch *AA*, and belonging to *S* after their firſt reflection, will all be received upon the arch *bcd*, and be thence reflected to ϖ; and by turning the arch *cad* about the axis *cT*, the ſpeculum *aca* will be increaſed by a zone of the breadth *ad*, and will receive all the rays which flowed from a circular object deſcribed by *PQ* turned about the ſame axis *QC*. Now by the ſimilar figures *Aad*, *ATS*, we have *ad* : *TS* :: (*Aa* : *AT* ::) *Cc* : *CT*.

633. A circle *hkmk* deſcribed with the center *K* and ſemidiameter *Kh* equal to the ſine of the arch *cd*, is now equal to the ſhadow of the leſſer ſpeculum, caſt upon the larger by the principal pencil; and when caſt by the collateral pencil flowing from *P*, its edge *m* will touch the edge of the hole *BB*. For *Bh* is nearly equal to *ad*, becauſe the angles *Bah*, *QAP*, *aAd* are equal, and alſo the diſtances *Ba*, *aA* very nearly. Hence every point of the laſt image is equally deprived of as many rays as the leſſer ſpeculum intercepts and therefore is uniformly bright. But every point is leſs bright than the center ϰ was, before the addition of the ſaid zone, by the deduction of as many rays as the zone intercepts.

634. *Corol.* 1. Hence the former brightneſs of the center ϰ is to the latter, as $\overline{CA}^2 - \overline{ca}^2$ to $\overline{CA}^2 - \overline{cd}^2$; I mean as the differences of the ſquares of the ſines of theſe arches.

635. *Corol.* 2. The leſſer ſpeculum *dad* being convex, if the ſemidiameter of the laſt image ϖϰ be not rather leſs than *ca* or that of the former hole *CB*, ſome of the outermoſt rays of the moſt oblique pencil, now reflected from *ad*, will be ſtopt at *B*.

636. *Corol.* 3. But if the leſſer ſpeculum *dad* be concave, the laſt image ϖϰ may continue equal to, or rather bigger than the ſaid hole *BC*; becauſe the ſame ray *b*ϖ is ſtill the outermoſt of the moſt oblique pencil; the additional rays reflected from *ad* to ϖ being here on the inſide of that pencil.

637. *Corol.* 4. If the breadth of the hole be made equal to the increaſed breadth *dd* of the leſſer ſpeculum, whether concave or convex, the laſt image tho' leſs than this hole, will be rather brighteſt in its center; becauſe thoſe rays of the oblique pencils which fall on the lunar parts of the hole, explained above, will be loſt in them. But this loſs is almoſt inconſiderable, in compariſon to what was loſt before by the larger lunar ſpaces *AGDGA*.

LEMMA III.

638. Inſtead of a ſingle eye-glaſs, adapted to theſe teleſcopes, to ſubſtitute two, which, without altering the apparent magnitude of objects, ſhall ſomewhat enlarge the viſible area. Fig. 173.

639. Suppoſing the focal diſtance *l*ϰ of the ſingle eye-glaſs *lk* to be given, if towards the ſpeculums we take *lm* = 2 *l*ϰ and *ln* = ⅓ *lm*, and inſtead of the glaſs *lk* we place two other convex ones at *m* and *n*, whoſe focal diſtances are *lm* and *ln* reſpectively, the teleſcope will magnify juſt as much as before; but the viſible area will be ſomewhat diſtincter and freer from colours about its margin, and therefore may be ſomewhat enlarged if it were ſufficiently diſtinct before.

640. For biſecting *mn* in *q*, we have *qn* = *nl* by conſtruction, and taking *mf* = *ml*, we have ϰ*f* to ϰ*m* and ϰ*m* to ϰ*q* as 3 to 1. Therefore the rays of the principal pencil, which by reflection tended towards ϰ, will be refracted through the glaſs *m* to *q* *, and paſſing on to the glaſs *n*, will emerge from it in parallel lines as requiſite for diſtinct viſion. * Art. 239.

641. Hence by the glaſs *m* the image ϖϰ will be contracted to *pq*, terminated at *p* by the line *m*ϖ. Therefore joining *pn*, we have a triangle *mpn* iſoſcelar and ſimilar to *m*ϖ*l*, and therefore *pn* parallel to ϖ*l*. Conſequently, to the eye at any place *o*, the object will appear through the glaſſes *m*, *n* under an angle equal to *pnq* or ϖ*n*ϰ, that is, of the ſame magnitude as uſually through the ſingle glaſs *l*.

642. Biſecting *ln* in *o*, the eye at *o* will ſee the largeſt area at one view. For each pencil of rays reflected from the larger ſpeculum to the leſſer, takes up nearly its whole breadth *aca* and therefore a ray of each, reflected from its middle point *c*, is nearly the middlemoſt ray of each pencil; conſequently the eye will receive all theſe middle rays, if it be placed at their focus after refractions through both glaſſes, which focus might eaſily be found. But ſuppoſing *ag* to be that ray of any oblique pencil, which falls on the glaſs *m* in a line parallel to its

its axis; after refraction it will tend towards l, the principal focus of that glaſs, till meeting the glaſs n, it will emerge from it in a line ho parallel to pn, and will biſect ln in o; and ſince all the rays of that pencil will emerge parallel to ho and exceedingly near to it, we may take this point o for the place of the eye, or rather of a little hole in a thin plate hereafter to be mentioned.

643. Now ſuppoſing the glaſſes m, n were taken away, the ſaid parallel ray $ag\varpi$ would fall on the ſingle eye-glaſs kl at k, and be refracted into ki parallel to ϖl, to which all the reſt of that pencil would alſo go parallel.

644. But the viſion cauſed by the ſame rays is diſtincter to the former eye at o than to this at i, becauſe the ratio's of the focal diſtances to the reſpective apertures of the glaſſes m, n, that is, of lm to mg and ln to nh, are ſeverally double* the ratio of the focal diſtance to the aperture of the glaſs l, that is, of li or $l\varkappa$ to lk.

* Ratio dupli.

645. The diſtinctneſs will be farther promoted by making the glaſſes m, n plano-convex and placing their plane ſides towards the eye; ſo that their ſecond refractions of the rays, into air, which contribute much more to the production of colours than their firſt, may be leſs than they would have been in a contrary poſition of the glaſſes. For in the common experiment with a priſm*, if it be turned about its axis that way which makes the rays emerge more obliquely out of the ſecond refracting ſurface, the ſun's coloured image on the wall will ſoon become an inch or two longer, or more; and if it be turned the contrary way, ſo as to make the rays fall more obliquely on the firſt ſurface the image will ſoon become an inch or two ſhorter; the reaſon of which might eaſily be ſhewn by theory*. The proportions and places of the glaſſes m, n might be variouſly altered but with little difference in their effect.

* Rem. 392.

* See Newt. Opt. Lect. P. I. ſec. 2. n. 29.

646 *Corol.* 1. Let the outermoſt ray $arqs$, of the principal pencil, cut the glaſſes m, n at r and s, and ſince $qn = qm$ we have $ns = (mr =) \frac{\varkappa m}{\varkappa c} \times ac$. Now it is eaſy to underſtand, that ns is the ſemidiameter of the little hole to be placed at o, through which the principal pencil and all others, as having originally the ſame breadth or baſe at the ſpeculum aca, will juſt paſs; and conſequently, that all foreign rays which paſs by the circumference of the ſpeculum aca, conſidered as belonging to pencils of a broader baſe than aca, will fall wide of this hole and be ſtopt by the plate from coming to the eye. But to receive all the rays of the pencils made to diverge a little upon the eye, eſpecially of a ſhort-ſighted perſon, the hole at o muſt be ſomewhat enlarged tho' very little.

Hole in the plate next the eye.

Hole in the plate that limits the angle of viſion.

647. *Corol.* 2. By placing the center of a larger hole at q, whoſe ſemidiameter is pq, the ambient plate will intercept ſome ſtragling rays reflected from the imperfect margins of the ſpeculums; and by circumſcribing the laſt image, will limit the viſible area, and angle of viſion, to ſuch a ſize as the glaſſes will permit, without ſhewing it too much coloured about the margin.

Application of the theory to a given teleſcope.

648. *Corol.* 3. Hence in any given teleſcope we may find the half angle of viſion pnq (in a groſs manner) by meaſuring the diameter of the hole which circumſcribes the image at q, and the diſtance between the glaſſes, m, n, whoſe half is qn, or, rather by finding the focal diſtance qn of the glaſs n, by the ſun's rays. Then is $\frac{pq}{qn}$ the tangent of the angle pnq to the radius 1. But this angle may be found more accurately by the time of a ſtar's tranſit over the teleſcope when fixt.

649. *Corol.* 4. Hence alſo we have the place l and focal diſtance $l\varkappa$ of a ſingle eye-glaſs which will magnify juſt as much as the two given glaſſes m, n. For $\varpi\varkappa$ being equal to ac is given by meaſuring ac, and $\varkappa l : \varkappa\varpi :: qn : qp$, that is, in the ratio of the radius to the tangent of the half angle of viſion found before, and $ml = 2\varkappa l$ by conſtruction. But this ſingle glaſs may be found more exactly by the magnifying power as ſhewn hereafter.

650. *Corol.* 5. Hence alſo we have $T\varkappa$ the diſtance of the image $\varpi\varkappa$ from the focus of the object-metal, by meaſuring TC and Cm: becauſe $T\varkappa = TC + Cm + m\varkappa$.

LEMMA IV.

651. In a given teleſcope of Mr. *Gregorie's* or *Caſſegrain's* form to find the angle of aberration.

652. Put d for the focal diſtance CT of the object-metal, s for the ſine AS of its ſemiaperture AC, e for the focal diſtance of the ſingle eye-glaſs l, n to 1 for the ratio of $T\varkappa$ to TC, r to 1 for that of $c\varkappa$ to cT; then ſuppoſing the outermoſt ray $QAqar\chi$, that comes parallel to the axis, to croſs it in q and r and the laſt image $\varpi\varkappa$ in χ, and a be the tangent of the required angle of aberration $\varkappa l\chi$, we ſhall have, in *Gregorie's* teleſcope, $a = \frac{s^3}{8dde} \times \overline{r + n - \frac{n}{rr}}$, and in *Caſſegrain's*, $a = \frac{s^3}{8dde} \times \overline{r - n + \frac{n}{rr}}$.

Fig. 174, 175.

653. For ſince Tq and $\varkappa r$ are the ſucceſſive aberrations of the outermoſt ray from the firſt and ſecond focus's T, $\varkappa$ of the innermoſt rays, taking k for the conjugate focus to q of other rays reflected from the innermoſt points of the leſſer ſpeculum ac, we have $tk : tc :: tc : tq$* and alſo $t\varkappa : tc :: tc : tT$*, and conſequently* $tk : t\varkappa :: tT : tq$, and disjointly $tk : k\varkappa :: tT : Tq$.

Fig. 174.

* Art. 207.

* Eucl. VI. 14.

: Tq, and alternately $kz : Tq :: (tk : tT :: tz : tT$ * $:: \overline{tz}^2 : \overline{tc}^2 ::) rr : 1$; because $tz : tc :: tc : tT :: (tz + tc$ or $cz : tc + tT$ or $cT :) r : 1$ by construction. Therefore $kz = rr \times Tq$.

t. 204.

654. Let a ray za come parallel to the axis, and let tu be its aberration from the focus t, and since kr is the aberration of the ray qar from the focus k, we have $kr : tu :: \overline{r-1}^2 : 1$*; because we had $tz : tc$ or $te :: r : 1$, and disjointly ez or $ek : et :: r - 1 : 1$. Hence $kr = \overline{r-1}^2 \times tu$.

em.

655. Therefore since kr and es, the versed sines of ca, tend contrary ways from k and c*, in *Gregorie*'s telescope we have the whole aberration $zr = kz + kr = rr \times Tq + \overline{r-1}^2 \times tu$

656. In the construction we put $Tz : TC$ or $d :: n : 1$, whence $Tz = nd$; we also put $cz : cT :: r : 1$ and disjointly $Tz : cT :: r - 1 : 1$, whence $cT = \frac{nd}{r-1}$ and $cz = \frac{ndr}{r-1}$, and by the

tt. 204.

similar figures qac, qAC*, we have $ac : AC$ or $s :: (cq : Cq ::) tc : cT :: r : r + 1$, whence $ac = \frac{ns}{r-1}$. We had also $tc : tT :: r : 1$ and conjointly $tc : cT$ or $d :: r : r + 1$. Whence $tc = \frac{r}{r+1} \times cT = \frac{ndr}{\overline{r-1} \times \overline{r+1}}$.

em.

657. Now the aberration $Tq = \frac{\overline{CA}^2}{8CT}$ * $= \frac{ss}{8d}$ and likewise $tu = \left(\frac{\overline{ca}^2}{8ct} = \frac{nnss}{\overline{r-1}^2} \times \frac{\overline{r-1} \times \overline{r+1}}{8ndr}\right) = \frac{nss}{8dr} \times \frac{r+1}{r-1}$. Therefore the longitudinal aberration $zr = (rr \times Tq + \overline{r-1}^2 \times tu = \frac{ss}{8d} rr + \frac{nss}{8dr} \times \overline{rr-1} =) \frac{ss}{8d} \times \overline{rr + nr - \frac{n}{r}}$.

658. Lastly, by the similar figures $z\chi r$, car we have $z\chi : zr :: (ca : cr :: ca : cz$*, that is, $:: \frac{ns}{r-1} : \frac{ndr}{r-1} :: s : dr$; whence the lateral aberration $z\chi = \left(\frac{s}{dr} \times zr = \right) \frac{s^3}{8dd} \times \overline{r + n - \frac{n}{rr}}$ and $a = \left(\frac{z\chi}{zl} = \right) \frac{s^3}{8dde} \times \overline{r + n - \frac{n}{rr}}$. Q. E. J.

175.

659. By revising the computation with regard to the other figure and changing a sign or two, it will appear, in *Cassegrain*'s telescope, that $a = \frac{s^3}{8dde} \times \overline{r - n + \frac{n}{rr}}$. For now $zr = kz - kr$, because the versed sine cs and consequently the aberration kr* are moved to the other sides of c and k. Q. E. J.

* Rem. 146.

660. *Corol.* For the magnifying power of the telescope put m to 1, then $m = \frac{rd}{e}$*, and in *Gregorie*'s, $a = \frac{ms^3}{8d^3} \times \overline{1 + \frac{n}{r} - \frac{n}{r^3}}$ and in *Cass.* *segrain*'s, $a = \frac{ms^3}{8d^3} \times \overline{1 - \frac{n}{r} + \frac{n}{r^3}}$.

* Rem. 612.

PROBLEM.

To compose a telescope of Mr. Gregorie's *or* Cassegrain's *form, which being of a given length, shall have a given angle of vision, and shew objects with a given degree of brightness and distinctness, and magnified as much as these given conditions can permit.*

661. The algebraick expressions of the given degrees of apparent brightness and distinctness may be taken from a given telescope of Sir *Isaac Newton*'s form, or rather of either of the forms proposed. In the given telescope let A be the half breadth of the larger speculum, B that of the lesser, that is, half the least breadth of the oval plane if *Newton*'s telescope be made choice of, and M to 1 the ratio of its magnifying power; and for the quantity $\frac{\overline{A+B} \times \overline{A-B}}{MM}$ put b, which being as the apparent brightness in the given telescope*, may be retained in the telescope required, or be increased or diminished in the same ratio as this given degree of apparent brightness is intended to be increased or diminished. In the telescope required, to the radius 1 let a be the given tangent of the proposed angle of aberration, which likewise may be the same and consequently the distinctness the same as in the given telescope, or different at discretion; and v the tangent of half the proposed angle of vision; d the given focal distance CT of the larger speculum ABC, n the given place of the last image formed by the lesser speculum ac, and n to 1 the given ratio of Tz to TC.

* Art. 348.

Fig. 171. 172.

662. *Cas.* 1. In *Gregorie*'s telescope having computed a number $c = \frac{64n^4 aa}{bv^4} dd$, find the greatest affirmative root r of this equation, $\overline{rr \times \overline{r + n - \frac{n}{rr}}^2} \times \overline{r-1}^4 - c \times \overline{1 + \frac{n}{r-1}} \times \overline{1 - \frac{n}{r-1}} = 0$, and in CT produced taking $Tc = \frac{nd}{r-1}$, and back again $ct = \frac{r}{r+1} \times Tc$, the point c will be the vertex and t the princi-

Solution.

Fig. 172.

pal

pal focus of the ſmaller concave ſpeculum ac. The ſemiaperture of the larger concave will be $d\sqrt{\frac{8na}{v \times r-1 \times r+n-\frac{n}{rr}}} = CA$, and that of the leſſer will be $\frac{n}{r-1} \times CA = ca = CB = \varpi x$, and the focal diſtance of the eye-glaſs will be $\frac{1}{v} \times \varpi x = xl$. This teleſcope will ſhew objects as required and magnified in diameter in the ratio of $r \times d$ to xl.

663. *Caſ.* 2. In *Caſſegrain*'s teleſcope having computed the number $c = \frac{64n^4aa}{bv^4}dd$, find the greateſt affirmative root r of this equation, $rr \times \left|r-n+\frac{n}{rr}\right|^2 \times \overline{r+1}^4 - c \times 1+\frac{n}{r+1} \times 1-\frac{n}{r+1} = o$, and in TC taking $Tc = \frac{nd}{r+1}$ and back again $ct = \frac{r}{r-1} \times Tc$, the point c will be the vertex and t the principal focus of the convex ſpeculum ca. The ſemiaperture of the concave ſpeculum will be $d\sqrt{\frac{8na}{v \times r+1 \times r-n+\frac{n}{rr}}} = CA$, and that of the convex will be $\frac{n}{r+1} \times CA = ca = CB = \varpi x$ and the focal diſtance of the eye-glaſs will be $\frac{1}{v} \times \varpi x = xl$. This teleſcope will ſhew objects as required and magnified in the ratio of $r \times d$ to xl.

Fig. 171.

Analyſis. Fig. 171.

664. *Caſ.* 1. For ſince $1 : n :: TC$ or $d : Tx$, we have $Tx = nd$; we have alſo tx, tc, tT continual proportionals in ſome unknown ratio, ſuppoſe of r to 1. Hence $cx : cT :: r : 1$ *, and diſjointly $Tx : Tc :: r-1 : 1$, whence $Tc = \frac{nd}{r-1}$. Again, ſince $tc : tT :: r : 1$, conjointly we have $tc : Tc :: r : r+1$, whence $tc = \frac{r}{r+1} \times Tc$.

* Euclid. V. 22.

665. Let the required ſemiaperture $CA = s$, then by the ſimilar * figures acT, ACT, made by the reflection of the outermoſt ray ATa of the principal pencil, we have $ac : cT :: (AC : CT ::) s : d$, whence $ac = \frac{ns}{r-1}$. Taking this for the half breadth of this ſpeculum, and the hole's half breadth CB equal to it, the center of the viſible area will be the brighteſt poſſible *, and the quantity of light in it will be as $\overline{AC}^2 - \overline{ac}^2 = (\overline{AC+ac} \times \overline{AC-ac} = \overline{s+\frac{ns}{r-1}} \times \overline{s-\frac{ns}{r-1}} =) 1+\frac{n}{r-1} \times 1-\frac{n}{r-1} \times ss$.

* Art. 204.

* Rem. 617.

666. Suppoſe $xl = e$ and m to 1 as the apparent magnitude to the true, that is, in a ratio compounded of TC to xl and tx to tc *; then we have $m : 1 :: r \times d : 1 \times e$, whence $m = \frac{rd}{e}$.

* Rem. 144. or 611.

667. Now the apparent brightneſs of the center of the viſible area, is as $\frac{\overline{AC}^2 - \overline{ac}^2}{mm}$ * $= 1+\frac{n}{r-1} \times 1-\frac{n}{r-1} \times \frac{eess}{rrdd}$. For the rule here quoted, being not confined to any certain magnitude of the viſible area, holds good of its center conſidered as an area diminiſhed to a phyſical point. Therefore putting this value of the apparent brightneſs equal to b, as ſettled in the *data*, we have $b = 1+\frac{n}{r-1} \times 1-\frac{n}{r-1} \times \frac{eess}{rrdd}$, which I ſhall call the firſt equation.

* Art. 34.

668. For ſhortneſs of notation putting $t = r+n-\frac{n}{rr}$, we have the tangent of the propoſed angle of aberration, that is, $a = \frac{s^3t}{8dde}$ *, our ſecond equation.

* Rem. 651.

669. By the *data* we have $\varpi x : xl$ or $e :: v : 1$, whence $\varpi x = ev$. Now ſince we took the hole's half breadth $CB = ca$, we may chuſe any magnitude for the image ϖx not much exceeding CB *. For the angle ϖlx being equal to the given angle of viſion, and the breadths of ſimilar eye-glaſſes being as their focal diſtances, the refractions of the rays flowing from ϖ upon the margin of theſe glaſſes will alſo be ſimilar and equal, and conſequently the indiſtinctneſs and colours about the margin of the viſible area will be nearly the ſame whatever ſize we pitch upon for the image ϖx; we may therefore put $\varpi x = ca$, that is, $ev = \frac{ns}{r-1}$, which is our third equation.

* Rem. 630.

670. Hence $e = \frac{ns}{v, r-1}$ and $ee = \frac{nnss}{vv, \overline{r-1}^2}$, which values being ſubſtituted in

the

the firſt and ſecond equations, give $b = 1 + \frac{n}{r-1} \times 1 - \frac{n}{r-1} \times \frac{nns^4}{ddvvrr,r-1^2}$ and $a = \frac{sstv,r-1}{8ndd}$, whence $ss = \frac{8andd}{tv,r-1}$ and $s^4 = \frac{64aannd^4}{ttvv,r-1^2}$; therefore $b = 1 + \frac{n}{r-1} \times 1 - \frac{n}{r-1} \times \frac{64n^4ddaa}{v^4ttrr,r-1^4}$.

Laſtly by reſtoring $r - n + \frac{n}{rr}$ for t and putting $c = \frac{64n^4ddaa}{bv^4}$, we have $rr \times \overline{r - n + \frac{n}{rr}}^2 \times \overline{r-1}^4 = 1 + \frac{n}{r-1} \times 1 - \frac{n}{r-1} \times c$. We muſt have the greateſt affirmative root of this equation in order to magnify the more *, which root being found, the ſolution of the problem will appear very evident by looking back upon the analyſis. *Q. E. F.*

671. *Caſ.* 2. By reviſing the analyſis of the firſt caſe with regard to the other figure, the analogous changes will appear ſo plain as to need no repetition. *Q. E. F.*

672. I obſerved above that in this ſolution the viſible area will have the propoſed degree of brightneſs at its center only. But its diminution at the circumference, when computed by the corollaries to the lemma's, will be found ſo inconſiderable that I ſee no ſort of inconvenience in it, but rather ſome ſmall advantages. Firſt the teleſcope can magnify ſomewhat more than it would do if the zone abovementioned * were added to the area of the leſſer ſpeculum, becauſe there is more light left. Secondly this zone would intercept an annulus of the innermoſt rays that are reflected the moſt accurately to every part of the image; inſtead of which we ſhould take in that lunar figure * of the exterior rays which are reflected the moſt inaccurately, and that only to the exterior parts of the laſt image. Thirdly the interior parts of the image, being diſtincter than the reſt, will bear and therefore deſerve a ſtronger light than the leſs diſtinct and tinctured parts about the margin, which in a ſtronger light would only become more offenſive.

673. This is the reſult of the theory; but in practice ſome ſmall addition ſhould be made to the breadth *aa* of this little ſpeculum, by reaſon of the difficulty of placing and moving it ſo exactly along the axis of the teleſcope as to receive all the reflected rays; none of which, as being very much condenſed by their firſt reflection, ſhould be ſuffered to paſs by the little ſpeculum. But no addition ſhould be made to the breadth of the hole in the large concave.

674. By multiplying the factors in the equation found above, and ranging the terms of the product in the order of the dimenſions of r, the reſult will be a compleat equation of twelve dimenſions, whoſe reſolution by the common rules would be ſo tedious, that I choſe to perform in the following manner. The equation of Caſ. 1. being put into this fractional form $\frac{rr \times \overline{r + n - \frac{n}{rr}}^2 \times \overline{r-1}^4}{nn} = 1$, at firſt I reject the ſmall fractions $\frac{n}{rr}$ and $\frac{nn}{r-1^2}$ and ſeek the root of this ſimpler equation $\frac{rr \times \overline{r+n}^2 \times \overline{r-1}^4}{c} = 1$, as follows. Reſolution of the equation.

675. Having computed the logarithm of the denominator c, I firſt aſſume $\frac{1}{8}$ of it for the log. of r, and thence compute the log. of the numerator $rr \times \overline{r+n}^2 \times \overline{r-1}^4$. Then I take the difference of the logarithms of the numerator and denominator, and, if the former exceeds the latter, I ſubſtract $\frac{1}{8}$ of their difference from the log of r, otherwiſe I add it. With this corrected log. of r I repeat the ſame operation two or three times, till I find a log. of the numerator agreeing with that of the denominator in 3 or 4 of their firſt figures excluſive of the index; then the corrected log. of r gives me 3 or 4 figures of the root of this latter equation; and this root will be the greateſt of all if being increaſed a little, it conſiderably increaſes the value of the numerator, which will generally be evident enough by reviſing their variations in the ſeveral operations.

676. Laſtly with the log. of r laſt found I compute the logarithms of the denominator and numerator of the equation $\frac{rr \times \overline{r + n - \frac{n}{rr}}^2 \times \overline{r-1}^4}{c \times 1 + \frac{n}{r-1} \times 1 - \frac{n}{r-1}} = 1$, as firſt propoſed, and by $\frac{1}{8}$ of their difference I correct the log. of r as before, which after two or three operations gives me the root r ſufficiently exact. A little attention will eaſily diſcover the reaſon of this method, whoſe ſucceſs is chiefly owing to the largeneſs of c and the ſmalneſs of n.

677. In order to give ſome examples of this Problem, I procured from Mr. *Short* of *Edinburgh* Examples of the ſolution of the problem.

burgh the following dimenſions of his beſt teleſcope of Mr. *Gregorie*'s form, to ſerve as a model for calculating others of any given lengths.

	Inches
The focal diſtance of the larger ſpeculum	9. 6
Its breadth or aperture	2. 3
Focal diſtance of the leſſer ſpeculum	1. 5
Its breadth	0. 6
Breadth of the hole in the larger ſpeculum	0. 5
Diſtance between the leſſer ſpeculum and the next eye-glaſs	14. 2
Diſtance between the two eye glaſſes	2. 4
Focal diſtance of the eye-glaſs next the metals	3. 8
Focal diſtance of the eye-glaſs next the eye	1. 1

678. By experiment the teleſcope was found to magnify 60 times in diameter *, and by the time of the tranſits of ſtars over the viſible area, to take in an angle of 19 minutes at the naked eye: therefore the magnified angle of viſion was $60 \times 19' = 19$ degrees.

* Rem. 485.

679. Before we can calculate the dimenſions of other teleſcopes, the given one muſt firſt be reduced to the form propoſed in the problem, by finding a ſingle eye-glaſs with which it ſhall magnify juſt as much as with the two given eye-glaſſes; and by calculating the mathematical aperture aa of the leſſer ſpeculum, as poſſeſſed by the principal pencil, whoſe breadth will be ſomewhat leſs than the breadth of this ſpeculum; then we may deduce the logarithms of the numbers b, a, v, n abovementioned, which (excepting n) ſhall be kept unaltered in all the other teleſcopes in the following table.

Fig. 173.

680. Firſt then, we have $mq = mn - nq =$ 1. 3, whence I collect * $mx = \frac{mq \times mf}{mf - mq} =$ 1. 9760, and $cx = cm + mx =$ 16. 1760 and $tx = cx - ct =$ 14. 6760. Now putting r to 1 for the ratio of the continual proportionals tx, tc, tT, we have $r = \frac{tx}{tc} =$ 9. 7840, $tT = \frac{tc}{r} =$ 0. 15331, $Tx = tx - tT =$ 14. 5227 and $n = \frac{Tx}{TC} =$ 1. 51278, whoſe log. is 0. 17977. 58.

* Rem. 639, &c.

681. Secondly, we have $cT = ct + tT =$ 1. 6533 and $ca = \frac{cT}{TC} \times CA =$ 0. 19805 $= CB$ $= \varpi x$ according to the ſolution of the problem; and ſince $M = 60$ we have $xl = \frac{r \times CT}{M}$ * $=$ 1. 5654, whence log. $v = 1 : \frac{\varpi x}{xl} = \overline{1}.10213.$ 36 and therefore 9. 10213. 36 is the log. tangent of 7°. 12'. 40". This is the ſemiangle of viſion in the teleſcope reduced to the form propoſed in the problem, and this we ſhall retain in all the teleſcopes in the following table, till we enlarge it to 9°. 30', by two eye-glaſſes, as in Mr. *Short*'s.

* Rem. 666.

682. Thirdly, we have the log. $a = 1 :$ $\frac{CA^3}{8\overline{CT}^2 \times xl} \times r + n - \frac{n^*}{rr} = \overline{2}.17217.14$ and therefore 8. 17217. 14 is the log. tangent of 51'. 05", which angle of aberration, being two or three times greater than uſual in catoptrick teleſcopes of Sir *Iſaac Newton*'s form, would cauſe an intolerable degree of indiſtinctneſs, were it not diminiſhed by correcting the ſpherical figure of the large ſpeculum, and inclining it towards a parabolick *, which Mr. *Short* takes conſtant care to do.

* Rem. 652.

* Art. 8

683. Fourthly, we have the log. $b = 1 :$ $\frac{\overline{CA + ca} \times \overline{CA - ca}}{MM} = \overline{4}.55201.77.$

684. From theſe *data*, according to the ſolutions in Caſ. 1 and 2, I computed the following Tables, where the meaſures of the given teleſcope are alſo inſerted; and have added the value of the root r belonging to each teleſcope for the ſatisfaction of ſuch as ſhall pleaſe to examine the calculations.

A Table

Example of the ſolution of the problem.

A Table of the dimenſions and magnifying powers of ſome teleſcopes of Mr. Gregorie's *form.*

Fig. 171.

CT	Cx	Tc	ct	CA	$ca=CB$	xl	*mag. pow.*	r
5. 65	2. 987	1. 131	1. 106	0. 773	0. 155	1. 223	39. 69	8. 5589
9. 60	4. 923	1. 653	1. 5	1. 15	0. 198	1. 565	60.	9. 7840
15. 50	7. 948	2. 343	2. 148	1. 652	0. 250	1. 973	86. 46	11. 0090
36.	4.	3. 724	3. 432	3. 132	0. 324	2. 561	165. 02	11. 7408
60.	6.	5. 391	5. 012	4. 605	0. 414	3. 271	242. 94	13. 2426

A Table of the dimenſions and magnifying powers of ſome teleſcopes of Mr. Caſſegrain's *form.*

Fig. 172.

CT	Cx	Tc	ct	CA	$ca=CB$	xl	*mag. pow.*	r
15. 5	7. 948	1. 992	2. 196	1. 769	0. 227	1. 797	92. 91	10. 7720
15. 5	3.	1. 766	1. 974	1. 761	0. 201	1. 585	92. 65	9. 477
36.	4.	3. 253	3. 569	3. 286	0. 297	2. 347	173. 28	11. 2966
60.	6.	4. 786	3. 173	4. 804	0. 383	3. 028	253. 44	12. 7913½

685. I have calculated for the focal diſtances of 5. 65 and 15. 5 inches as belonging to the ſizes of Mr. *Short*'s tools, who tells me that the teleſcopes made with them perform very nearly according to this calculation. What tryals have been made of Mr. *Caſſegrain*'s form I know not, but by the tables it appears to have the advantage of Mr. *Gregorie*'s, (at leaſt for aſtronomical uſes where the inverſion of the object is not regarded,) as being ſhorter by twice the focal diſtance of the leſſer ſpeculum, and yet magnifying more. The two calculations for the $15\frac{1}{2}$ inch teleſcope are founded on different lengths of Cx, to ſhew that the magnifying power is ſcarce ſenſibly altered thereby. The length of the eye-piece, as being troubleſome in high obſervations, ſhould therefore be no longer than is neceſſary for holding the eye-glaſſes, unleſs, as Mr. *Short* imagines, it may contribute to ſecure the eye-glaſſes from foreign light which paſſes by the ſides of the leſſer ſpeculum through the hole in the larger; for which reaſon he alſo chuſes to make that ſpeculum ſomewhat broader than the hole. But this point had better be ſecured if poſſible, by taking due care of the ſize and place of the little hole in the plate next the eye.

686. The following tables give the poſitions and focal diſtances ml, nq of the two eye-glaſſes m, n, together with the place and ſemi-diameter pq of the hole in the plate that limits the viſible area and magnified angle of viſion to 19 degrees, as in Mr. *Short*'s teleſcope. Fig. 173.

For Gregorie's *teleſcopes.*

CT	Cm	mn	ml	nq	no	pq
5. 65	1. 764	1. 631	2. 446	0. 815	0. 408	0. 136
9. 6	3. 358	2. 087	3. 130	1. 043	0. 522	0. 174
15. 5	5. 975	2. 631	3. 946	1. 315	0. 658	0. 220
36.	1. 439	3. 415	5. 122	1. 707	0. 854	0. 286
60.	2. 783	4. 289	6. 434	2. 144	1. 072	0. 359

Fig. 173.

For Caſſegrain's *teleſcopes.*

CT	Cm	mn	ml	nq	no	pq
15. 5	6. 151	2. 396	3. 594	1. 198	0. 598	0. 200
15. 5	1. 415	2. 113	3. 170	1. 057	0. 528	0. 177
36.	1. 653	3. 029	4. 694	1. 565	0. 782	0. 262
60.	2. 972	4. 037	6. 056	2. 019	1. 010	0. 338

687. The breadth of the hole at *o* in the plate next the eye muſt be $\frac{1}{50}$ inch in the teleſcopes of Mr. *Gregorie*'s form and $\frac{1}{60}$ in Mr. *Caſſegrain*'s.

Compared with Sir *Iſaac Newton*'s teleſcope. * Rem. 631.

688. A 60 inch teleſcope of Sir *Iſaac Newton*'s form, made in as great perfection as Mr. *Hauksbee*'s ſtandard *, will magnify 313 times, and therefore excels Mr. *Caſſegrain*'s, of the ſame length, in the ratio of 6 to 5 nearly, and is more commodious for high obſervations.

689. If according to Lemma 2, we would have the viſible area uniformly bright, to the half breadth *ac* of *Gregorie*'s little ſpeculum we muſt add $ad = \frac{dv}{m} \times 1 + \frac{n}{r-1}$ which is ſcarce 0.022 of an inch when $d = 5.65$ inches, and but 0,034 when $d = 60$ inches. To *Caſſegrain*'s *ac* we muſt add $ad = \frac{dv}{m} \times 1 - \frac{n}{r+1}$, which is but 0.032 of an inch when $d = 60$ inches. Theſe ſmall additions will alter the apparent brightneſs ſcarce ſenſibly, and the apparent diſtinctneſs not at all.

Fig. 171.

690. The reaſon of the rule is this. In the analyſis of the problem we had $m = \frac{rd}{e}$ and $\varpi x = ev = \frac{rdv}{m}$, and by the ſimilar triangles STe, ϖex we have $ST : \varpi x :: Te : ex :: 1 : r$, whence $ST = \frac{dv}{m}$; and by the ſimilar triangles adA, STA we have $ad : ST :: CT + Tc : CT :: d + \frac{nd}{r-1} : d :: 1 + \frac{n}{r-1} : 1$, whence $ad = \frac{dv}{m} \times 1 + \frac{n}{r-1}$ in *Gregorie*'s. But in *Caſſegrain*'s, $ad : ST :: CT - Tc : CT :: d - \frac{nd}{r+1} : d :: 1 - \frac{n}{r+1} : 1$, whence $ad = \frac{dv}{m} \times 1 - \frac{n}{r+1}$. Fig. 172

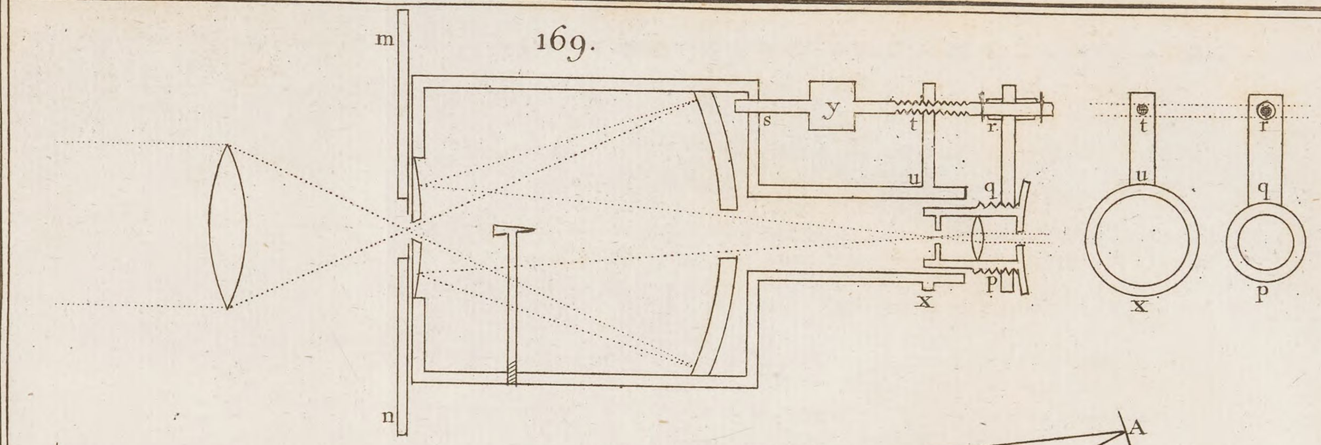
691. If d, s, m, v and n were all given, we might find the other dimenſions of *Gregorie*'s teleſcope by taking $r = 1 + \sqrt{n + \frac{nms}{dv}}$, $Tc = \frac{nd}{r-1}$, $ct = \frac{r}{r+1} \times Tc$, $xl = \frac{rd}{m}$, $cd = CB = \varpi x = v \times xl$. For by putting $\varpi x = cd = ca + ad$, that is, $\frac{rdv}{m} = \frac{ns}{r-1} + \frac{dv}{m} \times 1 + \frac{n}{r-1}$, and by algebraick reduction, the value of r will come out as above.

692. Theſe theorems, by ſubſtituting for r its value and putting $n = 1$, will come to the ſame as Mr. *Hadley*'s *, who has given ſome very uſeful tables of the dimenſions of *Gregorie*'s teleſcopes of different lengths from thoſe in mine.

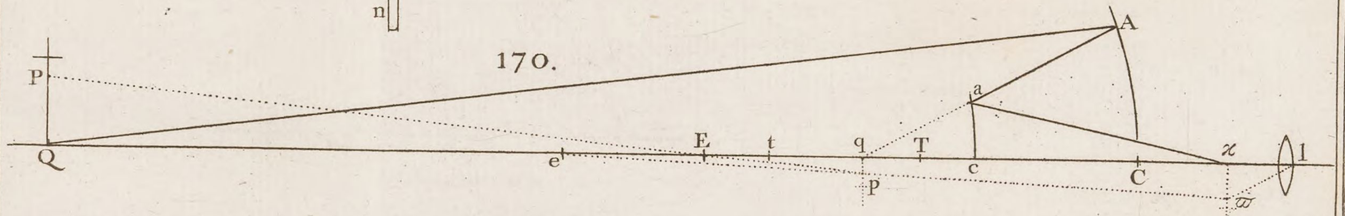
* Appendix to *Gregorie*'s Elements of opticks pub. by Dr. *Deſaguliers* pag. 286.

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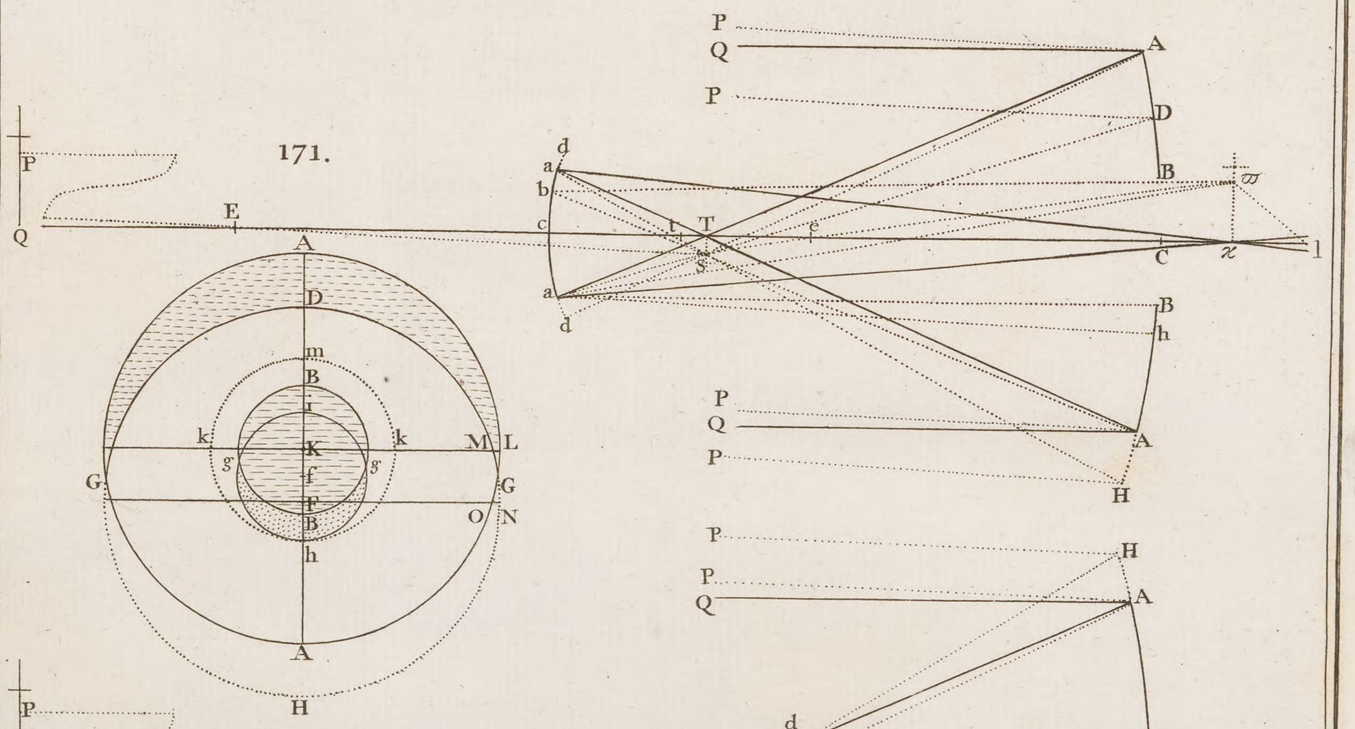
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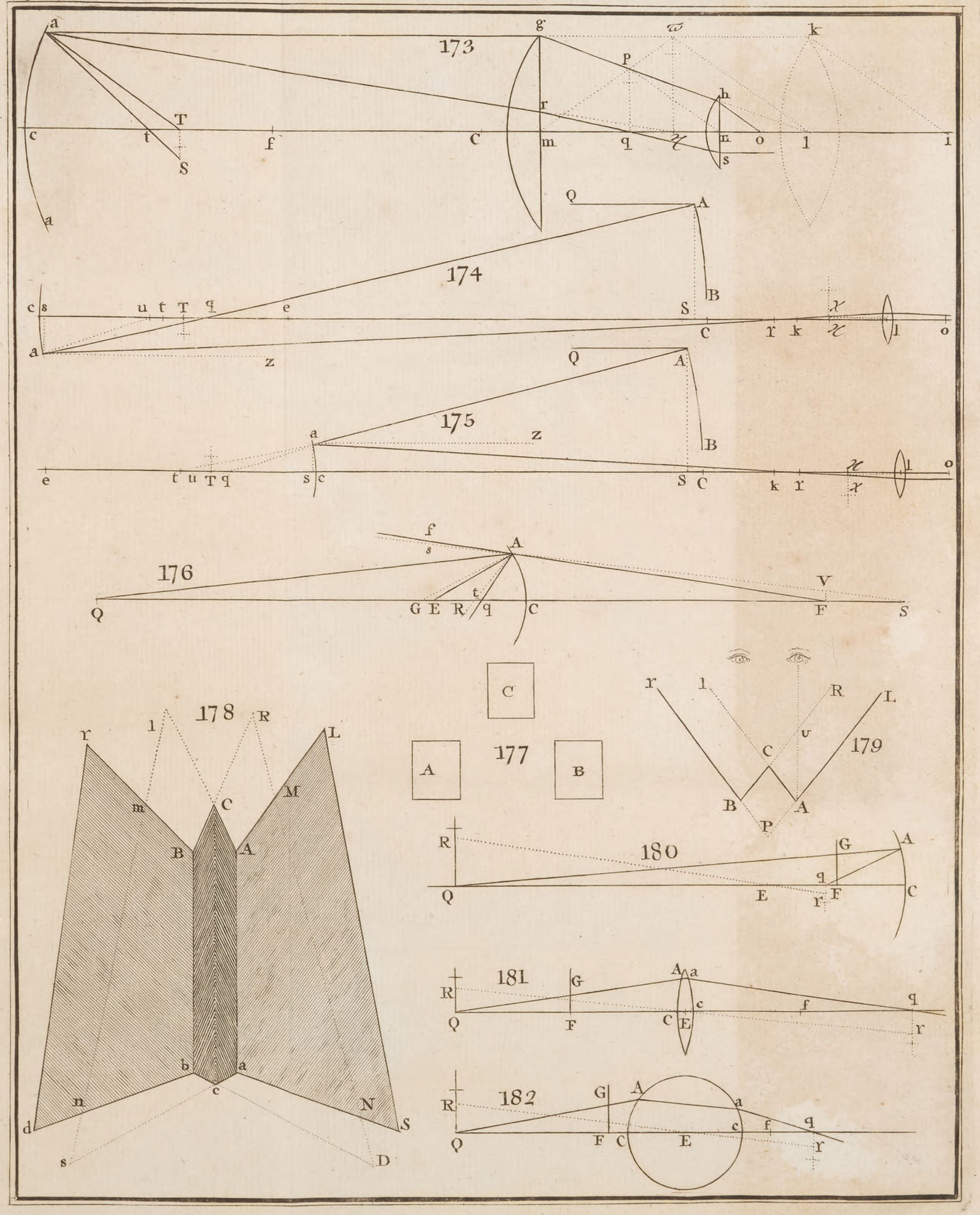
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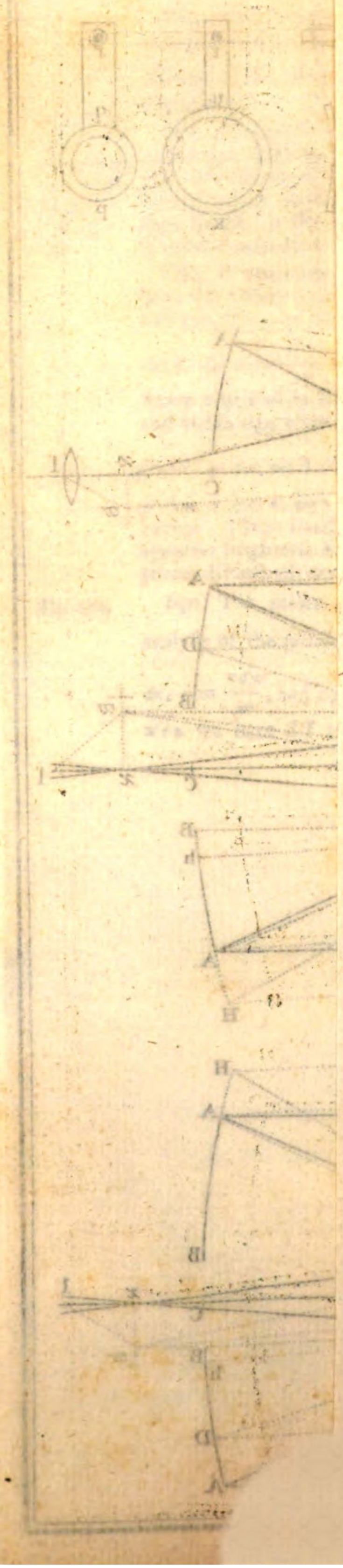


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THE AUTHOR'S REMARKS UPON BOOK III.

Upon ART. 806.

How to clean a tarnished speculum.

MR. *Hearne*'s way of cleaning a tarnished speculum is this. Get a little of the strongest sope-lye from the sope-maker; and having laid the speculum on a table with its face upwards, put on as much of the lye as it will hold and let it remain about an hour; then rub it softly with a silk or muslin till the lye is all gone; then put on some spirit of wine and rub it dry with another part of the silk or muslin. If after this the speculum will not perform well, it must be new polished. A few spots of faint tarnish may be rubbed off with spirit of wine only.

Upon ART. 825.

693. To these methods of rectifying telescopick sights Mr. *Molyneux* has added an account of the controversy about them between *Hevelius* and *Hook* *, and shews that *Hevelius*'s opposition to the use of those sights arose chiefly from his misapprehension of the nature of them, in taking the line of collimation to be no longer than from the cross-hairs to the eye, whereas its real length is from the hairs to the object-glass; and that Dr. *Hook* should have demonstrated this to him in order to induce him to use these sights instead of his plain ones.

* Dioptr. Part. II. chap. 5.

Upon ART. 915.

Aberrations of rays by a bad polish. Fig. 176.

694. Mr. *Molyneux* means, that the angular aberration of a ray from its true course, when reflected from an irregular particle of a surface, is about five or six times greater than it would be, if the same ray were refracted through that particle. Let AC represent the regular surface, E its center, $QGRqCFS$ its axis, QA an incident ray, EA the perpendicular of incidence, Aq the regular reflection, and AF the regular refraction of that ray. Then imagine the particle of the surface about A to be so changed, that AG becomes perpendicular to it; and let AR and AS be the irregular reflection and refraction of the same ray QA, making the angular aberrations qAR by reflection and FAS by refraction. Through A produce FA towards f and SA towards s. Then by reflection we have the angle $QAq = 2QAE$ and also $QAR = 2QAG$ and consequently their difference $qAR = 2EAG$. Likewise in small refractions from the perpendicular, the angle $QAf = \frac{1}{2} QAE$ and also $QAs = \frac{1}{2} QAG$ nearly, (the ratio of the sines being 3 to 2) and consequently the difference fAs or $FAS = \frac{1}{2} EAG$. Therefore the former difference qAR, is to the latter FAS, as ($2EAG$ to $\frac{1}{2} EAG$, as) 4 to 1. But in refractions towards the perpendicular the like ratio is 6 to 1.

695. Hence if the angle EAG, which measures the irregularity of the particle at A, be given in different surfaces, the aberration of the ray in the image at the focus of a reflecting telescope, will be to the like aberration in a refracting one, in a ratio compounded of their focal distances directly and of 5 to 1 taking one refraction with another at the two surfaces.

Upon ART. 974 —— 978.

Binocular telescope.

696. A binocular telescope of the dioptrick kind being heavy and cumbersome, a more commodious one might be made with a couple of small reflecting telescopes of Mr. *Gregorie*'s form. The phænomenon of the enlarged circle of the visible area, explained in these articles, may also be seen very plainly in looking at distant objects through a pair of spectacles, removed from the eyes about as far as the length of the hold-fasts (made for fixing them to the temples) and held steady at that distance. The two innermost of the four apparent rings, that hold the glasses, will then appear united in one larger and more distant ring, than the two outermost, which will hardly be visible unless the spectacles be farther removed.

Dr. *Jurin*'s experiments, to find how much brighter an object appears to both eyes than to one alone.

" In order to find, whether an object seen with both eyes appears brighter, or stronger, or larger than when seen with one alone, I made the following experiments.

697. I laid a slip of clean, white paper directly before me upon a table, and applying the side of a book close to my right temple, so as that the book advanced considerably more forward than my face, I held it in such manner, as to hide from my right eye that half of the paper, which lay to my right hand, while the left half of the paper was seen by both eyes without any impediment.

Then looking at the paper with both eyes, I observed it to be divided from top to bottom by a dark line, and the half of the paper

which lay to the right hand of this line, to appear conſiderably darker than the half which lay to the left hand.

698. In looking at other objects in the ſame manner, as at the wainſcot, or the cieling, I conſtantly found that part which was ſeen by one eye only, to appear manifeſtly darker, than that which was ſeen by both eyes.

When the book was applied to my left temple inſtead of the right, the ſame difference was obſerved, which ſhewed my two eyes to be of equal goodneſs.

699. When I looked in this manner upon a page of a book divided into two columns, I found the column, that was ſeen with both eyes to be much plainer and more legible than that which was ſeen with one only.

This difference was more conſpicuous, when, in making the experiment by candle-light, the book was at ſuch a diſtance from the candles, as that there was ſcarce light enough to read with both eyes. For then the column, which was ſeen by one eye only, was not at all legible: but I could read the other, though with ſome difficulty.

700. Being now fully ſatisfied, that an object ſeen with both eyes, appeared brighter and ſtronger than when viewed with one, I next endeavoured to find to what degree this exceſs of brightneſs amounted: particularly, whether there was as much difference in the brightneſs of an object when ſeen with one eye, and with both, as when illuminated by one candle, and by two.

To this end, I ſet upon a table two candles of equal height, and burning to appearance with equal light, at about a foot diſtance beyond a ſlip of white paper lying before me, and about four inches from one another, ſo as that the diſtance between the candles was parallel to the ſlip of paper.

Then I ſet a book upon one end, between the right hand candle and the paper, ſo as to caſt a ſhade from that candle upon the right hand half of the paper: Thus the left half of the paper was illuminated by the two candles, and the right by one only: conſequently, the left half was twice as luminous as the right; and the boundary between thoſe two halves was pretty well defined by the edge of the ſhade.

I then took another book, and applyed it to my left temple, in ſuch manner as to hide the left and brighter half of the paper from my left eye; ſo that the left half was ſeen by my right eye only, while the other half was ſeen by both eyes: Now I expected, that the right half of the paper, having the light of one candle only thrown upon it, but being ſeen by both eyes, would appear as luminous as the left half, which had twice as much light caſt upon it, but was ſeen by one eye only. In which I found my ſelf miſtaken For the left half appeared much whiter and brighter than the right. Conſequently, an object ſeen with both eyes is nothing near twice as luminous, as when ſeen with one only.

701. Being deſirous to know the quantity of this exceſs of brightneſs more exactly, I fixed a ſlip of white paper flat againſt the wainſcot by the help of pins, and at a yard diſtance I ſet a candle, ſo as that the flame was about the ſame height with the paper, and nearly oppoſite to the middle of it, but rather inclining to the right ſide At two yards diſtance from the paper I placed another candle with its flame at the ſame height, and oppoſite to the middle of the left half of the paper. Then I ſet up a book ſo as to cut off the light of this ſecond candle from the left half of the paper; which half therefore being illuminated by one candle only, appeared conſiderably darker than the right half of the paper, on which both candles ſhone without interruption.

The difference in brightneſs of the two halves of the paper is eaſily eſtimated, by conſidering that the ſecond candle, being at twice the diſtance, muſt throw upon the right half of the paper juſt a quarter of the light, that was caſt upon the ſame half by the nearer candle*; and conſequently that the luminouſneſs of the right half of the paper was to that of the left half, as five to four. * Art. 58.

Things being thus diſpoſed, and the candles burning with equal brightneſs, I applyed a book to my right temple, ſo as to hide the right half of the paper from the right eye. Then looking at the paper with both eyes, the right half of it, which had five degrees of light, and was ſeen by the left eye only, appeared manifeſtly whiter than the left half, which had four degrees of light, and was ſeen by both eyes. Conſequently, an object ſeen with both eyes, is not a quarter part more luminous than when ſeen with one only.

702. After the ſame manner, by ſetting the ſecond candle at three yards diſtance from the paper, I found, that the right half of the paper, which then had ten portions of light thrown upon it, and was ſeen by one eye, appeared ſomething whiter than the left half, which was illuminated by nine portions of light, and was ſeen by both eyes.

703. When I removed the ſame candle one foot farther, ſo that the diſtances of the two candles from the paper were reſpectively as 3 and 10; and conſequently, the quantities of light they threw upon the right half of the paper were as 100 and 9 reſpectively, *i. e.* nearly as 11 and 1; the right half of the paper ſeen with

with one eye only, ſeemed ſtill a little whiter than the left half ſeen with both.

704. When the ſecond candle was ſet at the diſtance of four yards from the paper, the right half of the paper ſeen with one eye, appeared a little darker than the left half ſeen with both eyes.

705. Hence it follows, that, when the ſecond candle is about 11 foot diſtant from the paper, the right half ſeen with one eye, and the left half ſeen with both eyes, muſt appear of an equal whiteneſs.

Conſequently, an object ſeen with both eyes, appears brighter than when ſeen with one only, by about a thirteenth part. But it would be difficult to make the experiment exactly.

If a perſon, who ſquints, ſhould go about to make theſe experiments, I ſuppoſe he would find none of them to ſucceed, but that in the firſt experiment particularly, both halves of the paper would appear equally luminous. If ſo, this would prove, that he ſaw the paper with one eye only, in confirmation of what was laid down in Remark 178 &c.

whe- larger th eyes to one e.

706. I proceeded next to enquire, whether an object ſeen with both eyes, appeared any thing bigger than when ſeen with one alone.

In order to which, applying the ſide of a book to one of my temples as before, I looked at two prints of equal ſize hanging beſide one another, at two panes in a window, two ſmall pannels in a door, and ſeveral other ſuch objects; but could perceive no difference in the apparent magnitude, when one of them was ſeen with both eyes, and the other with one.

707. Not content with theſe tryals, I proceeded to make the experiment with ſmaller objects, as the letters of a printed book, and found, that though they appeared brighter and ſtronger, yet they did not ſeem at all larger, when viewed with both eyes, than when ſeen with one only. But the tryal which gave me the fulleſt ſatisfaction, was that which follows.

708. Upon a narrow rectangular piece of paper I drew two lines with ink, parallel to one another, and likewiſe to the long ſides of the paper. Then laying the paper directly before me, ſo as that my eyes were equidiſtant from the black lines and from the edges of the paper, I applyed a book to one of my temples, ſo as that one half of the paper and of the parallel lines upon it, was ſeen with one eye only, and the other half was ſeen with both eyes.

I then carefully conſidered the parallel lines, particularly at the joining of the two ſeveral halves: but could obſerve no difference between thoſe halves, except that the half ſeen by one eye only, did not make ſo ſtrong an appearance as the other. The lines and the edges of the paper appeared perfectly ſtraight; which they could not have done, if that half of the paper, or of the ſpace intercepted between the black lines, which was ſeen by both eyes, had appeared at all larger than that which was ſeen by one eye only.

From which I conclude, that an object does not appear larger to both eyes than to one, unleſs by reaſon of ſome particular circumſtances, as in the obſervation of *Leonardo da Vinci* *, or in the caſe of the binocular teleſcope Art. 976; or of concave ſpecula Rem. 520.

* Rem. 244, 245.

Some ſurpriſing phænomena in double viſion.

709. Among the ſeveral experiments I made in order to find the difference between the brightneſs and largeneſs of objects, when viewed with both eyes, and when ſeen with one eye only, there was one, which, though no way neceſſary to be mentioned for that purpoſe, after thoſe I have already related, yet was attended with ſome phænomena ſo ſingular and ſurpriſing, as on that account to deſerve a particular recital.

Before three prints, equal in ſize, and hung up againſt a wall in the ſituation repreſented by Fig. 177, I ſet a table with two candles upon it, at about four foot diſtance, ſo as to throw a ſtrong light upon the prints. Then ſtanding about four foot from the candles, with my face towards them and the prints beyond them, I opened a thin quarto book to an angle of about 40 degrees, and held it directly before me, at ſuch a diſtance, as that I could but juſt ſee the whole book though very indiſtinctly with each eye, the back of the book inclining obliquely downwards and towards my breaſt, by which poſition the direct light of the candles was kept off both from my eyes, and from the inſide of the book. Then directing my eyes towards the prints, I held the book ſo, as that the left hand print *A* was hid from my left eye by the left ſide of the book, and the right hand print *B* was hid from my right eye by the right ſide of the book: while the print *A* was ſeen by my right eye, the print *B* by my left eye, and the print *C* by both eyes.

Fig. 177.

Things being in this poſture, I obſerved the prints *A*, *B*, and *C* to appear, as near as I could judge, exactly of the ſame ſize. Nor could I well ſatisfy my ſelf, whether the print *C*, which was ſeen with both eyes, appeared clearer or ſtronger than *A* or *B*, which were ſeverally ſeen by one eye only; this experiment not being ſo well adapted to that purpoſe, as ſome of thoſe above related.

710. But obſerving the wainſcot to appear unequally luminous in different parts, I turned my ſelf about ſtill holding the book in the ſame poſture with regard to my face, and directed my eyes firſt to another part of the wainſcot free from prints, where the appearance I am going to deſcribe was more perfect; and then

to

to the cieling, where the ſurface being white and plain, the phænomenon was ſtill more conſpicuous.

Fig. 178. In Fig. 178 part of the cieling is repreſented by the ſpace *LMABmrlRL*, whercin the line *AL* repreſents the projection upon the cieling, of the upper right hand edge of the book from my left eye; *BCR* the projection of the ſame edge from my right eye; *ACl* the projection upon the cieling of the upper left hand edge of the book from my left eye; *Br* the projection of the ſame edge from my right eye; *RM* the projection of part of the right hand fore edge of the book from the right eye; *lm* the projection of part of the left hand fore edge from the left eye.

By theſe lines the viſible part of the cieling was divided into ſix different portions, *LMR*, *RCl*, *lmr*, *AMRC*, *BmlC*, and *ACB*. Of theſe, the three firſt portions *LMR*, *RCl*, and *lmr*, were the brighteſt, and were equally bright one with another: the portions *AMRC* and *BmlC* were equal in brightneſs to one another, but conſiderably darker than the three firſt, and the portion *ACB* was the darkeſt of all.

711. The cauſe of this phænomenon is eaſily ſeen, being no other than what follows. By the deſcription given above it appears, that the portion of the cieling *AMRCB* is entirely hid from the right eye by the interpoſition of the right ſide of the book; and that the portion *BmlCA* is entirely hid from the left eye by the interpoſition of the left ſide of the book: that is, the portion *ACB* is hid from both eyes, and conſequently exhibits the darkeſt appearance from the dark inſide of the book projected on it by both eyes; *AMRC* is ſeen by the left eye only, *BmlC* by the right eye only; and *LMR*, *RCl*, and *lmr* are ſeen by both eyes, which therefore appear the brighteſt of all.

712. But the moſt extraordinary part of this appearanee is yet to come. For the dark part *ACB* does not appear like a plane, as might be expected on conſidering it as a portion of the cieling; but is ſeen as a pyramid extending it ſelf from the book to the point *C* in the cieling. And if, inſtead of the cieling, the eyes be directed to any other object how remote ſoever, either upwards or downwards, or horizontally; the apex *C* of this pyramid will appear to be extended to, and to touch the very remoteſt object in view; provided that object be ſituated in the interſection of two planes, one drawn through the line *BCR* and the right eye, the other through *ACl* and the left eye. In which reſpect this pyramid is analogous to the middle point of the compaſſes, in the appearance deſcribed in Art. 977.

713. It is farther obſervable, that there appears another dark body *ABbcaA*, in form of a wedge, from which the pyramid *ACB* ſeems to ariſe and to be a continuation of it: but this wedge-like dark body appears within the book it ſelf, between the binding and the eyes.

714. In order to explain theſe appearances, it muſt in the firſt place be remembred, that the axes of my eyes were at that time not directed to the book it ſelf, in which caſe all theſe phænomena would immediately have vaniſhed; but that they were pointed at the cieling, as I ſaid before. Now, in this caſe, it is obvious, that the book and every thing within it muſt appear double; becauſe the images formed in the two eyes will not fall upon correſponding parts of the retina*. We are therefore to conſider the ſeveral portions of thoſe images, and to ſee how they contribute to the production of the phænomena abovementioned.

* Art.

715. Firſt then, the juncture of the leaves, or the line in which they meet at the binding, appearing double, the image of this juncture formed in the right eye will appear to the left hand, as the line *bB* Fig. 178, and the image of the ſame juncture formed in the left eye will appear to the right hand, as the line *aA*.

Conſequently, the image of the left hand leaf formed in the right eye, as *bBrd*, muſt appear adjoining to the left hand of the juncture *bB* ſeen by the right eye: And the image of the right hand leaf formed in the left eye, as *aALS*, muſt appear adjoining to the right hand of the juncture *aA* ſeen by the left eye. Theſe two images will therefore be intirely diſtinct one from the other, being ſeparated by the ſpace *bBAa*.

716. But the image of the left hand leaf ſeen by the left eye, as *aAls*, muſt appear adjoining to the left hand of the juncture *aA* ſeen by the left eye.

And the image of the right hand leaf ſeen by the right eye, as *bBRD*, muſt appear adjoining to the right hand of the juncture *bB* ſeen by the right eye.

Conſequently, theſe two images will coincide with one another in the ſpace *aACBbca*; and will coincide with the two former images, (Rem. 715.) in the ſpaces *bBmn*, and *aAMN*, reſpectively. But in order to ſee more particularly, how theſe two laſt images contribute to the production of the phænomena above related, it will be neceſſary to diſtinguiſh them into ſeveral parts.

717. The image *aAls* of the left hand leaf ſeen by the left eye may be conceived to be divided into the four parts *bBmn*, *BClm*, *bcsn*, and *bBCAacb*.

718. Of theſe four parts, the firſt *bBmn* lying to the left hand of the juncture *bB* ſeen by the right eye, will conſequently coincide with part of the image of the ſame leaf *bBrd* formed

ed in the right eye, and will be obliterated by that image. For, as the two images coincide, that formed by the right eye, which is ſtronger and more diſtinct as being ſeen more directly and from a greater diſtance, will ſwallow up and efface that formed by the left eye, as being ſeen more obliquely, and much nearer.

719. The ſecond and third parts of this image, as *BClm*, and *bcsn*, will fall without the image of the ſame leaf formed in the right eye; and will be ſpread like a faint ſhade over part of the outward objects, as the cieling, the walls, or the floor ſeen by the right eye.

720. The fourth part of the image, as *bBCAacb*, having been already obſerved to coincide with part of the oppoſite image, it will be proper to conſider theſe two parts together by and by.

721. In like manner, the image of the right hand leaf *bBRD*, ſeen by the right eye, may be conceived to be divided into the four parts *aAMN*, *ACRM*, *acDN*, and *aACBbca*.

722. The firſt of theſe *aAMN* will coincide with and be obliterated by part of the image of the ſame leaf *aALS*, formed in the left eye, for the reaſon already given in the like caſe, in Rem. 718.

723. The ſecond and third parts *ACRM*, and *acDN*, will fall without the image of the ſame leaf formed by the left eye, and will be ſpread like a faint ſhade over part of the outward objects ſeen by the left eye.

724. The fourth part of this image, namely *aACBbca*, will coincide with a like part of the oppoſite image, which two oppoſite parts we come now more particularly to conſider; in order whereto we ſhall diſtinguiſh this fourth part *aACBbca* into the two portions *bBCc*, and *CcaA*; and ſhall diſtinguiſh *bBCAacb* the fourth part of the oppoſite image into the two portions *aACc*, and *CcbB*.

725. Since the book was held directly before me, not inclining to one ſide more than to the other, and was opened to an angle of about 40 degrees, it follows, that the image of the two leaves ſeen by the right eye, as *bBRD* and *bBrd*, muſt appear to contain nearly the ſame angle of 40 degrees; and likewiſe the image of the two leaves ſeen by the left eye, as *aAls*, *aALS*, muſt appear to contain nearly the ſame angle of 40 degrees. That is, *bBCc* being part of the image of the right hand leaf ſeen by the right eye, muſt appear as making an angle of about 40 degrees with *bBrd* the image of the left hand leaf ſeen by the ſame right eye; and likewiſe *aACc* being part of the image of the left hand leaf ſeen by the left eye, muſt appear as making an angle of about 40 degrees with *aALS* the image of the right hand leaf ſeen by the ſame left eye.

Conſequently, theſe two firſt portions *bBCc* and *aACc* muſt appear inclined to each other in an angle of about 40°, or muſt appear as two ſides of a wedge containing ſuch an angle.

For let *LPr*, Fig. 179, repreſent the angle of about 40°, to which the book was opened; and let *LACl*, and *RCBr* repreſent the double image of a ſection made by a plane paſſing perpendicularly through both leaves. Then will *Br* ſtand for the image of the left hand leaf ſeen by the right eye; *BC* for that portion of the image of the right hand leaf, which in Fig. 178 is expreſt by *bBGc*, ſeen by the ſame right eye; *AL* for the image of the right hand leaf ſeen by the left eye; and *AC* for that portion of the image of the left hand leaf ſeen by the ſame left eye, which is expreſſed in Fig. 178, by *aACc*. Fig. 179.

Now, as we have already taken notice, that the angle *LPr* is about 40°; and that the two angles *rBC*, *CAL* are each of them nearly of 40°; it follows, that the lines *Br*, *AC*, and likewiſe the lines *BC*, *AL* are nearly parallel. Conſequently, the angle *ACB* muſt alſo be nearly of 40°: that is, the portions *aACc*, and *bBCc* of the images abovementioned, muſt appear to make two ſides of a wedge, containing an angle of about 40°, which is the wedge-like appearance we were to explain.

726. It remains for us to ſhew what becomes of *CcaA* the ſecond portion of the fourth part of the image of the right hand leaf ſeen by the right eye; as likewiſe of *CcbB* the ſecond portion of the fourth part of the image of the left hand leaf ſeen by the left eye; neither of which we have as yet accounted for.

In order to which we muſt obſerve.

1. That the firſt of theſe, *CcaA*, coincides with *aACc* the firſt portion of the fourth part of the image of the left hand leaf ſeen by the left eye, and conſtituting the right hand ſide of the wedge; and that the ſecond, *CcbB* coincides with *bBCc* the firſt portion of the fourth part of the image of the right hand leaf ſeen by the right eye, and conſtituting the left ſide of the wedge.

2. That ſince the book was too near the eyes to have any of theſe images diſtinct, that part of each leaf which was farther from the eye it was ſeen by, muſt form a diſtincter, and therefore a ſtronger image than that part which was nearer to the eye.

Hence it follows, that of the two coinciding images *CcaA* and *aACc*, the latter, which is nearer to the juncture *aA*, and conſequently farther from the left eye by which it was ſeen, muſt be diſtincter and therefore ſtronger than the firſt, which is farther from the juncture *bB*, and conſequently nearer to the right eye by which it was ſeen.

Therefore

Therefore the firſt of theſe *CcaA* muſt be entirely obliterated and effaced by the latter.

727. In like manner it may be ſhewn, that *CcbB* is entirely obliterated and ſwallowed up by *bBCc*.

728. The truth of this explication is confirmed by the following obſervation.

If one of the leaves, *ex. gr.* the left leaf be blank and the right be print, then *CcaA*, ſeen by the right eye, though more remote from the juncture *aA*, and conſequently nearer to the eye it is ſeen by, and therefore leſs diſtinct, yet, as it makes a ſtronger appearance on account of the printing, will obliterate and ſwallow up *aACc*, ſeen by the left eye, though nearer to the juncture, and conſequently more remote from the eye it is ſeen by, and therefore more diſtinct, but making a weaker impreſſion: And the letters of the right hand page will appear to be continued from the left hand ſide of the wedge *bBCc* acroſs the left ſide *cCAa* of the ſame wedge. This is eaſily ſeen to be effected by the projection of *C v* Fig 179 upon *CA*.

729. Likewiſe, when neither page is blank paper, both ſides of the wedge will appear imprinted with the letters or figures of one and the ſame page, provided they make a ſtronger impreſſion than thoſe of the other, either by reaſon of their greater ſtrength or magnitude, or, even in caſe of equality, by our attending more particularly to them. In which latter caſe we can vary the phænomenon at pleaſure, by fixing our attention, even without changing the poſture of our eyes, firſt upon one page, and then upon the other. From which conſideration ſome other phænomena of coinciding images may be explained, which ſeem to be otherwiſe inexplicable.

Fig. 178. 730. It remains for us to ſhew, why the fartheſt part of the wedge, as *ABC*, inſtead of appearing within the book, as the reſt of the wedge, is ſeen without it, and extended to a conſiderable diſtance beyond it; alſo, why it appears in form of a pyramid; and laſtly, why its extremity *C* ſeems to touch the moſt diſtant object, towards which the book and the eyes are directed.

Here it is to be obſerved, that the diſtincteſt and ſtrongeſt image of the right hand leaf being that ſeen by the left eye, or *ALSa*, and the diſtincteſt and ſtrongeſt image of the left hand leaf being that ſeen by the right eye, as *Brdb*, we look upon the edges of theſe images, *AL* and *Br*, as the edges of the book.

731. Conſequently, *BR* the image of the farther edge of the right hand leaf ſeen by the right eye, and *Al* the image of the farther edge of the left hand leaf ſeen by the left eye, muſt both appear without the book: that is, the lines *BC* and *AC*, part of thoſe edges, and ſuch part of the wedge as lies between theſe lines, as *ABC*, muſt appear without the book.

732. And as the objects, upon which the edges *BR*, *Al* are projected in looking horizontally or obliquely downward, are much more remote than thoſe objects upon which the edges *AL* and *Br* are projected; it is manifeſt, that the edges *BR*, *Al*, and the parts of them *BC*, *AC*, or the part of the wedge *ACB* muſt appear to be projected to a conſiderable diſtance beyond the edges *AL*, *Br*, i. e beyond the book. And the point *C* appearing on this account to be farther extended beyond the book than any other part of the wedge, the part *ACB* muſt neceſſarily put on the appearance of a pyramid having *C* for its vertex.

733. Laſtly, as the point *C* is ſeated in the plane drawn through the right eye and the line *BR*, and likewiſe in the plane drawn through the left eye and the line *Al*; i. e. in the interſection of thoſe two planes; it muſt neceſſarily appear in the ſame line with any remote object likewiſe ſituated in the ſame interſection.

Therefore the image of *C*, and that of the object, muſt fall upon contiguous parts of the retina in each eye, and conſequently muſt appear in the ſame place, or the point *C* muſt appear to touch the object.

Theſe experiments will ſucceed by daylight, and with a book of any ſize, and opened to any angle, *mutatis mutandis*." So far Dr. *Jurin*.

Upon ART. 1016.

734. A compariſon of different ways of illuminating microſcopical objects, pictures in a magick lantern &c. and of the burning powers of glaſſes and ſpeculums.

PROPOSITION.

Of a luminous object QR *let* qr *be the image formed by reflection from a concave ſurface, or by refraction through a convex lens, or ſphere* AC; *whoſe center is* E, *principal focus* F, *axis* QEFC *and ſemiaperture* AC; *and let a perpendicular* FG, *to the axis, cut the outermoſt ray* QA *in* G; *I ſay the brightneſs of the ſeveral pictures* qr, *will be very nearly as* $\overline{FG}^2$ *directly and* $\overline{FE}^2$ *inverſely.* Fig. 180. 181. 182.

735. For, not regarding the ſmall loſſes of light by the ſeveral reflections and refractions, the quantity collected to the point *q* is very nearly as $\frac{\overline{AC}^2}{\overline{CQ}^2}$ *, and conſequently the quantity in the area of the whole picture *qr*, as $\frac{\overline{AC}^2}{\overline{CQ}^2} \times \overline{QR}^2$ or $\frac{\overline{FG}^2}{\overline{FQ}^2} \times \overline{QR}^2$. But the area of the picture

* Art. 5

picture is as $qr^2 = \frac{\overline{Eq}^2}{\overline{EQ}^2} \times \overline{QR}^2 = \frac{\overline{FE}^2}{\overline{FQ}^2} \times \overline{QR}^2$. Becauſe, in the reflecting concave, we have $Fq : FE :: FE : FQ$ * and conſequently $Eq : EQ :: FE :: FQ$; and in the lens and ſphere we have $Qq : QE :: QE : QF$ * and conſequently $Eq : EQ :: FE : FQ$. Therefore the brightneſs of the picture, or the denſity of the rays in its area, being as their quantity directly and the area inverſely, is as $\frac{\overline{FG}^2}{\overline{FE}^2}$ very nearly; and the more exactly as the aperture is ſmaller and the object farther off.

* Art. 207.

* Art. 239.

736. *Corol.* 1. In a given ſpeculum, lens or ſphere, the brightneſs of the picture of a given object, is as $\overline{FG}^2$; and therefore increaſes continually with the diſtance of the luminous object from the focus *F*.

737. *Corol.* 2. If the luminous object be very remote, and the apertures of ſeveral ſpecula, lens's and ſpheres be equal to one another, the degrees of brightneſs of the ſeveral pictures formed by them are reciprocally as the ſquares of their reſpective focal diſtances very nearly.

738. *Corol.* 3. Conſequently if the ſeveral apertures be equal portions of equal ſpheres, the degrees of brightneſs of the ſeveral pictures formed by a concave ſpeculum, a double-convex glaſs, a glaſs ſphere and a plano-convex glaſs, are reſpectively as the ſquares of the decreaſing muſical progreſſion 12, 6, 4, 3. Becauſe the reſpective focal diſtances are $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$ of the diameter of the given ſphere, by Art. 205, 235, 227: and the reciprocals of an arithmetical progreſſion are called a muſical progreſſion.

739. *Corol.* 4. Therefore the concave ſpeculum has greatly the advantage of the ſphere and lens's, for illuminating microſcopical objects, and alſo for burning things in the ſun-ſhine, tho' not in ſo great a proportion, as will appear by the next corollary.

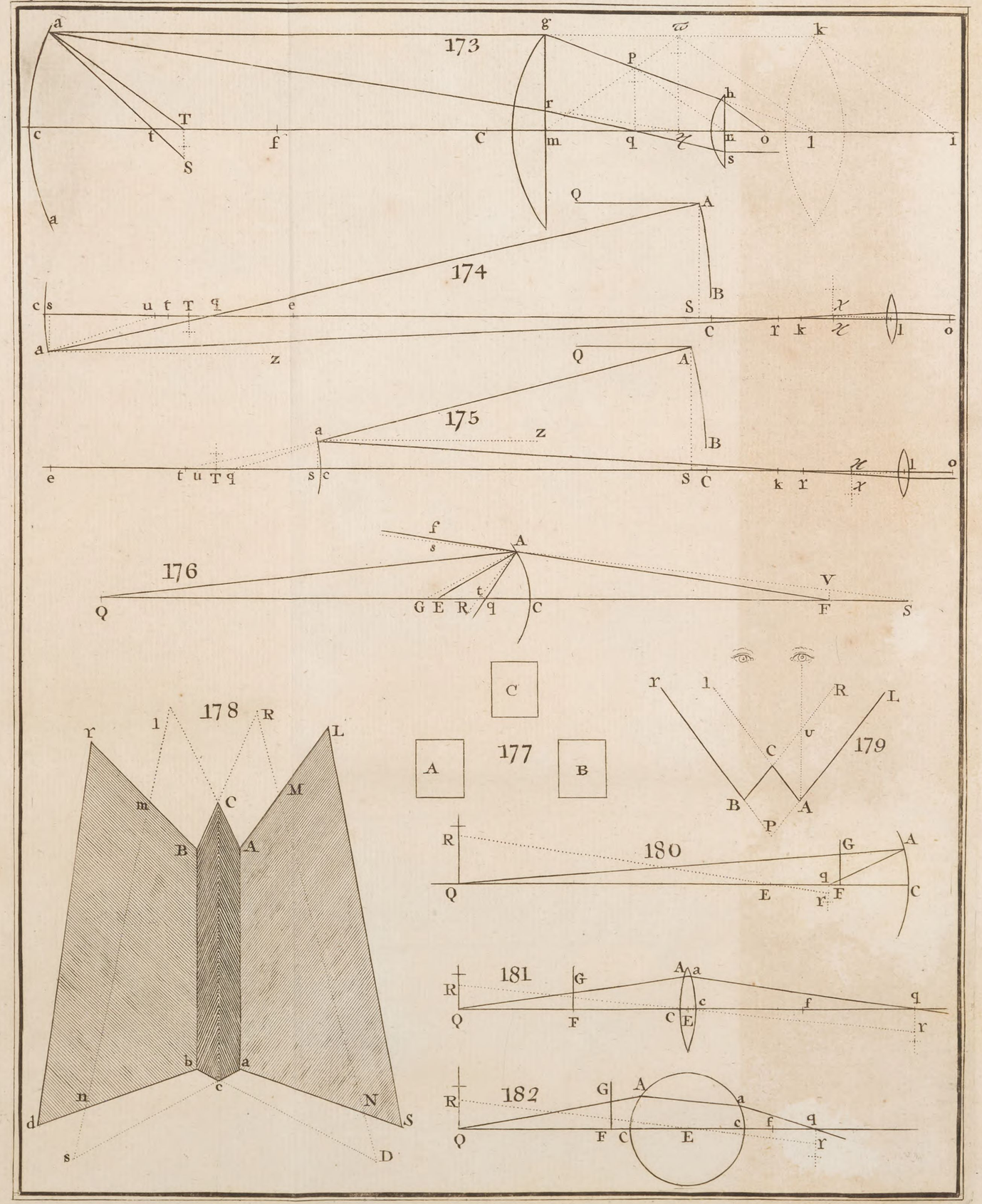
740. *Corol.* 5. Though the rays in two pictures of the ſun formed by ſimilar ſpecula, be equally denſe *, yet the picture formed by the larger ſpeculum, being proportionably larger, will burn things more vehemently than the ſmaller; becauſe the burning particles of matter communicate and propagate their heat to one another. And when the ſpecula are ſimilar, the aberrations of the rays, from the peripheries of the pictures, are alſo ſimilar *.

* Rem. 375.

* Art. 337.

End of the REMARKS.




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AN ESSAY UPON DISTINCT AND INDISTINCT VISION.

TO Dr. SMITH.

SIR,

As my first entrance upon the following speculation was principally owing to the hints and observations contained in your Book, the greatest part of which I have had by me in print for several years, and I was encouraged to pursue it thus far by the admirable facility of those methods of computation which you have therein delivered, I look upon whatever may be of any value in these papers, as more yours than mine. To you therefore I send them, to make what use of them you think proper.

I might have added a great many other uncommon and hitherto unobserved appearances, particularly in looking thro' small perforations in cards or thin plates, but that I found it would take up too much time to digest them into order and insert them in their proper places; and to retard the publication of your long expected work for any thing of mine, would be doing a sensible injury to the publick. I am,

SIR,

your most affectionate friend,

and very humble servant,

JAMES JURIN.

Objects when seen distinctly.

1. AN object is said to be seen distinctly, when its outlines appear clear and well defined, and the several parts of it, if not too small, are plainly distinguishable, so as that we can easily compare them one with another, in respect to their figure, size and colour. For instance, the words of this book are distinctly seen, when the letters appear well defined, and their shape and the intervals between them are plainly perceived

and diſtinguiſhed, ſo as that the book may be read with eaſe. A ſingle letter alſo is diſtinctly ſeen, when the ſeveral parts of the letter, the connexion of thoſe parts, and the intervals between them are clearly perceived and diſtinguiſhed.

2. In order to ſuch diſtinct Viſion, it has hitherto been commonly thought, that all the rays of a pencil flowing from a phyſical point of an object, muſt be exactly united in a phyſical, or at leaſt in a ſenſible point of the *Retina*.

Exact union of rays not neceſſary.

3. But that ſuch an exact union of the rays is not neceſſary to diſtinct viſion, will manifeſtly appear upon making the following tryals.

Experiments to prove it.

4. Take a title-page of a book, in which there is print of three or four different ſizes: And firſt, place the book at ſuch a diſtance, as that every ſort of print may without any ſtraining of the eye, appear perfectly diſtinct. In this caſe it may reaſonably be preſumed, the rays of every pencil flowing from the letters are accurately collected into ſo many ſeveral phyſical, or at leaſt, ſenſible points upon the *Retina*.

5. Afterwards bring the book by degrees ſo near, as that the letters of the ſmalleſt print may now begin to appear a little confuſed, and cannot by any endeavour or ſtraining of the eyes be rendred ſo diſtinct as they were before. Then, keeping the book at that ſame diſtance, look at a print ſomewhat larger than the former, and that larger print ſhall ſeem perfectly diſtinct without any the leaſt appearance of confuſion.

Here, it is manifeſt from the leſs diſtinct appearance of the ſmaller print, that at this diſtance the rays of each pencil are not accurately united in a ſenſible point of the *Retina*, notwithſtanding which the larger print appears diſtinct.

6. If the book be brought ſtill nearer, the ſmalleſt print will now be quite confuſed, and the larger print will begin to appear indiſtinct: But keeping the book at this ſame nearer diſtance, a print ſtill larger will appear diſtinct.

In this caſe the rays are ſtill leſs accurately collected into points; and yet the largeſt of theſe prints appears as diſtinct as the two ſmaller prints had done in the former tryals.

7. Theſe experiments may be made the contrary way likewiſe, by putting on a pair of ſpectacles of a ſufficient convexity, or by a ſhortſighted perſon without any ſpectacles. In this caſe the book muſt firſt be held at ſuch a diſtance, as that every ſort of print may appear as diſtinct as poſſible: And muſt afterwards be removed ſucceſſively to ſuch greater diſtances from the eye, as that the ſmaller prints may one after another begin to appear confuſed, while the larger preſerve their diſtinctneſs.

8. To thoſe, who ſhall make theſe experiments with ſufficient care, it will manifeſtly appear, that ſometimes we have diſtinct viſion, when the rays of each pencil iſſuing from the object are not accurately collected by the eye into a ſenſible point upon the *Retina*; and that this ſort of diſtinct viſion, without an exact union of the rays, is not to be diſtinguiſhed, even by an attentive eye, from that other ſort of diſtinct viſion where the rays are the moſt accurately collected into ſenſible, or even into phyſical points.

Two ſorts of diſtinct Viſion.

9. Diſtinct Viſion may therefore not unfitly be divided into the two following ſorts, or ſpecies, namely, Viſion perfectly diſtinct, or *Perfect Viſion*, and Viſion imperfectly diſtinct, which I ſhall uſually call, ſimply, by the name of *Diſtinct Viſion*.

Perfect Viſion.

10. Viſion perfectly diſtinct, or *Perfect Viſion*, is that, in which the rays of a ſingle pencil are collected into a ſingle phyſical, or ſenſible point of the *Retina*, as in ART. 4.

Diſtinct Viſion.

11. Viſion imperfectly diſtinct, or ſimply, *Diſtinct Viſion*, is that, in which the rays of each pencil are not collected into a ſenſible point, but occupy ſome larger ſpace upon the *Retina*, yet ſo as that the object is diſtinctly perceived, as the larger point in Art. 5 and 6.

12. *Perfect Viſion*, in a given eye, and a given diſpoſition of that eye, depends only upon the diſtance of the object. It has no dependance upon the magnitude of the object, except only that it is requiſite the object ſhould not be ſo very ſmall as on that account not to be perceivable. This excepted, at whatever diſtance any one object is ſeen perfectly diſtinct, all other objects do likewiſe appear perfectly diſtinct.

13. *Diſtinct Viſion*, in a given eye, and a given diſpoſition of that eye, depends upon the diſtance and magnitude of the object jointly. In Art. 5. the leſſer print appears confuſed, becauſe it is too near: But, at a greater diſtance, as in Art. 4. it appeared diſtinct.

And

And the larger print in Art. 5. appears diſtinct becauſe of its magnitude: But it will appear confuſed at a leſs diſtance, as in Art. 6.

14. It appears therefore, that there is a real difference between *Perfect Viſion*, and what we call *Diſtinct Viſion*, and we deſign to enquire particularly into the reaſon, Why an object may be ſeen diſtinctly without *Perfect Viſion*: But in order thereto, it will firſt be neceſſary to take notice of the principal *Phænomena* when objects are confuſedly or indiſtinctly ſeen, and to examine into the cauſes of thoſe appearances.

In doing which, we ſhall generally ſpeak of the object we conſider, as being white or luminous upon a dark ground, though the *Phænomena* would be the ſame, and from the ſame cauſe, if the object were dark or black upon a white ground.

15. If a circular object be viewed at a diſtance proper for *Perfect Viſion*; its picture upon the *Retina* will be circular, and proportional in diameter to the angle which the object ſubtends at the eye; its limb will be well defined, and all parts of the circular picture will be equally ſtrong. Conſequently, the idea thereby excited in us, will be that of a circle equally ſtrong in all parts, and well defined.

Phænomena of indiſtinct Viſion in a circular object.

16. If the ſame circular object be viewed at a diſtance much too ſmall for *Perfect Viſion*; its picture upon the *Retina* will be circular, but the diameter will be greater than in proportion to the angle which the object ſubtends at the eye: Nor will the picture be equally ſtrong in all its parts, but the middle part will generally be the ſtrongeſt, and will be ſurrounded with a *penumbra* growing gradually fainter towards the outer edge, whereby the limb will appear indiſtinct and ill defined. Conſequently, the idea thereby excited in the mind will be that of a circle too large, and too faint and indiſtinct towards the limb. That is, inſtead of the appearance *A* Fig. 1. we ſhall have the appearance, *B*, or *C*, or *D*.

17. The cauſe of this is not difficult to diſcover. For ſince, by our ſuppoſition, the diſtance of the circular object from the eye is too ſmall for *Perfect* or *Diſtinct Viſion*, it is obvious that the rays of each pencil iſſuing from the object cannot be united but at a point beyond the *Retina*; conſequently, the rays of each pencil will occupy a circular ſpace upon the *Retina*.

Let therefore, the circle *A B D C*, Fig. 2. repreſent that circular ſpace upon the *Retina*, which the image of the object would take up, if that image were perfectly diſtinct: Or, which comes to the ſame thing, let *A B D C* repreſent that circular ſpace upon the *Retina*, which is occupied by the centers of all the pencils of rays belonging to the indiſtinct image of the circular object. This circular ſpace upon the *Retina*, repreſented by *ABDC*, we ſhall call the image of the object, and ſometimes, to diſtinguiſh it from other appearances, we ſhall call it the *true image*. Alſo let the circle *f g h c*, having its center *c* in the circumference of the former circle *A B D C*, repreſent that circular ſpace upon the *Retina*, which is taken up by one of the extreme pencils of rays iſſuing from the object. This circle *f g h c* we ſhall call the *circle of diſſipation*, becauſe the rays of a pencil, inſtead of being collected into the central point *c*, are diſſipated all over this circle: And the radius of this circle, *c f*, or *c g*, we ſhall for the ſame reaſon call the *radius of diſſipation*. And let the circle *a b d f C*, concentrick with the firſt circle, or the *true image*, *A B D C*, touch the circle *f g h c*, in the point *f*.

True image what.

Circle of diſſipation what.

Radius of diſſipation what.

18. Then, I ſay, 1. that part of the image of the circular object, which is repreſented by the circle *a b d f C*, will be equally ſtrong in all parts; and will be of the ſame ſtrength, as if the image of the object had been perfectly diſtinct.

To prove this, let the circles *A B D C*, *a b d C*, Fig. 3. repreſent the ſame things as before, and taking any point at pleaſure, as *c*, within the circle *a b d C*, from that point as a center, with the *radius of diſſipation c f*, draw the circle *f g h c*. Then it is manifeſt, that, as the pencil of rays whoſe center is the point *c*, is diſſipated or ſcattered over the whoſe circle *f g h c*, and thereby helps to illuminate all the points or centers of pencils ſituated within that circle; ſo on the other hand, the point *c* muſt receive light from every pencil whoſe center is ſituated within the ſame circle: Or, the point *c* beſtows a part of its light upon every other point of the circle *f g h c*, and receives an equal portion of light from each of thoſe points; ſo that it is juſt as much illuminated, as if the rays of its pencil had never been diſſipated, but had been all collected into that one point *c*, as they would have been, if the image had

had been perfectly distinct. And as this is true of every point within the circle *abdC*, it is manifest, that this whole circle must be as strongly illuminated, as if the image had been perfectly distinct, and must be equally strong in all its parts.

False image what.

This part of the *true image*, represented by the circle *Cabd*, which loses none of its light by dissipation, but is as strongly illuminated as if it were seen by *Perfect Vision*, and is equally luminous in all parts of it; we shall, for distinction sake, call the *false image*.

19. I say, 2. The circular ring *ABDdba*, comprehended between the circumferences of the two circles *ABD*, *abd*, Fig. 2, 3, whose breadth is equal to the *radius of dissipation*, or that part of the *true image ABDC* which lies without the *false image abdC*, will not be so strongly illuminated as the *false image abdC*, and will gradually grow fainter towards its extremity.

For, let the circles *ABDC*, *abdC*, Fig. 4. represent the same things as before, and taking two points within the circular ring, *ABD dba*, one more inward as *c*, and the other more outward as *m*, from the centers *c* and *m*, with the radii *cf*, *mn*, each equal to the *radius of dissipation*, draw the two circles *ch fg*, *mno*, cutting the circumference *ABD* in the points *h* and *f*, *n* and *o*, respectively.

Then is it plain, that the pencil, whose center is the point *c*, will dissipate its rays into the whole circle *chfg*; but that it will not receive light from every point in that circle, but only from those points of it, that are likewise situated within the circle *ABDC*. All those points therefore, that are comprehended within the *Lunula hf*, return no light to the point *c* in recompence of what they receive from it. The point *c* therefore gives away more light than it receives, and will consequently appear darker than any point within the circle *abdC*; and this excess of darkness will be measured by the area of the *Lunula hf*.

In like manner it will be found, that the point *m* must appear darker than any point in the circle *abdC*, and that this excess of darkness is measured by the *Lunula no*. But the *Lunula no* is greater than the *Lunula hf*, and consequently the point *m* which is nearer to the outside of the ring, is darker than the point *c* which is more inwardly situated. The whole ring therefore is darker than any part of the circle *abdC*, and grows gradually darker towards its outer edge. And at the very extremity it has not half the light of any part of the circle *abdC* as is manifest by the inspection of the circle *gfhc*, Fig. 2.

20. I say, 3. Besides the ring last described, which is darker than the *false image*, or the circle *abdC*, there is another ring of equal breadth, situated without the *true image*, or the circle *ABDC*, which is still darker, and whose light gradually diminishes towards the outside, till it becomes insensible, and at last vanishes away into nothing.

For, let *ABDC*, Fig. 5. represent the same thing as in Fig. 2, and from the center *c* taken any where in the circumference *ABD*, with the *radius of dissipation cg*, draw the *circle of dissipation cghf*: and from the center *C* describe the circle *GFHC*, touching the *circle of dissipation* in the point *g*. Then, between the two circumferences *ABD* and *GFH* will be comprehended a new ring *ABDHFG*, of the same breadth with the former ring *ABD dba*, Fig. 2, 3, 4. This new ring will receive light from the pencils, whose centers lie within the former ring; but it will be darker than any part of the former, and its light towards the outer edge will gradually diminish till it vanishes into nothing. This is manifest from the inspection of the figure, where the light received by the point *c* situated on the inner edge of this ring is measured by the circular segment *hf*, and the light received by the point *m* situated near the outer edge is measured by the much smaller circular segment *no*.

Annular penumbra what.

This second ring, which lies without the *true image*, and gradually decreases in light towards its outside, we shall take leave to call for distinction sake, the *annular penumbra*.

From what has been said it appears, that if the radius *CA* of the circle *ABDC*, or of the *true image*, be called *r*, and the radius of the *circle of dissipation cfgh* be called ϱ, the radius of the *false image*, or of the circle *abdC*, will be the difference of those two radii, or $r - \varrho$, the breadth of each of the rings *ABDdba*, Fig. 2, and *FGHDBA*, Fig. 5, will be ϱ, and the radius of the whole appearance or of the circle *CFGH* will be the sum of the radii of the *true image* and of the *circle of dissipation*, or $r + \varrho$.

Having shown that a circular object, when viewed at too small a distance for *Perfect* or

Distinct

Diſtinct Viſion, muſt appear more ſtrongly illuminated in the middle and fainter towards the edge, I proceed now to conſider ſome particular cauſes of this phœnomenon, which I ſhall have occaſion to apply hereafter to other purpoſes.

21. If in Fig. 2, the *radius of diſſipation* be increaſed, the *falſe image*, or the circle *abdC*, which preſerves its full quantity of light as much as if the image were perfectly diſtinct, will grow leſs, and the ring *ABD dba* and the *annular penumbra* without it, *ABDHFG*, will both grow proportionably broader, as in Fig. 6, and 7, ſo that the circular object, from appearing as *B*, Fig. 1, will come to the appearance *C*, or to ſome other where the *nucleus*, which is fully illuminated, will be ſtill ſmaller, and the ſhade ariſing from the two rings abovementioned, will be broader.

22. If the *radius of diſſipation* be equal to the radius of the *true image*, as in Fig. 8, the *falſe image*, or the circle *C a b d*, whoſe radius is $r-\varrho$, will now vaniſh, and the ring *ABD dba*, Fig. 6, 7, will degenerate into and take up the whole circle *ABDC*. Conſequently, there will now be no part of the image, that will have its full quantity of light as if the image were perfectly diſtinct, except the central point alone which will receive light from every other point; nor will there be any two points at different diſtances from the center, that will be equally illuminated: but the light decreaſing all the way from the center will grow gradually fainter towards the extremity, as is manifeſt from Fig. 9, 10, in each of which the meaſure of light received by the point *c*, nearer to the center in the one, and more remote in the other is expreſſed by the gibbous ſegment *g h*. For in the firſt of theſe Figures, where the point *c* is nearer to the center C of the image, the gibbous ſegment *g h* and conſequently the light thrown upon the point *c*, is greater than in Fig. 10, where the point *c* is more remote from the middle of the image.

23. If the *radius of diſſipation* exceed the radius of the *true image*, as in Fig. 11, we ſhall again have a portion of the *true image*, which will be more ſtrongly illuminated than the reſt, though not ſo ſtrongly as by *Perfect Viſion*, and which will be equally luminous in all parts. And this portion will be ſeated in the middle of the *true image*, and will be ſurrounded by an appearance leſs luminous and decreaſing gradually in light towards the outer edge.

For in Fig. 11. let the circle *CABD* repreſent, as before, the *true image* of a circular object upon the *Retina*: And taking a point as *c* in the circumference *ABD*, from the center *c*, with the *radius of diſſipation c b*, greater than *C c* the radius of the *true image*, draw the *circle of diſſipation cfgh*. Then from the center *C*, with the radius *C b*, equal to the difference between the *radius of diſſipation bc*, and the radius of the *true image Cc*, deſcribe the circle *Cabd*, touching on the inſide the *circle of diſſipation* in the point *b*. Alſo, from the ſame center *C*, with the radius *Cg* equal to the ſum of the *radius of diſſipation cg*, and the radius of the *true image Cc* added together, draw the circle *CFGH*, touching the *circle of diſſipation* on the outſide in the point *g*.

24. Then, I ſay, 1. that part of the *true image*, which is repreſented by the circle *C a bd*, will be equally luminous in all parts. For the point *b* in the circumference of this circle will be diſtant from *c*, the moſt remote point of the *true image*, by juſt the *radius of diſſipation bc*, and will therefore receive light from that point *c*. Much more will the point *b* receive light from every other point of the *true image* leſs remote from it than the point *c*. And much more ſtill will any other point within the circumference *abd*, receive light from every point of the *true image*.

Since therefore each point of this circle *Cabd* receives light from every point of the *true image*, this circle muſt be equally illuminated in all parts.

25. I ſay, 2. this circle *Cabd* will not be ſo ſtrongly illuminated, as if it were ſeen by *Perfect Viſion*.

For, ſince any point in this circle, as *C*, ſcatters its light into a circle as large as the *circle of diſſipation*, and receives light only from the circle *CABD*, leſs than the *circle of diſſipation*, it is manifeſt, that this point gives away more light than it receives back, and conſequently muſt be leſs luminous than if it were ſeen by *Perfect Viſion*.

26. I ſay, 3. that this circle *Cabd* will be ſurrounded by an appearance leſs luminous, and gradually decreaſing in light towards its outer extremity.

For around this circle will appear the ring *abdDBA*; and without that ring will appear the

the *annular penumbra ABDFGH*, in the ſame manner and for the ſame reaſons, as in Art. 19, 20.

Faint falſe image what.

27. This circle *Cabd*, which is equally luminous in all parts, and brighter than all the reſt of the appearance, though not ſo bright as if ſeen by *Perfect Viſion*, we ſhall for that reaſon call the *faint falſe image*.

From what has been ſaid, it is manifeſt, that if, as before, we call the radius of the *true image*, r, and the *radius of diſſipation*, ϱ, the radius of the *faint falſe image* muſt be $\varrho - r$, the breadth of the firſt ring without the *faint falſe image* will be $\varrho - 2r$, the breadth of the ſecond ring or *annular penumbra* will be ϱ, and the radius of the whole appearance, or of the circle *CFGH*, will be $r + \varrho$, theſe two laſt the ſame as in Art. 20.

And if the *radius of diſſipation cb* be encreaſed, the *faint falſe image Cabd* will increaſe and approach nearer to the magnitude of the *true image CABD*; and thereby the ring *abdDBA* will grow narrower.

28. If the *radius of diſſipation* be equal to the diameter of the *true image*, as in Fig. 12, 13, the *faint falſe image*, or the circle *Cabd*, whoſe radius is $\varrho - r$, or $2r - r = r$, will now be equal to and coincide with the *true image CABD*. And this image will be equally illuminated in all parts, but each point will have but a quarter part of the light, and the whole image will be but a quarter part as ſtrong, as if it had been ſeen by *Perfect Viſion*.

For any point, as *c*, taken in the extremity of the image *CABD*, Fig. 12, will diſſipate its light into the circle *gfhc*, and will receive light only from the circle *ABDC*, which is but a quarter part of the *circle of diſſipation*. And another point as *c* taken in the middle of the image *CABD*, Fig. 13, will likewiſe diſſipate its light into the circle *gfhc*, and will receive light only from the circle *ABDC*, which is juſt a quarter part of the *circle of diſſipation*. The middle therefore and the extremities of the image *ABDC* will be equally illuminated, having each a quarter of the light they would have had, had the image been perfectly diſtinct.

29. The ring *abdDBA*, Fig. 11, whoſe breadth was $\varrho - 2r$, which was comprehended by the circumferences of the *faint falſe image* and the *true image*, will now vaniſh by the coincidence of thoſe circumferences.

30. But there will ſtill appear the *annular penumbra*, incompaſſing the image, which will be fainter than the image, and will gradually grow weaker towards the extremity, till it vaniſh and decline into nothing. And the breadth of this *annular penumbra* will be equal to the *radius of diſſipation*.

For, in Fig. 14, 15, 16, let the circle *ABDC* repreſent, as before, the *true image* of the circular object upon the *Retina*, and with the radius *CG*, compoſed of *CA*, the radius of the *true image*, and the *radius of diſſipation* added together, draw the circle *GFHC* concentrick with the circle *ABDC*: Then will the ring *GFHDBA* comprehended between the two circumferences *GFH* and *ABD*, be equal in breadth to the *radius of diſſipation*.

Likewiſe, from any point of this ring as *c*, with the radius *cg* equal to the *radius of diſſipation*, draw the *circle of diſſipation cfgh*.

31. Then, I ſay, 1. this ring will form a *penumbra* fainter than the image.

For the point *c* will receive light only from the pencils whoſe centers are ſituated in the circular ſegment *fg*. Therefore this point *c*, and conſequently every point in the ring, will be leſs illuminated than every point within the image; ſince every point within the image, does, as we have already ſeen, (Art. 28.) receive light from every other point within the image.

32. I ſay, 2. this *penumbra* will grow gradually fainter towards its outer edge, and at the very extremity will dwindle into nothing.

For, if the point *c* be gradually removed farther towards the outer edge, as in Fig. 14, 15, it is manifeſt from the inſpection of the figures, that the circular ſegment *fg*, and conſequently the light derived from that ſegment to the point *c*, will gradually diminiſh: And when the point *c* arrives at the very extremity of the ring, as in Fig. 16, the ſegment *fg* and the light derived from it muſt vaniſh into nothing, the *circle of diſſipation* then only touching the circle *ABD*.

33. If the *radius of diſſipation* exceed the diameter of the *true image*; then to the *true image* will be joined a ring of ſcattered light, which ring will be illuminated equally with the *true image*, ſo that the *true image* itſelf and this ring joined together will form one appearance, or *faint falſe image*, equally ſtrong in all parts, but of a far leſs ſtrength than if the image were perfectly diſtinct.

Alſo

Alſo round this appearance, or *faint falſe image*, will be formed an *annular penumbra*, the light of which will gradually diminiſh towards its outer edge, and at the very extremity will dwindle away into nothing.

For in Fig. 17, 18, 19, let the circle *ABDC*, as before, repreſent the *true image* of the circular object upon the *Retina*, and with the radius *CG*, compoſed of *CA* the radius of the *true image* and the *radius of diſſipation* added together, deſcribe a new circle *FGHC*, concentrick with the *true image ABDC*. Alſo, with the radius *Cb*, equal to the difference between the *radius of diſſipation* and the radius of the *true image*, deſcribe a third circle *Cabd* concentrick with the two former.

34. Then, I ſay, firſt, the ring *ABDdba*, comprehended between *ABD* the circumference of the *true image* and the circumference *abd*, will be equally illuminated with the *true image ABDC*.

For let any point as *c* be taken in the outer edge of this ring, Fig. 17, and with the radius *co*, equal to the *radius of diſſipation*, draw the circle *cofgh*. This circle muſt touch the circumference of the *true image* in the point *o* oppoſite to *c*, becauſe *co* is the *radius of diſſipation*, and *cC*, by the conſtruction, was equal to the difference between that radius and the radius *Co*. The point *c* therefore muſt receive light from every pencil whoſe center lies in the *true image ABDC*, and muſt conſequently be illuminated equally with any point within the *true image*. Much more muſt any other point of the ring, ſituated more inwardly than *c*, be equally illuminated with any point within the *true image*. Conſequently, the whole ring will be equally illuminated with the *true image*, and will form one uniform appearance with it, without any diſtinction, that is, a *faint falſe image*, like that deſcribed in Art 23, *&c.*

35. I ſay, ſecondly, this whole appearance, or *faint falſe image*, compoſed of the ring *ABDdba* and the *true image CABD* will be of far leſs ſtrength than the *true image* alone would have been, were it formed by viſion perfectly diſtinct. For the light of all the pencils belonging to the *true image* would in the caſe of *Perfect Viſion* be confined to that image only, whereas now it is ſcattered thro' the whole appearance, or *faint falſe image*, compoſed of the ring and *true image*, and conſequently muſt be much weaker. Beſides, a part of this light is farther ſcattered beyond the ring. For,

36. I ſay, thirdly, the ring *abdHGF*, comprehended between the two circumferences *abdc* and *HGF*, will form a *penumbra* round the *faint falſe image* above mentioned, which *annular penumbra* will gradually decreaſe in ſtrength towards its outer edge, till at the very extremity it vaniſhes to nothing.

For the point *c*, Fig. 17, at the inner edge of this ring, will be equally luminous with any point in the *true image*, as has been already ſhown, in Art. 34. And at the point *c*, Fig. 18, ſituated in the outer edge of this ring, the light will quite vaniſh, the circle of diſſipation *cfgho* only touching the *true image* in the point *o*. Therefore all the intermediate points of the ring, as *c*, Fig. 19, muſt be illuminated with intermediate degrees of light decreaſing towards the outer edge of the ring; that is, the ring will form a *penumbra* decreaſing gradually in ſtrength towards the outer edge, and vaniſhing away at the outer edge itſelf.

37. From this conſtruction it appears, that if, as before, we call the radius of the *true image r*, and the *radius of diſſipation* ϱ, the radius of the *faint falſe image* will be $\varrho - r$, the radius of the whole appearance will be $\varrho + r$, and conſequently the breadth of the *annular penumbra* will be $2r$.

38. If the radius of the *true image* be exceeding ſmall in reſpect of the *radius of diſſipation*, the *faint falſe image*, or the circle *Cabd*, whoſe radius is $\varrho - r$, Fig. 17, 18, 19, will be very nearly equal to the *circle of diſſipation*. Alſo the breadth of the *annular penumbra abdHGF*, whoſe breadth is $2r$, by Art. 37, will be exceedingly ſmall, ſo that the whole *penumbra* will be utterly inſenſible.

39. It may not here be amiſs to make one or two general remarks with regard to theſe *annular penumbræ* of circular objects, when the *radius of diſſipation* is given. Remarks upon *annular penumbræ*.

1. The *annular penumbra* will be ſenſible nearer to its outer edge, when the object is brighter, than when it is leſs luminous.

For the light in a given point of the *annular penumbra* will, *cæteris paribus*, be proportional to the luminouſneſs of the object. Therefore, if at a given diſtance from the

outer extremity of the *annular penumbra* of a brighter object, the light is but juſt ſtrong enough to affect the ſenſe; it will not at all affect us at that ſame diſtance, when the object is leſs luminous.

2. The *annular penumbra* will be ſenſible nearer to its outer edge, when the object is larger, than when it is ſmaller.

For the light in a given point of the *annular penumbra* will, *cæteris paribus*, be proportional to the circular ſegment cut off from the *true image* by the *circle of diſſipation*, of which the given point is the center. Let therefore the point *m*, in Fig. 5, be taken ſo near the outer extremity of the *annular penumbra*, as that the light in that point may be but juſt ſtrong enough to affect the ſenſe. Then will the *circle of diſſipation* belonging to this point *m*, cut off from the *true image ABDC*, a circular ſegment *no*, of a given depth; and the light in the point *m*, will be proportional to that ſegment, *no*. But if the circular object, or the *true image*, *ABDC* be diminiſhed, the circular ſegment *no*, whoſe depth is given, will have a leſs breadth, and conſequently a leſs magnitude. Therefore the light in the point *m* will not now be ſtrong enough to affect the eye.

40. As the reaſon of all theſe *phænomena* is only deduced from this one principle, that a pencil of rays iſſuing from the object is not collected into a point upon the *Retina*, but occupies a circular ſpace thereon, it is manifeſt that in all caſes where a pencil takes up a circular ſpace upon the *Retina*, the *phænomena* will be the ſame.

But when an object is too remote for *Perfect Viſion*, the rays of a pencil iſſuing from it will converge to and unite in a point, before they arrive at the *Retina*, and by diverging from that point will take up a circular ſpace upon the *Retina*.

Therefore all the above recited *phænomena* may ariſe from a circular object placed at too great a diſtance for *Perfect Viſion*.

41. Now, the human eye, as we ſhall have occaſion anon particularly to ſhew, when not too much flatted by age, is only fitted to ſee diſtinctly at moderate diſtances from our bodies, and is not at all accomodated to ſee objects very remote, tho' at much leſs diſtances than thoſe of the Planets or Fixed Stars, with perfect diſtinctneſs. Therefore a Planet or Fixed Star muſt to our eyes exhibit ſome of the *phænomena* abovementioned, and it will not be amiſs to conſider the principal of them.

Appearance of the full Moon.

42. The full Moon will appear broader than a circular object ſubtending an equal angle ſeen by *Perfect Viſion*.

For, by Art. 20, if the circle *ABDC*, Fig. 5, repreſent the *true image* of the full Moon upon the *Retina*, or the image of a circular object ſeen by *Perfect Viſion*, and ſubtending an equal angle with the full Moon; without this circle there will appear a *penumbra* repreſented by the ring *ABDHFG*, which being added to the circle *ABDC* will conſtitute the whole appearance *GFHC*, which is greater than *ABDC*.

43. If the Moon inſtead of being a globe, were only a plane disk, ſomething ſcabrous, ſo as to reflect the light equally every way from every part of it; it would then exhibit all the *phænomena* deſcribed in Art. 18, 19, 20.

For inſtance, if this plane disk were to ſubtend at the eye an angle of 32′, and conſequently its *true image ABDC*, Fig. 2, upon the *Retina* were ſuch, as correſponds to an angle of 32′ ſubtended by this disk at the eye; and the *radius of diſſipation* in a human eye be ſuch as correſponds to an angle of 2′ ſubtended at the eye by a very diſtant object, as I think it generally is, in what are reckoned good eyes, upon viewing celeſtial bodies; then by Art. 18, a part of the *true image*, repreſented by the circle *abdC*, would be brighter than the reſt, and the diameter of this part would be 28′: And by Art. 19. another part of the *true image*, repreſented by the ring *abdDBA*, 2′ in breadth, would gradually diminiſh in brightneſs till at its outer extremity *ABD* it had leſs than half the brightneſs of the part *abdC*: And this ring by Art. 20, would be ſurrounded by the *annular penumbra ABDHFG*, Fig 5, likewiſe 2 minutes in breadth, which from having an equal brightneſs with the outer edge of the ring within it, would gradually grow weaker towards its outer edge, and there would vaniſh to nothing. So that the breadth of the whole appearance compoſed of the *true image* and this *annular penumbra* would be 36′, except that a little of the outer part of the *annular penumbra* might be too faint to be diſcerned, and conſequently the breadth of the whole appearance might be ſomething leſs than 36′.

44. But

44. And farther, If the ſurface of the moon, although ſpherical, yet were neither poliſhed, nor conſiderably uneven, but a little ſcabrous all over, ſo as to reflect the light equally every way from all parts of it; then this ſurface ought to reflect light to the eye from every part of it, juſt in the ſame manner as the correſponding parts of a plane ſurface, underſtood to be likewiſe a little ſcabrous, into which the parts of the ſpherical ſurface are projected by the eye. Conſequently, this ſcabrous ſpherical Moon ought to exhibit all the ſame *phænomena* as the ſcabrous plane disk of the preceding article.

45. But, in fact, the Moon does not exhibit all theſe *phænomena*. The middle part does not appear brighter than the limb. On the contrary, the limb of the Moon for a digit or two in breadth appears as bright, if not brighter than the middle. What ſhould be the cauſe of this appearance, which from our reaſoning above ſeems ſo little to be expected?

46. This appearance proceeds, as I apprehend, from the two following cauſes.

1. The middle part of the Moon is in great meaſure taken up with the faint reſemblance of the human face, which is darker than the other parts, and being likewiſe more depreſſed than the reſt, is therefore by ſome Philoſophers taken for ſeas in the Moon: But the outer part of the Moon ſeems to have a much leſs proportion of thoſe dark parts. This is one reaſon, why the outer part ſhould appear brighter than the middle.

2. Although the outer part of the Moon had an equal proportion of what is taken for ſea, with the middle part; yet it muſt appear brighter than the middle part. For the ſea in the middle part lies directly expoſed to the eye, and conſequently is ſeen of its full magnitude in proportion to the land: But the ſeas towards the limb muſt be either wholly, or in great part hid from the eye by the prominence of the land, by reaſon of the convexity of the ſpherical figure; juſt as the cliffs at *Dover*, to an eye raiſed to a great height in the air and viewing them obliquely, might be ſo projected, as to appear adjoining to thoſe at *Calais*, and by that means to hide the intervening ſea; or as the water of a river is hid from an eye placed upon a hill, by the ſeeming union of its oppoſite banks.

47. Theſe two cauſes may have ſuch an effect as to render the limb of the Moon, notwithſtanding the diſſipation of its light by indiſtinct viſion, yet ſtill conſiderably ſtronger than the middle part, where no light is loſt by diſſipation.

For inſtance, if the limb by diſtinct viſion be above twice as bright as the middle part, it may notwithſtanding it loſes half its light by the diſſipation, be ſtill brighter than the middle of the Moon.

48. The ſecond of theſe cauſes may give us the reaſon, why the *annular penumbra*, which by our theory ought to encompaſs the Moon, may decreaſe very ſlowly in light towards its outer edge, ſo as that the inequality of brightneſs between its inner and outer part may be leſs ſenſible than it would otherwiſe be.

For, if a point as *c*, Fig. 5, be taken near the inner edge of this *penumbra*, and another point as *m*, be taken near its outer edge, the light thrown upon the firſt point *c* will bear a much leſs proportion to the light thrown upon the ſecond point *m*, than the proportion between the circular ſegments *fh*, *no*, which are cut off from the *true imoge ABDC* by their reſpective *circles of diſſipatiou cfgh*, *mno*, by reaſon that the ſegment *no* does in proportion to its magnitude give a greater quantity of light than the ſegment *fh*.

49. And if to this conſideration we add, that the outer edge of the *annular penumbræ* will have the advantage of having its light heightned by compariſon with the adjoining darkneſs of the ſky, it will appear upon the whole, that the degrees of light towards the outer edge of this *penumbra* ought hardly to be ſenſible.

50. But now another difficulty preſents itſelf. If the Moon appears to our eyes 36′ in breadth, when in fact its *true image* is but 32′, the ancient aſtronomers, and thoſe of the moderns likewiſe, who obſerved by plain ſights, as the noble *Hevelius*, muſt have made the apparent mean diameter of the Moon much larger than it is found to be by teleſcopick ſights, that is, by *Perfect* or at leaſt by *Diſtinct Viſion*.

51. I anſwer, that the obſervations, which are handed down to us from antiquity, are probably thoſe of the moſt ſkilful and moſt experienced obſervers. But thoſe perſons, who have been much accuſtomed to contemplate very remote objects, as muſt be the caſe of the moſt experienced aſtronomers,

have thereby acquired a facility, or habit rather, of ſo altering the conformation of their eyes, as to ſee thoſe remote objects much more diſtinctly than the reſt of mankind, who have not been ſo accuſtomed; juſt as thoſe, who are much employed in viewing ſmall objects very near, as engravers, painters in miniature, *&c.* do thereby acquire the facility and habit of ſeeing thoſe near and ſmall objects much more diſtinctly than other perſons. We muſt therefore ſuppoſe, that though the *radius of diſſipation* in viewing the ſtars or planets be 2′ in the eyes of the generality of Mankind, yet to practical aſtronomers it may be leſs than one minute, poſſibly not above half a minute, and then the diameter of the Moon obſerved by plain ſights will exceed the diameter taken by teleſcopick ſights by about 1′ only.

52. But farther, it is poſſible, that the obſervations of the ancient aſtronomers which are handed down to us, are not the product of their younger days, but made when they were grown famous and were advanced in years, at which time, no doubt, ſome of them might correct the obſervations they had made when younger. Conſequently, their eyes being now ſomewhat flatted by age, might be thereby the better fitted for ſeeing diſtinctly at a great diſtance; and by this means the *radius of diſſipation* would be rendered ſtill leſs, and their obſervations would come nearer to the truth. At leaſt, that this was the caſe of *Hevelius*, I ſhall have occaſion anon to ſhew.

Appearrance of the New Moon.

53. In the new Moon of three or four days old, the illuminated part muſt appear too broad in proportion to the obſcure part, and likewiſe muſt ſeem to extend more outwards, or to have a greater diameter than the obſcure part.

For, in Fig. 20. let *CABDE* repreſent the *true image* of the whole Moon, *ABDIA* the *true image* of the enlightned part, *AIDEA* the *true image* of the obſcure part, *abd* the outer edge of the *falſe image* of the obſcure part, *FGH* the outer edge of the *annular penumbra* of the whole Moon, *LGMKL*, the whole appearance of the enlightned part conſiſting of the *true image* and the *penumbra* added to it.

54. Then this whole appearance of the illuminated part is manifeſtly too broad in proportion to the obſcure part, for as much as to the *true image* of the illuminated part is added not only the *annular penumbra LGM DBAL* on the outſide, whoſe breadth *BG* is equal to the *radius of diſſipation*, but alſo a like *penumbra* on the inſide of the ſame breadth, *LKMDIAL*; whereas the obſcure part is leſſened by the ſpace which this laſt *penumbra* takes up.

55. Likewiſe, as by Art. 19, 20, the light of the obſcure part diminiſhes outwards from the edge of the *falſe image abd*, it can hardly be viſible much more outwardly than that edge, eſpecially where it is contiguous to the ſtronger light of the cuſps. Conſequently, it muſt appear of a leſs diameter than the illuminated part, the ſtronger light of which is ſenſible farther outward.

56. For the ſame reaſon as the illuminated part of the New Moon appears too broad in proportion to the obſcure part; in an eclipſe of the Sun or Moon the bright part muſt appear too broad in proportion to the dark part, or the eclipſe muſt appear leſs than it really is. Conformable to this is the obſervation of our famous countryman, Mr [a] HORROX. *Nudi oculi defectum ſemper juſto minorem exhibent: at Teleſcopium veram exhibet tum defectus tum diametri lunaris quantitatem.*

That the effect we have been ſpeaking of, ariſes from the cauſe here aſſigned, and not from the principle that has ſometimes been made uſe of to account for ſuch like *phænomena*, viz. that a light object affects the *Retina* to a greater diſtance than a dark one, will plainly appear by the following experiment.

Let a repreſentation of the new Moon, Fig. 21, or the circle in Fig. 22, half black, half white, be viewed at a diſtance proper for *Perfect Viſion*, and theſe two figures will appear in their juſt proportion. Then let them be viewed at a diſtance too ſmall for *Perfect Viſion*, and the light part will appear to intrench upon the dark part, and likewiſe to extend outwards beyond the dark part. Afterwards let them be removed to a diſtance too great for *Perfect Viſion*, which is eaſily done by a ſhortſighted perſon, or by one longſighted, by applying a convex-glaſs to his eye, and the white part will again intrench upon the dark part, and will extend outwards beyond the dark part.

N. B.

[a] Venus in ſole viſa. Cap. XVI.

N. B. To make this experiment ſucceed clearly, the figure muſt not be bounded with a black line, as is here repreſented, but the whole circle muſt be cut in white paper, the dark part blackened with ink, or a black-lead pencil, and the paper laid upon a blacker ground.

Appearance of the leſſer Planets.

57. The other Planets, which by reaſon of their greater diſtance appear much ſmaller than the Moon, will appear fainter but much larger by indiſtinct, than by *Perfect Viſion:* and their diameters will appear enlarged in a much greater proportion than that of the Moon.

1. For inſtance, if the apparent diameter of *Jupiter*, at his mean diſtance from the earth, be about 38″ when ſeen by *Perfect Viſion*; and in ſeeing him indiſtinctly the *radius of diſſipation* be 2′, or 120″, as was before ſuppoſed; then by Art. 37, the circle *ABDC*, Fig. 17, which repreſents the *true image* of *Jupiter*, or the image of *Jupiter* upon the *Retina* when he is ſeen by *Perfect Viſion*, will be 38″ in diameter; and the diameter of the *faint falſe image*, *Cabd*, equally bright in all parts, will be 202″, that is, above five times the diameter of the image of *Jupiter* ſeen by *Perfect Viſion*. And alſo, around this *faint falſe image* of *Jupiter* there will be an *annular penumbra* repreſented by the *annulus abdHGF*, of 38″ in breadth, which *annulus* being added to the *faint falſe image* already mentioned, will give 278″, or 4′, 38″ for the breadth of the whole phænomenon, that is, above ſeven times the diameter of the image of *Jupiter* ſeen by *Perfect Viſion*. But of this ſomething muſt be abated, becauſe the *annular penumbra* cannot be ſenſible to its very extremity, by Art 39, the image of *Jupiter* being much ſmaller than that of the Moon.

2. If the apparent diameter of *Mars*, at his mean diſtance from the earth, be 6″ when ſeen by *Perfect Viſion*, the breadth of the *faint falſe image*, equally bright in all parts, will be 234″, that is 39 times the diameter of *Mars* ſeen by *Perfect Viſion*.

3. If the apparent diameter of *Venus*, at her mean diſtance from the earth be 18′, the breadth of the *faint falſe image*, equally bright in all parts, will be 222″, that is above 12 times the diameter of *Venus* ſeen by *Perfect Viſion*.

By this means therefore we may account for the *radii adventitii*, which Mr.[a] HORROX ſays impoſed ſo far upon all the aſtronomers before him, as to make them think the apparent diameters of the planets 9 or 10 times bigger than they really are.

58. When any of theſe leſſer planets is at the greateſt diſtance from the earth, and conſequently has the leaſt apparent diameter, ſeen by *Perfect Viſion*; its *faint falſe image* is the largeſt. For, by Art. 37, the diameter of the *faint falſe image* is $2\varrho - 2r$: But when the apparent diameter is the leaſt, the image of that diameter, or $2r$, is the leaſt, and 2ϱ is a conſtant quantity: Therefore $2\varrho - 2r$ is then greateſt, or the diameter of the *faint falſe image* is then greateſt.

59. When the apparent diameter of a planet ſeen by *Perfect Viſion* is greatly altered in magnitude between the leaſt and mean diſtance of the planet from the earth, the change in its apparent magnitude ſeen by indiſtinct viſion may be inconſiderable.

1. For inſtance, let us ſuppoſe the mean diſtance of *Jupiter* from the earth, to bear to his leaſt diſtance the proportion of 5 to 4. This will make a change in the apparent diameter of *Jupiter* ſeen by *Perfect Viſion*, from 38″ to 48″ nearly; and this addition of 10″ to 38″ is very conſiderable, being a quarter part. But the diameter of the *faint falſe image* of *Jupiter*, by Art. 37, at his mean diſtance is 202″, and at his leaſt diſtance is 192″, the difference being $\frac{1}{19}$ part only. And the diameters of the whole appearance of *Jupiter*, taking in the *annular penumbra*, are at thoſe diſtances, 278″, and 288″, reſpectively, the difference between which is only $\frac{1}{28}$ part.

2. If the mean diſtance of *Mars* from the earth bear to his leaſt diſtance the proportion of 3 to one, and the apparent diameter of *Mars* at his mean diſtance be 6″, his apparent diameter at his neareſt diſtance will be three times as much, or 18″. But the diameter of the *faint falſe image* of *Mars* at his mean diſtance will, by Art. 37, be 234″, and at the leaſt diſtance will be 222″, the difference between them being only 12″ in 222″, which is leſs than $\frac{1}{18}$ part.

Why Mars and Venus appear round, when gibbous or horned,

60. When *Mars by Perfect Viſion* would appear gibbous, and *Venus* by *Perfect Viſion* would appear gibbous, or dichotomiſed, or horned, the appearance of theſe planets by indiſtinct viſion will be much the ſame as if their whole diſks were illuminated.

For

[a] Venus in ſole viſa. Cap. XVI.

For inſtance, when *Venus* at her mean diſtance from the earth is dichotomiſed, her *faint falſe image* will be an oval tranverſe to the line of ditchotomy, whoſe ſhorteſt diameter will be $2\varrho - 2r$, that is 240″—18″, or 222″: And the longeſt diameter will be $2\varrho - 2r$, that is 240″—9″, or 231″, as may eaſily be collected from Art. 37. And when *Venus* is a creſcent ever ſo narrow, this longeſt diameter muſt be ſomething leſs than 240″. But the difference of 9″, or 18″, in 222″, is leſs than $\frac{1}{24}$, or $\frac{1}{12}$ part. And an oval, whoſe tranſverſe diameters are 11 and 12, can hardly be diſtinguiſhed from a perfect circle, eſpecially when very ſmall.

Appearance of the fixed Stars.

61. A fixed Star appears to the eye under ſuch an angle, as anſwers to the diameter of the *circle of diſſipation*.

For by Art. 37, the diameter of the *faint falſe image* of a ſtar is $2\varrho - 2r$.

But by the obſervations of the beſt aſtronomers, the apparent diameter of the brighteſt fixed ſtars is ſo exceedingly ſmall, that even in looking thro' the longeſt Teleſcope no eſtimate can be made of its magnitude, the ſtars appearing only as lucid points. Therefore, neglecting the apparent diameter of the ſtar, or the diameter of the *true image*, $2r$, the diameter of the *faint falſe image* which is equally bright in all parts, or $2\varrho - 2r$, Art. 37, will be very nearly 2ϱ.

62. The *faint falſe image* of a ſtar, or the appearance of a ſtar to the eye, is attended with no ſenſible *penumbra*.

For by Art. 37, the diameter of the whole appearance of a circular object in the eye, taking in the *annular penumbra* ſurrounding it, is $2\varrho + 2r$, and the breadth of the *annular penumbra* is $2r$. But by reaſon of the exceeding ſmallneſs of the apparent diameter of a ſtar, $2r$, the *annular penumbra* ſurrounding it will be utterly inſenſible, and the whole appearance in the eye will to our ſenſe be the ſame as that of the *faint falſe image*, whoſe diameter we have ſhown to be the ſame with that of the *circle of diſſipation*.

63. The diſtance between two ſtars will appear to the eye leſs than it really is, by twice the *radius of diſſipation*.

Let the two circles *cfgh*, *mnop*, Fig. 23. whoſe radii *cf*, *mn* are ſeverally equal to the *radius of diſſipation*, Art. 61, repreſent the images of two ſtars upon the *Retina*; let the line *fn* repreſent the diſtance between thoſe images, and the line *cm* the diſtance between their centers. Then it is plain, that the apparent diſtance *fn* will be leſs than *cm* the diſtance between the centers of the two ſtars, by *cf* and *mn* added together, that is by twice the *radius of diſſipation*.

From this it evidently follows, that when the diſtance between two ſtars is not more than twice the *radius of diſſipation*, the two ſtars muſt appear contiguous.

64. If the diſtance between two ſtars be leſs than twice the *radius of diſſipation*, the two ſtars will appear to the eye as one ſtar, brighter than either of the two taken alone.

For ſince each ſtar appears under ſuch an angle as anſwers to twice the *radius of diſſipation*, by Art. 61, if two ſtars are leſs aſunder than that angle, the two radii will meet in the middle, and part of the two *faint falſe images* will coincide, and where they coincide will be nearly twice as luminous as the reſt of the image, ſo that the two ſtars will have the ſame appearance as if one brighter ſtar appeared in the middle of the ſpace, which is taken up by the two ſtars.

65. After the ſame manner two ſmall circular objects ſeen very near, as two points in printing, may appear to run into one another, and compoſe a ſtronger image in the middle, when the *radius of diſſipation* much exceeds the diameter of the objects. As the two points *a*, *b*, in Fig. 24, when ſeen very near, put on the appearance *A*, Fig. 25, or *B*, Fig. 26.

A Star ſeen within the limb of the Moon.

66. A ſtar approaching very near to the edge of a planet may appear within the limb of the planet, juſt as if the planet were tranſparent and the ſtar were ſeen through it.

Let the circle *ABD* Fig. 27, repreſent the Moon, and let *c* be a fixed ſtar very near it, as within 8″ or 10″ of the edge of the Moon.

Then, I ſay, the ſtar will appear within the limb of the Moon, as in Fig. 28, where *c* repreſents the ſtar and *FGH* the limb of the Moon.

For in Fig. 29, let the circle *CABD* repreſent the *true image* of the Moon upon the *Retina*, and concentrick with this draw the circle *CFGH*, at the diſtance *AF* equal to the *radius of diſſipation*. Then by Art. 20, the

the circumference *FGH* will appear as the limb of the Moon. And if the point *c* be taken very near to the circumference *ABD*, and from this center, with the *radius of diſſipation cf*, be drawn the circle *cfgh*, that circle, by Art. 61, will repreſent the image of the ſtar upon the *Retina*. But that image will be almoſt wholly comprehended within the limb *GFH*. Therefore the ſtar muſt appear within that limb.

As for the circular ſegment *fg*, which is without the limb, it will be inſenſible to the eye, both becauſe of its ſmallneſs, and becauſe the limb of the Moon itſelf is not even.

But as the *faint falſe image* of the ſtar, when invironed with ſo ſtrong a light as that of the Moon, will for reaſons hereafter to be given, Art. 220, contract itſelf from the circle *cfgh* to a leſſer circle as *cmno*, the ſtar will appear not only within the limb of the Moon, but at a conſiderable diſtance within it.

67. This accounts for the Obſervations [a] SCHICKARD ſpeaks of in the following paſſage. *Cynthia enim, quando ſtellis appropinquat, cernitur advenientes amplecti & aliquantulum intra peripheriam perſpicuam admittere; ultrinſecus vero exeuntes viſui reddere prius quam pervenerint ad oram: quod* Maeſtlinus *exemplo* Martis, *item Cordis* Scorpionis *animadvertit Anno* 1595. Diſput. de paſs. Planet. Theſ. 158, *unde collegit, quodam diaphano velut aere ambiri: ſed hæc experientiæ ulteriori relinquo.*

68. After the ſame manner we may account for *Mars* or *Venus*, &c. appearing within the limb of the Moon.

69. The ſame appearance of a ſtar within the limb of the Moon may happen in viewing the Moon thro' a teleſcope, if the teleſcope be not good, or the eye of the obſerver have not the exact conformation neceſſary for *Perfect Viſion*.

For though the object appear ſufficiently diſtinct thro' the teleſcope, yet if there be not *Perfect Viſion*, which by Art. 8 may eaſily happen, there muſt be a *circle of diſſipation*, and the radius of the *faint falſe image* of the ſtar muſt not be greater than the *radius of diſſipation*, by Art. 61, and the ring between the *true image* of the Moon and its apparent limb muſt be equal to the ſame radius. Conſequently the image of a ſtar may appear within that ring, that is, within the apparent limb of the Moon, as well as if the ring and the *faint falſe image* had both been greater.

This may account for the Obſervation of [b] PERE FEUILLEE, and likewiſe for that of [c] MONS[r] DELAHIRE, who both ſaw a ſtar within the Moon's limb thro' a teleſcope, and the laſt named Gentleman has touched upon the true cauſe of that *phænomenon*, as Mr. HORROX, and more eſpecially the incomparable GALILEO had done before him.

70. If a rectangular object be viewed at a diſtance proper for *Perfect Viſion*; its picture upon the *Retina* will be rectangular, (I mean as far as the ſpherical figure of the *Retina* will admit) and will be proportional to the angle which the object ſubtends at the eye; its limb will be well defined; and all parts of the rectangular picture will be equally ſtrong. Conſequently, the idea thereby excited in the mind will be that of a rectangle equally ſtrong in all parts, and well defined.

Phænomena of indiſtinct Viſion in rectangular objects.

71. If the ſame rectangular object be viewed at a diſtance much too ſmall, or much too great for *Perfect Viſion*; its picture upon the *Retina* will ſtill be rectangular: but the length and breadth will be greater than in proportion to the angles ſubtended at the eye by the length and breadth of the object. Nor will the picture be equally ſtrong in all its parts, but the middle part will be the ſtrongeſt, and will be ſurrounded with a *penumbra* growing gradually fainter towards the outſide, whereby the limb will appear indiſtinct and ill-defined. Conſequently, the idea thereby excited in the mind will be that of a rectangle too large, and too faint and indiſtinct towards the limb. That is, inſtead of the appearance *A*, Fig. 30, it will have the appearance *B*, or *C*, or *D*.

For let the rectangle *ABDC*, Fig. 31, repreſent that rectangular ſpace upon the *Retina*, which the image of the object would take up if that image were diſtinct: Or, which comes to the ſame thing, let *ABDC* repreſent that rectangular ſpace upon the *Retina*, which is occupied by the centers of all the pencils of rays belonging to the indiſtinct image of the rectangular object. This rectangle *ABDC* we ſhall call the *true image* of the object. Alſo, let the circle *fghe*, having its center *e* in the perimeter of the rectangle *ABCD*, repreſent that circular ſpace upon the *Retina*,

[a] Reſponſ. ad Gaſſendum. [b] Hiſt. de l'Academie Royale des Sciences. 1699. [c] Ibid. Item Art. 1088 of this Book.

Retina, which is taken up by one of the extreme pencils of rays iſſuing from the object, that is, let the circle *fghe* be the *circle of diſſipation*, and its radius *ef* the *radius of diſſipation*. Then draw the line *bd* parallel to *BD*, touching the *circle of diſſipation fghe* in the point *f* within the rectangle, and compleat the rectangle *abdc* having its ſides parallel to thoſe of the rectangle *ABDC*, and every where diſtant from them by the *radius of diſſipation ef*.

72. Then I ſay, 1. That part of the rectangular object, which is repreſented by the rectangle *abdc*, will be equally ſtrong in all parts, and will be of the ſame ſtrength, as if the image of the object had been perfectly diſtinct.

To prove this, let the rectangles *ABDC*, *abdc*, Fig. 32, repreſent the ſame things as before, and taking any point at pleaſure, as *e*, within the rectangle *abdc*, from that point as a center, with the *radius of diſſipation ef*, draw the *circle of diſſipation efgh*.

Then it is plain, that the point *e* muſt receive light from every point of the circle *efgh*, into which it ſcatters its own light, and conſequently muſt receive back the ſame quantity of light as is loſes by diſſipation. It muſt therefore be as ſtrongly illuminated, as if there had been no diſſipation of light, but the image had been perfectly diſtinct. And as this is true of every point in the rectangle *abdc*, that whole rectangle muſt be as ſtrongly illuminated as if the image had been perfectly diſtinct, and muſt be equally ſtrong in all parts. This rectangle, *abdc*, we call the *falſe image*.

73. I ſay, 2. The rectangular area, or border *ABDCcabd*, comprehended between the perimeters of theſe two rectangles, will not be ſo ſtrongly illuminated as the *falſe image abdc*, and will grow gradually weaker towards its outer extremity.

For let the rectangles *ABDC*, *abdc*. Fig. 33, repreſent the ſame things as before, and and taking two points in the rectangular area *ABDCcabd*, one more inward as *e*, and the other more outward as *m*, from the centers *e* and *m*, with the radii *ef*, *mn*, each equal to the *radius of diſſipation*, draw the two circles *cfgh*, *mnop*, cutting the lines *AC*, *BD*, in the points *h* and *f*, *n* and *o* reſpectively.

Then it is manifeſt, that the points *e* and *m*, will receive light not from the whole circles of which they are the centers, but only from the ſegments *hgf*, *npo* reſpectively. Therefore each of theſe points will be leſs luminous than any point of the rectangle *abdc*. But alſo the ſegment *hgf*, which throws light upon the more inward point *e*, is larger than the ſegment *npo* which throws light upon the more outward point *m*. Therefore the more inward point *e* will be more ſtrongly illuminated rhan the more outward point *m*, or the rectangular area *ABDCcabd* will decreaſe gradually in light towards the outer extremity.

74. I ſay, 3. Beſides the rectangular area laſt deſcribed, which is darker than the *falſe image abdc*, there will appear another rectangular area, ſituated without the *true image*, or rectangle *ABDC*, which is darker than the former, and whoſe light does alſo diminiſh gradually towards the outſide, till it wholly diſappear and vaniſh.

For let the rectangle *ABDC*, Fig. 34, again repreſent the *true image*, the rectangle *abdc* the *falſe image*, and *efgh* the *circle of diſſipation*, having its center *e* in *BD* one of the ſides of the *true image*. Then parallel to *BD* draw the line *GF* touching the circle *efgh* in its outer point *f*, and compleat the rectangle *GFHI* diſtant every where from the rectangle *ABDC* by the *radius of diſſipation ef*.

Then between the two perimeters *ABDC* and *GFHI*, will be comprehended a new rectangular area, which will receive light from the pencils whoſe centers are ſituated in the former rectangular area *ABDCcabd*; but it will be darker than the former, and its light will gradually diminiſh towards the outer edge till it entirely diſappears and vaniſhes. This is manifeſt from the inſpection of the figure, where the light received by the point *e* ſituated in the inner edge of this area is meaſured by the ſegment or ſemicircle *hg*, and the light received by the point *m* ſituated near the outer edge is meaſured by the ſmaller ſegment *no*.

75. From what has been ſaid, Art. 71, 72, 73, 74. and from conſideration of Fig. 31, 32, 33, 34, it is plain that the rectangle *abdc*, or the *falſe image*, is the only part that has its full quantity of light, and that the two rectangular area's *abdc CABD*, and *ABDCHIGF*, do both decreaſe in light juſt after the ſame manner

manner from *abdc* to *IGFH*, for which reaſon, and becauſe we ſhall have no occaſion afterwards to conſider theſe two rectangular area's ſeparately from one another, we ſhall here look upon them as one area *abdcHIGF*, which we ſhall call by the name of the *rectangular penumbra*.

76. But it may not be amiſs to obſerve, that this *penumbra*. which we call rectangular, is not ſtrictly ſo, nor exactly as we have repreſented it, but cut off at the corners nearly in the following manner.

Let *cab* repreſent one angle of the *falſe image*, *CAB* one angle of the *true image*, and *HIG* one angle of the *rectangular penumbra*; and from the center *A* with the *radius of diſſipation Ag* draw the *circle of diſſipation Agfeh*, touching the lines *IG*, *IH*, in the points *g* and *h* reſpectively; and produce the line *ca* till it interſect *GI* in the point *l*, and produce the line *ba* till it interſect *HI* in the point *m*. Then it is manifeſt from the conſtruction, that theſe two lines *cl*, *bm* muſt touch the circle *ehgf* in the points *f* and *e* reſpectively.

Now it is manifeſt, that the point *I* being more remote from any even the neareſt point of the *true image*, as *A*, than by the *radius of diſſipation*, can receive no light from that image. And this is equally true of every point in the ſpace *gIh*, comprehended between the two tangents *gI*, *hI*, and the arc *gh*. Conſequently the *rectangular penumbra* does not extend to any part of that ſpace.

Farther, the point *f* receives light from a ſemicircle of the *true image*, as is plain from this Figure compared with Fig. 34: But the point *A* receives light only from a quadrant of the *true image*. Therefore the light of the *penumbra* from *f* to *A* muſt be weaker than from *f* towards *B*, and muſt gradually decreaſe all the way from *f* to *A*.

It is eaſy to ſee likewiſe, that the outer edge of the *penumbra*, which is every where equally luminous in the line *Gl*, does begin to decline about the point *l*, and decreaſes gradually from *l* to *g*, where it vaniſhes.

And alſo, it is plain from Art. 73, that the light, which is equally ſtrong along the edge of the *falſe image* from *b* to *a*, begins to decline at the point *a*, and grows gradually weaker from *a* to *e*.

It is therefore manifeſt, that in every parallel to the line *AB*, the *penumbra* grows weaker without the line *lfa*, or towards *b*, than within that line, or towards *B*.

In like manner it will be found, that in every parallel to the line *AC*, the *penumbra* is weaker without the line *mea*, or towards *g*, than within that line, or towards *C*.

Therefore, the lines *lfa*, *mea*, may be looked upon as the bounds of the *penumbra*, ſince that part of it which appears within the angle *lam*, is conſiderably weaker than what appears within the angles *lab*, and *mac*.

77. Hence, the *falſe image*, with the *penumbra* about it, muſt put on the appearance repreſented in Fig. 36, where the ſame letters denote the ſame things as before. And this in fact is nearly the caſe, when the corner of a diſtant high wall, or ſquare tower, is looked upon by a ſhort-ſighted perſon, or by one long-ſighted uſing a pair of ſpectacles, or a convex glaſs, againſt the ſky-light.

78. For much the ſame reaſons, a pyramid or ſpire of a church, will, under the ſame circumſtances, appear, as in figure 37, after the manner of three different ſpires one ſtronger and lower as *cab*, the other two fainter and more elevated as *Hmb*, *Glc*, as in fact will nearly be found by thoſe who make the tryal.

I ſay nearly, becauſe this appearance, as well as that of the preceding article, will be a little varied from a cauſe hereafter to be aſſigned. See Art. 199, 200, *&c.*

79. If the *radius of diſſipation* increaſe, the *falſe image abdc*, will decreaſe both in length and breadth; but the breadth will decreaſe more in proportion than the length; for as much as the equal radii *ef*, *ef*, are taken both out of the breadth and length, as in Fig. 38.

80. If the *radius of diſſipation* be equal to half the breadth of the *true image*, the *falſe image* will degenerate into the right line *ac*, Fig. 39.

81. If the *radius of diſſipation* exceed half the breadth of the *true image*, there will be no *faint falſe image* equally luminous in all parts, as in the caſe of the circular object, Art. 24, but the whole image of the rectangle will be one *penumbra*, ſtrongeſt in the middle, and decreaſing gradually in light on both ſides towards the outer edge.

For in Fig. 40, the point *e* ſituated in the middle of the breadth of the rectangle, receives light from every point in its *circle of diſſipation*, except the two ſegments *fg*, and *hi*.

The meaſure of its light therefore is the ſpace *fghi*.

The meaſure of the light thrown upon the point *k*, ſituated more towards the edge than the point *e*, is the ſpace *lmon*, leſs than *fghi*, as is eaſy to perceive.

The meaſure of the light thrown upon the point *p*, ſtill more outwardly ſituated than the point *k*, is the ſpace, or ſegment *rqs*, leſs than *lmon*.

And the meaſure of the light thrown upon the point *t*, ſituated in the very edge of the *true image*, is the ſegment or ſemicircle *vw*, ſtill leſs than *rqs*.

Therefore the light of the whole appearance will decreaſe from the middle both ways breadthwiſe.

Towards the ends of the *true image* *AB* and *CD*, the *penumbra* will decreaſe lengthwiſe alſo. For the point *x*, in the line *AC*, whoſe diſtance from *A* is *xA* equal to the *radius of diſſipation*, will be illuminated by the ſemicircle *Axy*; but the points between *x* and *A*, will be illuminated by leſs than a ſemicircle, and the point *A* will be illuminated only by a quadrant, and any point of the *penumbra* beyond *A* in the line *xA* produced, will be illuminated by leſs than a quadrant. Therefore the ends of the *penumbra* about *AB* and *CD* will be weaker than the other parts.

82. If the *radius of diſſipation* exceed the breadth of the *true image*, the *penumbra* will be fainter in proportion to that exceſs, but ſtill the middle will be the ſtrongeſt, and the light will decreaſe from the middle towards the outer extremity, as is manifeſt from the conſideration of Fig. 41.

83. When the *radius of diſſipation* is very large in proportion to the breadth of the *true image*, the *penumbra* will be very faint, but will decreaſe very ſlowly and almoſt inſenſibly, except very near the outer edge.

For in Fig. 41, let us imagine the two ſides of the *true image*, *AC* and *BD*, to approach very near together, and then it is plain, that the ſeveral portions of equal circles, *fgih*, *lmon*, *qrst*, and *yx*, whereby the points *e*, *k*, *p*, and *x* are reſpectively illuminated, will decreaſe very ſlowly in reſpect to the diſtances of the points *k*, *p* and *x* from the middle of the image.

84. Very near the edge the *penumbra* will decreaſe very faſt.

For in Fig. 42, the light thrown upon the point *e* very near the edge of the *penumbra* will be meaſured by the ſegment *fg*, and the light thrown upon the point *h* a little nearer that edge, will be meaſured by the ſegment *ik*. But the ſegment *ik* is much leſs than the ſegment *fg*.

85. That part of the *penumbra* near its edge, where the light decreaſes very faſt, is nearly equal to the breadth of the *true image*. This eaſily appears from the conſideration of Fig. 42. For if the point *e* were diſtant from the line *GF* by juſt the length of *CD*, its *circle of diſſipation* would touch the line *AC*; and by removing the point *e* more outwards, its ſegment of light *fg* would decreaſe very faſt.

86. When the *radius of diſſipation* is vaſtly great in proportion to the breadth of the *true image*, that part near the edge where the *penumbra* decreaſes very faſt, will be inſenſible, and the whole *penumbra* will decreaſe inſenſibly in light from the middle towards the outer edge.

87. A narrow line, as a ſtroke in this print, when ſeen very near, will appear very broad and faint, and almoſt equally faint throughout the whole breadth. This follows from Art. 83, 84, 85, 86.

88. Two narrow parallel lines drawn near to each other, when viewed very near, may put on the appearance of one line, with a *penumbra* on each ſide of it. The two lines *A*, Fig. 43, may, by viewing them very near, appear as *B*. Two parallel lines appearing as one line.

For, by Art. 87, each of theſe lines muſt appear as one broad faint line, and being near together their *penumbræ* will meet in the middle, and the part where they meet muſt appear nearly of a double ſtrength

89. Two narrow lines meeting in a ſmall angle, when ſeen very near, may form the appearance of an inverted wedge ſurrounded with a *penumbra*.

The two lines *C* in Fig. 44, may appear as *D*, when viewed very near.

This is eaſily proved in the ſame manner with Art. 88.

90. A circle upon white paper, bounded by a narrow black line as its circumference, when viewed very near, will appear leſs than if it were ſeen diſtinctly: But the narrow line that encompaſſes it, will appear broad and faint, and will be ſtronger on the inner Circular lines ſeen by indiſtinct Viſion.

or

or concave ſide, than on the outer or convex ſide; and the whole breadth of this faint appearance, or *penumbra*, will be equal to twice the *radius of diſſipation*, added to the breadth of the circular line. That is, inſtead of the appearance *A*, Fig. 45, we ſhall have the appearance *B*.

For, let *AB* in Fig. 46, repreſent a part of the circular narrow line bounding the circle deſcribed with the radius *EA*: And let *ehgi* be a *ſemicircle of diſſipation*, having its center *e* in the outer edge of the narrow circular line *AB*, and cutting the radius *Ee* produced in the point *g*. Alſo let *ehfi* be another *ſemicircle of diſſipation* having its center *e* on the inner edge of the ſame circular line *AB*, and cutting the radius *Ee* in the point *f*.

Then it is manifeſt, that the faint appearance, or *penumbra*, of the circular line *AB* muſt extend inwards to the point *f*, and outwards to the point *g*: Or, the whole breadth of the *penumbra* muſt be equal to the two *radii of diſſipation ef* and *eg* added to the breadth of the circular line *AB*.

Conſequently, the radius of the white circle, or *Ee* will in this appearance be diminiſhed to *Ef*, the part *fe* being taken up by the *penumbra*.

91. I ſay further, this *penumbra* will be ſtronger on its inner edge than on the outſide.

For in Fig. 47, let the narrow circular line be repreſented by the ring comprehended between the two circular lines *AB* and *CD*, which ring is here made very broad to render the parts we are going to ſpeak of the more diſcernible: And from the center *E* draw the radius *Eiq*, cutting the inner and outer edge of this ring in the points *i* and *q* reſpectively. Then ſetting off in this radius the two equal diſtances *ie* and *qm*, each leſs than the *radius of diſſipation*; from the centers *e* and *m*, with the *radius of diſſipation* draw the arcs *fhg*, *npo*, cutting the circular lines *CD*, *AB*, in the points *f* and *g*, *n* and *o*, reſpectively.

Then will the circular ſegment *figh* repreſent the quantity of ſhade thrown from the ring upon the inner point *e*.

And the circular ſegment *nqop* will repreſent the quantity of ſhade thrown from the ring upon the outer point *m*.

And if thro' the point *i* be drawn the right line *rs* perpendicular to the radius *Ei*, and thro' the point *q* be drawn the right line *vt* perpendicular to the ſame radius: the ſpace intercepted between theſe lines *rs*, *vt* may be conſidered as a rectangular object of the ſame breadth as the circular object, or ring, *ABDC*.

But by the inſpection of the figure the circular ſegment *figh* is greater than what would be cut off from the rectangular object by the *circle of diſſipation*, whoſe center is *e*, and whoſe radius is *eh*.

And the circular ſegment *nqop* is leſs than what would be cut off from the rectangular object, by the *circle of diſſipation*, whoſe center is *m*, and whoſe radius is *mp*.

Therefore the ſhade thrown upon the point *e* from the circular ſegment *figh*, is greater than what is thrown upon the point *m* from the circular ſegment *nqop*: that is, the inner point *e* will appear darker than the outer point *m*, or the *penumbra* will be ſtronger on the inſide than on the outſide.

A dark ſpot in the center.

92. Since the breadth of the *penumbra* on the inſide is equal to the *radius of diſſipation*, it is manifeſt that the more this radius increaſes, the more will the radius *Ef*, Fig. 46, of the white circle be diminiſhed: And when the *radius of diſſipation* becomes equal to the radius *Ee*, the white circle will intirely diſappear, and in the room of it a black ſpot will appear in the center *E*, ſurrounded with a *penumbra* whoſe radius is double to *Ee*, the radius of the circular line. And this is found true in fact, when ſuch a circular line is either much too near, or much too remote to be ſeen diſtinctly: For then the object *A* appears as *B*, *E* as *C*, and *I* as *D*, Fig. 48: but to make the experiment rightly, the radius of the circular line muſt be proportioned to the diſtance.

To meaſure the radius of diſſipation

93. Hence we have a good method of finding by experiment the *radius of diſſipation* at any diſtance without the limits of *Perfect Viſion*.

Upon white paper draw the circumference of a circle with a ſtrong black line, and place the paper by gueſs about the fartheſt diſtance at which your eye can ſee an object diſtinctly. Then retiring gradually farther from the paper, obſerve at what diſtance the white circle in *B*, Fig. 48, appears equal in breadth to the *penumbra* on either ſide of it. At that diſtance the *radius of diſſipation* is nearly equal to half the radius of the *true image* of the circle.

Again, retire ſtill farther from the paper,

till the white circle vaniſhes and the central black ſpot juſt begins to appear. Then the *radius of diſſipation* muſt be preciſely equal to the radius of the *true image* of the circle.

But the radius of the *true image* of the circle at any diſtance is eaſily found by Art. 380, of this Book of Opticks, and conſequently the magnitude of the *radius of diſſipation* at the ſame diſtance may be determined.

94. When the *radius of diſſipation* exceeds the radius of the *true image*, there will appear in the middle a circular ſpot equally ſtrong in all parts, ſurrounded with a *penumbra* fainter than the ſpot, ſo that this ſpot will be a *faint falſe image* of the circular line, like the *faint falſe image* of the circular object in Art. 23. And the breadth of this circular ſpot, or *faint falſe image*, will be equal to the difference between the *radius of diſſipation* and the radius of the *true image*.

This is eaſily ſhown after the ſame manner as we proved the 23d Article by Fig. 11, where the circumference *ABD* repreſents the *true image* of the circular line, whoſe radius is *r*; *cbfgh* the *circle of diſſipation*, whoſe radius is ϱ; the circle *Cabd* repreſents the central ſpot, whoſe radius is $\varrho - r$; the circle *CHGF* repreſents the whole appearance whoſe radius is $r + \varrho$, and the ring *abdFHG* repreſents the *penumbra* round the central ſpot, whoſe breadth is $2r$.

Another way to meaſure the ſame radius.

95. When the *radius of diſſipation* is equal to the diameter of the *true image* of the circular line, the central ſpot will then be equal in breadth to the *true image*, and equal in breadth to the *penumbra* encompaſſing the central point.

For $\varrho - r$ the radius of the ſpot will now be equal to $2r - r = r$, and its diameter will be $2r$ equal to the breadth of the *penumbra*.

Hence we have another way of finding the *radius of diſſipation*, which muſt be equal to twice the radius of the *true image*, when the *faint falſe image*, or central ſpot appears equal in breadth with the *penumbra*.

A purple ſpot in the center.

96. The central ſpot we have been ſpeaking of, does ſometimes appear of a purple colour, inſtead of black.

This, I think, happens for the following reaſon. At the time of this appearance, the *radius of diſſipation* exceeds the radius of the *true image* by a little more than the breadth of the circular line, as in Fig. 49, where *ABD* repreſents the circular line, and *Cfgh* repreſents the *circle of diſſipation*, which takes in a narrow ring of white paper beyond the circular line.

This ring of white will therefore throw a portion of its light upon the central ſpot, which portion of light being the fartheſt diſſipated from the centers of the pencils in the white ring, muſt conſiſt of the leaſt refrangible rays, or the red rays. But this red, mixingwith the bluiſh black of the central ſpot, muſt exhibit the appearance of purple.

97. When this central ſpot begins to appear, if the eye continue attentively fixed upon it, or if the ſun break out on a ſudden from a cloudy ſky, or if at night a candle be ſnuffed ſo as to burn more brightly, in any of theſe caſes the ſpot will diſappear, and a ſmall white circle will appear in the room of it.

For in all theſe caſes the pupil contracts to a narrower apperture, and the *radius of diſſipation*, which is always proportional to that aperture, is thereby leſſened, ſo that this radius, which at firſt was equal to the radius of the *true image*, now falls ſhort of it, and conſequently there muſt appear no ſpot, but only a ſmall white circle, by Art. 92.

Ovals indiſtinctly ſeen.

98. The circumference of an oval will exhibit much the ſame *phænomena*, as the circumference of a circle, and for the ſame reaſon. Only, the central ſpot will now be oval, and when there is no central ſpot, the *penumbra* will be ſtronger in the more concave part of the oval than in the leſs concave part.

The oval *A* will appear as *B*, or *C*, Fig. 50.

Letters of a book ſeen by indiſtinct Viſion.

99. From the preceding articles it is eaſy to ſee the reaſon, why a book of a ſmall print held very near the eye appears quite confuſed. For, as the letters therein conſiſt either of lines parallel to one another, as *m*, *n*, *il*, *it*, &c. or of lines inclined to each other, as v, w; or of circular, or oval, or partly oval lines, as o, *o*, *c*, *e*; or of a mixture of ſome of theſe, as *b*, *d*, *p*, *q*; it is plain from what has been above delivered, that when viewed very near, they will not only form large *penumbræ*, ſo as to render them ill defined, but will exhibit the appearance of foreign lines formed by the union of theſe *penumbræ* between the parallel or inclined lines of the letters, as alſo central ſpots in the circular, or oval, or mixed letters, ſo as to render

render the true ſhape of the letters utterly confuſed and indiſcernible.

100. But a print conſiderably larger than that of the preceding article, when viewed at the ſame diſtance, will not be attended with the appearance of theſe foreign lines or ſpots, becauſe the *penumbræ* will not now meet in the middle, the ſpace being too great between them, and conſequently the letters will appear only ill defined, and ſomewhat indiſtinct; but not confuſed.

101. If the ſmall print be viewed at a diſtance ſomewhat greater than in Art. 99, it will now appear only ill defined and ſomewhat indiſtinct on account of the *penumbræ*: But it will not appear quite confuſed, ſince it is not now attended with any foreign lines or ſpots, the pencils not ſpreading far enough for the *penumbræ* to coincide from oppoſite ſides and thereby to form thoſe lines or ſpots.

102. If the larger print be viewed from the ſame diſtance with the ſmaller in the preceding Article, it will appear leſs indiſtinct than the ſmaller, for two reaſons.

1. The lines being thicker will make a ſtronger appearance in the middle, by Art. 71, and this will in ſome meaſure efface the *penumbra*, eſpecially towards the edge, where it is weakeſt, ſo that the whole *penumbra* will appear both fainter and narrower, and conſequently will take up leſs of the void interval between the ſtrokes of the letters than in the ſmaller print.

2. That interval is in itſelf larger than in the ſmaller print, and conſequently appears more conſpicuous, and thereby ſhews the ſtrokes of the letters more ſeparate and diſtinct from each other.

103. If the two prints be viewed at a diſtance ſtill greater, the *penumbræ* will grow leſs, ſo that the ſmaller print will appear but little indiſtinct, and the greater not at all ſo, for much the ſame reaſons as were given in the preceding Articles.

And in this caſe, as the eye perceives no indiſtinctneſs in the larger print, this ſort of Viſion, although, on account that the rays of a pencil are not accurately collected into a point upon the *Retina*, we call it imperfectly diſtinct, cannot by the ſenſe be diſtinguiſhed from *Perfect Viſion*.

104. We have hitherto conſidered *Perfect* and *Diſtinct Viſion* only upon the foot of a given diſpoſition of the eye, in which caſe it has been ſhown, that *Perfect Viſion* depends upon the diſtance of the object ſolely, and that *Diſtinct Viſion* depends upon the diſtance and magnitude of the object jointly.

It remains to conſider how either of theſe ſorts of Viſion may be procured by a change in the diſpoſition of the eye, and here ſome very curious points offer themſelves to be diſcuſſed.

I. Whether *Perfect Viſion* in a given eye is confined to one determinate invariable diſtance, or may be obtained at different diſtances by a change in the conformation of the given eye.

II. Within what limits theſe different diſtances are comprehended, or what is the greateſt and leaſt diſtance at which *Perfect Viſion* can be obtained in a given eye, by changing its conformation.

III. What is that change, which is made in the conformation of the eye, in order to obtain *Perfect Viſion* at different diſtances.

IV. By what means Viſion, when it cannot be *Perfect*, is rendred *Diſtinct*, or at leaſt not ſo indiſtinct as it would otherwiſe be.

V. What change is made in the eye, either by habit and cuſtom, or by our growing into years.

VI. What is the leaſt object, or angle, that the eye is capable of perceiving.

Of all theſe I intend to treat in their order.

I.

Whether the eye changes its conformation for Perfect Viſion at different diſtances.

105. In the firſt of theſe points all Authors, that I know of, agree together, except only the famous Monſ^r^ De la Hire. He is of opinion,* that there is no accurate collection of the rays of a pencil into a point upon the *Retina*, that is, no *Perfect Viſion*, except when the object is at ſome one determinate diſtance ſuited to the eye of the obſerver. The others maintain, that the diſtance of the object may vary, and yet the rays of a pencil may be accurately united upon the *Retina*. The experiment upon which Monſ^r^ De la Hire founded his opinion, has been briefly related in the *Remarks*, Art. 15, *&c.* and the inſufficieney of his argument drawn from that experiment has been fully ſhown. The ſame thing has likewiſe been done by Dr Porterfield in the *Medical Eſſays* of *Edinburgh*,

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be exa-
ined.

* Traitè des differens accidens de la Veüe.

Edinburgh, Vol. 4, where this learned and judicious Gentleman by a very well contrived experiment has plainly demonſtrated the truth of the common opinion, that the eye has a power of altering its conformation ſo as to ſee diſtinctly at different diſtances.

I had myſelf, at leiſure times for ſeveral years paſt, made a great number of tryals, in looking thro' two and ſometimes more pin-holes, ſometimes thro' two narrow ſlits made near to each other in a card, ſometimes by a pin held directly before the eye, which anſwers the ſame purpoſe as the narrow interval between the pin-holes, or between the ſlits in a card. And the objects I looked at, were ſometimes round, as a pin-hole in a card ſet againſt a candle; ſometimes long, as a narrow ſlit in a card ſet againſt a wainſcot ſtrongly illuminated by a candle hid from the eye, or a ſtrong black line upon white paper ſet againſt a window. And by thoſe tryals I found, that the neareſt diſtance at which with one eye I could ſee any of theſe objects ſingle thro' the two pin-holes, or thro' the two ſlits, or on each ſide of the pin, was 40 inches; but that I could ſometimes ſee them diſtinct at much greater diſtances, as 50, 60, *&c.* to 90 inches or more. But as my eyes are now conſiderably decayed with regard to their power of ſeeing diſtinctly at ſmall diſtances, and as Dr PORTERFIELD's experiments ſeem to be better contrived, and were more methodically made than mine, I fully acquieſce in the proof he has given, that the eye has a power of altering its conformation ſo as to ſee objects perfectly diſtinct at different diſtances.

II.

Limits of Perfect Viſion.

106. We paſs therefore to the ſecond point, to enquire within what limits theſe different diſtances are comprehended, or what is the leaſt and the greateſt diſtance at which *Perfect Viſion* can be obtained.

The firſt of theſe limits, or the leaſt diſtance at which we can ſee with *Perfect Viſion*, is eaſily ſettled. For beſides Dr PORTERFIELD's experiment, who determines it to be about 7 inches to his own eye, we have the concurrent experience of the generality of perſons who are at their full growth, and whoſe eyes are no way impaired by age; for as much as in looking at minute objects, as the diviſions upon a *Gunter's* line, or thoſe of a diagonal ſcale for half an inch, or in examining the fineneſs of linnen and cambrick, or chuſing lace, *&c.* they hold the objects at 5, 6, or 7 inches from the eye. Whence it may reaſonably be preſumed, that the neareſt diſtance for *Perfect Viſion* is commonly 5, 6 or 7 inches. And this I ſhall afterwards prove by another method.

107. But as for the other of theſe limits, or the greateſt diſtance at which *Perfect Viſion* can be obtained, it is a matter of ſomewhat more difficulty.

This greateſt diſtance is by Dr PORTERFIELD determined for his own eye to be 27 inches. But that to the generality of eyes this diſtance is much greater, may reaſonably be preſumed from the diſtinctneſs which we ſee a ſmall miſling rain, when walking in a piazza, or coming out of a church and within not leſs than ſix or eight feet from the door; or the ſmall filaments of ſilk upon which ſpiders tranſport themſelves thro' the air, at a greater diſtance; or the ſtring of a boy's kite at a great height in the air. And to find what this diſtance is for a given eye, it may be proper to determine the *radius of diſſipation* at ſome great diſtance by Art. 93, or 95, or rather by Art. 63, or 64, and from thence to compute at what leſſer diſtance the *radius of diſſipation* muſt vaniſh, that leſſer diſtance being the utmoſt limit of *Perfect Viſion*. But in order to do this it will be neceſſary to conſider the meaſures of ſome of the parts of the eye.

108. In theſe meaſures I ſholl generally follow the famous Monſ. * PETIT, he having beſtowed more pains upon the examination of them than any other Author I know of; but ſhall reduce them from the *French* lines in which he has given them, to the tenths of the *London* inch and decimals of thoſe tenths, following herein the proportion eſtabliſhed by my ingenious Friend, Mr. GEORGE GRAHAM, between the *London* yard and the half toiſe of *Paris*, in ſome very accurate meaſures that have been mutually exchanged between the *Royal Society* and the *Royal Academy of Sciences* at *Paris*, namely that of 36 to 38, 355.

	tenths.
The radius of the convexity of the *cornea* is commonly	3, 3294

* Memoires de l' Acad. Royale 1728, 1730.

The

	tenths.
The radius of the anterior convexity of the cryſtalline at a medium from 26 eyes is	3, 3081.
The radius of the poſterior convexity of the cryſtalline at a medium from all the ſame eyes is	2, 5056.
The axis or greateſt thickneſs of the cryſtalline from all the ſame eyes at a medium is	1, 8525.
The joint axis of the *cornea* and the aqueous humour is commonly	1, 0358.

109. The refraction of the aqueous humour, or the proportion between the ſines of incidence and refraction, we here ſuppoſe to be the ſame with that out of air into water, namely that of 4 to 3.

110. The refraction of the *cornea* we ſuppoſe to be the ſame with that of the aqueous humour, ſo that the incident rays ſuffer no new refraction in paſſing from the *cornea* into the aqueous humour.

111. The proportion between the ſines of incidence and refraction, upon paſſing out of the aqueous humour into the cryſtalline, we take to be that of 13 to 12, upon paſſing out of the cryſtalline into the vitreous humour, that of 12 to 13.

Mr. *Hawkſbee* * from an oxe's cryſtalline make this proportion a little more; Dr *Pemberton* † ſeems to think it a little leſs, but ſays his experiment was not accurately made.

112. By computing upon theſe meaſures and refractions it will be found by Art. 369 of this Book of Opticks, that ſuch an eye will collect parallel rays into a point at the diſtance AM from the outer ſurface of the *cornea* of 8,9993 tenths, that fM is 23,9562, AL=5,3732, IL=2,0559, and the rectangle $IL \times fM$=49,2526.

113. Hence, if AP, the greateſt diſtance at which the rays of an object can by the eye be collected into a point, be 27 inches, or 270 tenths, MS will by Art. 370 of the Book of Opticks be found equal to 0,1861; and by ſo much muſt the *Retina* be ſituated behind the point M in order to have *Perfect Viſion* at the diſtance of 27 inches; and at all greater diſtances than 27 inches the pencils cannot be collected into ſingle points, but muſt each of them occupy a circular ſpace upon the *Retina*

114. Let us now examine what ſpace the image of a lucid point ſuppoſed at an infinite diſtance, as a fixed ſtar, will upon this ſuppoſition occupy upon the *Retina*, and what angle the ſtar muſt appear to ſubtend.

By Art. 386 of the Book of Opticks, when PL is infinite, $Xx = \frac{AB \times MX}{Z}$ and by Art. 380. of the ſame Book, $\frac{Pp}{Pl} = \frac{Xx}{V} \times \frac{nM}{nX}$. Therefore, ſuppoſing this Xx equal to the other Xx, $\frac{Pp}{Pl} = \frac{AB \times MX}{ZV} \times \frac{nM}{nX}$. And by Art. 388, $Z \times V = LI \times fM$. Hence $\frac{Pp}{Pl} = \frac{AB \times MX}{LI \times fM} \times \frac{nM}{nX}$. And if we neglect the *ratio* of $\frac{nM}{nX}$, $\frac{Pp}{Pl}$ is nearly equal to $\frac{AB \times MX}{LI \times fM}$.

Now, if a ſtar be conſidered as a lucid point, and MX be the diſtance of the *Retina* from the point M, the ſemidiameter of the image of the ſtar upon the *Retina* will be Xx. And if Pp be the diameter of an object ſeen at a very great diſtance, Pl, whoſe image upon the *Retina* is equal to that of the ſtar, it is plain that this object and the ſtar muſt appear of the ſame magnitude, and to ſubtend the ſame angle. But the half angle which the object ſubtends at the eye, or $\frac{Pp}{Pl}$ is meaſured by $\frac{AB \times MX}{LI \times fM}$: therefore this quantity is likewiſe the meaſure of the half angle under which the ſtar appears. Hence, if AB, or the half aperture of the pupil be one tenth of an inch, which I think by ſtar-light muſt be pretty near the truth, and MX be equal to MS, or by the preceding article, to 0,1861, we ſhall find the angle $\frac{Pp}{Pl}$ to be nearly 13′. So that a ſtar muſt appear to be about 26′ in diameter, or equal to between 8 and 9 digits of the full Moon, which is contrary to experience.

115. Likewiſe two ſtars, which are not more aſunder than 26 minutes, will by Art. 63, appear contiguous, which is alſo contrary to experience. For ζ *Bayeri*, or the middle ſtar in the tail of the great Bear is diſtant from *Alcor* but a little more than 12′, and yet the diſtance

* Phyſico-Mechanical Experiments. † Diſſertatio Phyſico-Medica.

diſtance between them is eaſily perceived.

116. Alſo the diſtance between the two ſtars, which compoſe the double ſtar α in the head of *Capricorn*, is little more than ſix minutes, yet that diſtance is eaſily ſeen by a common eye.

117. But farther, the interval between the two ſtars in the *Hyades*, η *Bayeri*, is very conſpicuous, and might, as I judge by my own eye, and am imformed by ſeveral other perſons, young and old, be plainly diſtinguiſhed, if it were leſs than it is, though it be only 5′. 40″.

118. From all which it follows, that the eye can diſtinguiſh a much leſs interval between two ſtars than that of 26′, and conſequently that the eye can have *Perfect Viſion* at a much greater diſtance than 27 inches.

119. And if we ſuppoſe a ſtar to appear under an angle of 6′, and conſequently that we can but juſt diſcern the interval between two ſtars that are a little more than 6′ aſunder, it will be found by computation, that the eye is ſuited to ſee an object by *Perfect Viſion* at no leſs a diſtance than 9 feet 7 inches.

120. But if a ſtar appears under an angle of 4′, and we can ſee the interval between two ſtars little more than 4′ aſunder, which by the tryals I have made, ſeems generally to be the caſe in good eyes, it will follow from computation, that the eye in ſeeing ſuch an interval is accomodated to ſee an object with *Perfect Viſion* at the diſtance of 14 feet five inches.

From what has been delivered in the preceding articles may be drawn a deciſive argument againſt the opinion of Monſr DE LA HIRE mentioned in Art. 105.

If the eye cannot alter its conformation ſo as to obtain *Perfect Viſion* at different diſtances, but is confined to ſome one determinate diſtance only, and at all other diſtances viſion is more or leſs indiſtinct, without any other help than the contraction of the pupil, let us ſuppoſe that one determinate, invariable diſtance for *Perfect Viſion* to be 27 inches.

Then by Art. 114, a ſtar muſt appear under an angle of 26′, and by Art. 115, the eye will not be able to perceive an interval between two ſtars that are 26′ aſunder, ſuppoſing the radius of the apperture of the pupil to be by ſtar-light one tenth of an inch.

And if we were even to ſuppoſe the diameter of the pupil to be by ſtar-light ſo ſmall as one tenth of an inch, which I take to be as ſmall a ſize, as it can be contracted to in moſt eyes by a moderate day-light, or a ſtrong candle-light, yet ſtill we ſhall not be able to perceive an interval between two ſtars that are not more than 13′ aſunder, which by Art. 115, 116, 117, is contrary to experience.

Alſo, by Art. 8 of the *Remarks*, by a moderate day-light, or ſtrong candle-light, a point in this print will at the diſtance of 13 ½ inches appear under an angle of 13′, that is, as large as a black circle of half a tenth of an inch in diameter ſeen at that diſtance by *Perfect Viſion:* And the *penumbræ* of two parallel lines in an *n*, or *m*, will meet in the middle of the ſpace between them, unleſs the intervals of thoſe lines be half a tenth of an inch aſunder, that is, the print of this *Eſſay*, or of the *Book of Opticks* itſelf, will not be at all legible at the diſtance of 13 ½ inches, which is alſo contrary to experience.

And, if inſtead of 27 inches a larger diſtance be pitched upon for the invariable diſtance of *Perfect Viſion*, this will a little help the matter with regard to the intervals of the ſtars; but will increaſe the confuſion at the diſtance we uſually read at. If a ſmaller diſtance be pitched upon, we ſhall read more eaſily at our uſual diſtance; but ſhall not ſee the interval between two ſtars, unleſs they are more than 13′ aſunder.

III.

How the eye alters its conformation.

121. We come now to the third point propoſed in Art. 104, namely, What is that change, which is made in the conformation of the eye, in order to obtain *Perfect Viſion* at different diſtances.

Concerning this point Anatomiſts and Opticians are divided into many opinions, ſuch of which as I can recollect either from reading or converſation, I ſhall here propoſe, and ſhall briefly examine.

122. One ſentiment is, that the eye in a ſtate of inaction, when both itſelf and the parts about it are in perfect reſt, is accommodated to ſee with perfect diſtinctneſs at the greateſt diſtance: And that in order to view nearer objects, the globe of the eye is compreſſed by its muſcles into an oblong figure,

figure, ſo as to render the axis ſo much longer as is neceſſary to unite the pencils into points upon the *retina*.

But to this opinion it may reaſonably be objected, that in many animals the *ſclerotica* is ſo hard, as not to be capable of changing its figure by this preſſure. And that even in men and other animals, where the *ſclerotica* is leſs hard, yet this preſſure of the muſcles can never be ſo equal as to affect the fibres of the *retina* alike in all parts, but that its fibres muſt in ſome places be crowded cloſer together, or more preſſed inwards, than in others, which muſt needs diſturb the viſion. To which we may add, that no ſlight preſſure will be ſufficient for the effect propoſed. In order to render viſion perfectly diſtinct at all diſtances from ſix inches to 14 feet five inches, it muſt be ſuch a preſſure as will lengthen the axis of the eye one tenth part, and to do this the *retina* muſt be reduced from its ſpherical figure to ſuch an oval, as very much to diſorder the range of its fibres.

123. A ſecond opinion is, that the eye, when at reſt, is fitted for ſeeing the neareſt objects diſtinctly; and that in order to diſtinct viſion of remote objects, it is preſſed againſt the back of the orbit, ſo as to render it flatter and its axis ſhorter.

But againſt this *hypotheſis* all the ſame objections, as we have urged againſt the former, may juſtly be made.

124. A third opinion is, that the eye, when at reſt, is ſuited to the moſt diſtant objects, and that in order to ſee the nearer ones diſtinctly, the cryſtalline humour is by means of the *ligamentum ciliare* drawn forwards, ſo as to increaſe the diſtance between its back ſurface and the *Retina* ſufficiently to unite the pencils into points upon that membrane.

But to ſee objects with *Perfect Viſion* from 14 feet five inches to ſix inches, it would be neceſſary that the cryſtalline ſhould be drawn forwards by about 0,87, which the *uvea* will not permit, there being no more than the diſtance of 0,22 at the moſt between the *uvea* and the cryſtalline.

125. A fourth *hypotheſis* is, that the eye, in a ſtate of reſt, is accomodated to the neareſt objects, and that in order to ſee diſtinctly the more diſtant ones, the *ligamentum ciliare* contracts, and thereby draws out the cryſtalline into a leſs convexity.

But to ſee objects perfectly diſtinct from ſix inches to 14 feet five inches, the alteration in the convexity of the cryſtalline muſt be very great. For the radius of each of its ſurfaces muſt increaſe more than $\frac{2}{5}$ of what they are at preſent. But the cryſtalline is of too firm a texture, and the *ligamentum ciliare* ſeems much too weak, for ſo great an effect to be expected.

126. A fifth opinion has been advanced by the Learned and Ingenious D^{r} * *Pemberton*, that in order to ſuit the eye to the neareſt objects, one ſurface of the cryſtalline is to be rendred more convex, while the other grows flatter. And that, to ſuit the eye to the more diſtant objects, one ſurface of the cryſtalline is rendred flatter, while the other becomes more convex. And this alteration is ſuppoſed to be made by certain muſcular fibres within the ſubſtance of the cryſtalline. But this ſentiment has not been ſo fully explained by the Learned Author, as we could wiſh.

We ſhall therefore only take notice, that if the eye is to be accommodated to near objects by rendring the anterior ſurface of the cryſtalline more convex, while the hinder ſurface grows flatter, which for many reaſons ſeems to be the moſt advantageous and convenient method; then in order to ſee an object with *Perfect Viſion* at a diſtance of 5 or 6 inches, the radius of that anterior ſurface muſt be leſſened from 3,3081 to 2 nearly, if the radius of the hinder ſurface be increaſed only from 2,5056 to 3. And in order to ſee an object with *Perfect Viſion* at the diſtance of 14 feet 5 inches, the radius of the anterior ſurface muſt be increaſed to 5 nearly, if the radius of the hinder ſurface be diminiſhed no more than to 2. So that while the hinder radius changes from 3 to 2, the anterior radius, in order to ſee perfectly at all diſtances from between 5 and 6 inches to 14 feet 5 inches, muſt be more than doubled. But this ſurely is too great a change for a ſubſtance of ſuch a conſiſtence as the cryſtalline humour to admit of.

I might add, that the demonſtrations, upon which that Learned Gentleman founds his opinion, are all built upon this ſuppoſition, that in viewing objects which appear confuſed, the pupil is always contracted to the

* Diſſert. Phyſico-Medica.

leaſt ſize it is capable of. For that this ſuppoſition is contrary to experience, will appear anon.

127. Meeting with no ſatisfaction in any of the *hypotheſes* above related, I have applied myſelf to a diligent conſideration of the parts of the eye, in order to find out, if poſſible, ſome power or powers ſeated within it, by which its conformation may be ſo altered, as adequately to anſwer the effects obſerved. And in order to enable the reader to judge how far I may have ſucceeded in this reſearch, I ſhall, before I lay down my own opinion, examine a little into thoſe parts of the eye, which I think ſubſervient to the effect in queſtion.

128. The *cornea* is a compreſſible and ſpringy membrane, eaſily giving way to any force external or internal, and eaſily reſtoring itſelf to its former figure by its own ſpring aſſiſted by the preſſure of the aqueous humour within it.

129. The *uvea* is a muſcular membrane, and as ſuch is capable of contracting itſelf into leſs dimenſions. It ariſes from a circular ridge or protuberance running all along the inſide of the *cornea* at its juncture with the *ſclerotica*, which ridge I do not remember to have ſeen hitherto taken notice of by any Anatomiſt.

That the *uvea* is furniſhed with a narrow ring of circular muſcular fibres on the edge next the pupil, is now generally agreed by Anatomiſts, though, I think, not ſo much from their being able to demonſtrate thoſe fibres, as from reaſon; for as much as the contraction of the pupil upon a ſtrong light, or upon attentively viewing a very near and ſmall object, is plainly viſible, and that contraction is juſtly preſumed to be owing to ſuch a muſcular ring. Mr. *Ruyſch* indeed has repreſented this ring of muſcular fibres in one or two of his figures, but he tells us at the ſame time, *Sculptor hic juſto diſtinctius repræſentavit, nam in objecto ipſo non ita luculenter viſuntur.* * And in another place he ingenuouſly declares, † *Fateor haſce fibras circulares non tam luculenter conſpici poſſe, quin oculi mentis in ſubſidium ſint vocandi.*

It is likewiſe an agreed point, that the *uvea* is furniſhed with ſtreight fibres inſerted into this ring, and having their origin from that part of the *uvea* which is connected to the inner edge of the *cornea*, and that theſe ſtreight fibres, which are put upon the ſtretch and drawn out into a greater length when the ring contracts, do again by their ſpring, or by their muſcular force, reſtore themſelves to their former dimenſions, and thereby ſerve to dilate the pupil, when the abovementioned muſcular ring ceaſes to contract, and is in a ſtate of inaction.

But it is here to be conſidered, that, when theſe ſtreight fibres are thus put upon the ſtretch by the contraction of the muſcular ring, they muſt neceſſarily draw the edge of the *uvea*, which is connected to the *cornea*, and likewiſe the edge of the *cornea* itſelf, a little inwards at the ſame time. And this edge of the *uvea* cannot be drawn inwards, without contracting into a leſs circumference than it had before. Muſt not therefore this edge of the *uvea*, which is next the *cornea*, be furniſhed with a ring of circular fibres, whereby it may contract itſelf into a leſs circumference, as well as that edge of the *uvea* which is next the pupil? To me this part of the *uvea* appears of ſuch a ſtrength, and to adhere ſo ſtrongly to the *cornea*, by the reſiſtance it makes in tearing them aſunder, that I make no doubt of its being muſcular, there ſeeming in this place to be no occaſion for a membrane of that ſtrength, unleſs it were to exert a muſcular force, and ſuch a one as might overcome a conſiderable reſiſtance. I ſhall therefore make no ſcruple of qualifying this limb of the *uvea* next the *cornea*, by the name of the greater muſcular ring of the *uvea*, to diſtinguiſh it from the other ring next the pupil, which I ſhall hereafter call the leſſer muſcular ring.

It will perhaps be objected to me, that the exiſtence of this ſuppoſed greater muſcular ring has not yet been proved by ocular demonſtration. I anſwer, neither has the exiſtence of the leſſer muſcular ring been yet proved in the ſame manner.

But it may be ſaid, although the exiſtence of the leſſer muſcular ring has not been demonſtrated by ocular inſpection, yet it is juſtly inferred from its effect, the contraction of the pupil, which is viſible, and is no other way to be accounted for, but by ſuppoſing the exiſtence of ſuch a muſcular ring.

* Theſ. Anatom. II. p. 87. † Ibid. p. 14.

I anſwer, the change of conformation, in adapting the eye to very near objects, is no leſs certain than the contraction of the pupil: And this change of conformation has not yet been adequately accounted for, but may be fairly made out by ſuppoſing the exiſtence of the greater muſcular ring, as I propoſe to ſhew by-and-by.

130. The cryſtalline humour is contained in a very fine membranous *capſula*, with a water between them, after the manner of the heart in the *pericardium*.

This I take from the obſervations of the late Anatomiſts, particularly the famous Monſ. * PETIT, from whom I muſt likewiſe obſerve, that the † back part of this *capſula*, or that part which inveſts the hinder ſurface of the cryſtalline humour, adheres to the membrane encloſing the vitreous humour, yet ſo as to be ſeparated from it without cutting: but that all along the limb or edge of the cryſtalline theſe two membranes adhere ſo firmly together, as not to be parted without the knife.

I muſt alſo take notice that, from the meaſures taken by this diligent and accurate Anatomiſt, as well as from autopſy, it appears that the figure of this compound body conſiſting of the cryſtalline humour, the water ſurrounding it, and the *capſula* containing them both, is ſuch as would ariſe from two ſegments of equal breadth, but of unequal ſpheres, clapt together on their plane ſides, and having the ſharp edge rounded off, ſo as to leave an obtuſe limb or edge of ſome conſiderable thickneſs, by which means the attachment of the edge of the *capſula* to the membrane of the vitreous humour all along that edge is rendred much ſtronger than it could otherwiſe be.

And to render this attachment ſtill ſtronger, I have obſerved the limb of the *capſula* to be indented all round with ſhallow tranverſe *ſulci*, or furrows, ſeemingly perpendicular to the limb, into which furrows I ſuppoſe the membrane of the vitreous humour is all along inſerted.

Thus much from obſervation, and till it ſhall be otherwiſe determined by experiment, I take leave for facility of calculation to ſuppoſe, that the *capſula*, the water within it, and the cryſtalline humour itſelf have all of them one and the ſame refractive power.

131. The *ligamentum ciliare* is a muſcle compoſed of longitudinal fibres, and is much weaker than the *uvea*. It ariſes cloſe behind the *uvea*, from the abovementioned circular ridge at the juncture of the *cornea* and *ſclerotica*, and running over the outer edge of the vitreous humour is inſerted all round the anterior ſurface of the *capſula*, upon which, ſays Monſ. ‡ PETIT, this ligament prolongs its fibres and the veſſels which it furniſhes to the *capſula*.

Now as that part of the *capſula*, into which theſe muſcular fibres and veſſels are inſerted, muſt thereby be rendred ſomething leſs diaphanous than the reſt, it is probable that this inſertion does not extend far enough towards the middle of the *capſula*, to be in the way of the rays that paſs the pupil in its greateſt dilatation.

132. If what is contained in the four preceding articles be allowed me, I think the change of conformation in the eye, to ſee objects diſtinctly at different diſtances, may be explained in the following manner.

When the eye is perfectly at reſt, no force, ſtrain, or effort of any kind being uſed by any of its parts, it is then ſuited to ſee with *Perfect Viſion* at ſome one determinate moderate diſtance.

This diſtance, I ſuppoſe, is for moſt eyes about 15 or 16 inches, the uſual diſtance for reading a print of a middle ſize. For it is likely, we uſually read at that diſtance, where viſion is perfect without any ſtraining of the eye, and at which conſequently we read with moſt eaſe, and can continue it longeſt.

Eye how altered for near objects.

133. When we view objects nearer than the diſtance of 15 or 16 inches, I ſuppoſe the greater muſcular ring of the *uvea* contracts, and thereby reduces the *cornea* to a greater convexity. And when we ceaſe to view theſe near objects, this muſcular ring ceaſes to act, and the *cornea* by its ſpring returns to its uſual convexity ſuited to 15 or 16 inches. In which condition the elaſticity of the *cornea* on the one ſide, and the tone of the muſcular ring on the other, may be conſidered as two antagoniſts in a perfect *equilibrium*.

How altered for remote objects.

134. When the eye is to be ſuited to greater diſtances

* Memoires de l'Acad. Royale 1730. † Ibid. p. 436. ‡ Ibid. p. 438.

diſtances than 15 or 16 inches, I ſuppoſe the *ligamentum ciliare* to contract its longitudinal fibres, and by that means to draw the part of the anterior ſurface of the *capſula*, into which theſe fibres are inſerted, a little forwards and outwards. And at the time this is done, the water within the *capſula* muſt neceſſarily flow from under the middle towards the elevated part of the *capſula*, and the aqueous humour muſt flow from above the elevated part of the *capſula* to the middle. Conſequently, the middle part of the anterior ſurface of the *capſula* muſt a little ſink, while the other is elevated, or the whole anterior ſurface within the inſertion of the *ligamentum ciliare* muſt be reduced to a leſs convexity. And when the contraction of the *ligamentum ciliare* ceaſes, the anterior part of the *capſula*, which has been put a little upon the ſtrain by that contraction, will by its elaſticity recover itſelf and return to its former figure. In which condition the elaſticity of the *capſula* and the tone of the ligament may alſo be looked upon as two antagoniſts perfectly *in æquilibrio* with one another.

This *capſula* as it is a very tender membrane, and contains a water between its inner ſurface and the cryſtalline, can readily obey the effort of ſo weak a muſcle as the *ligamentum ciliare*, which would not be ſufficient to flatten the cryſtalline itſelf, conſidering the firmneſs of its contexture. And hence appears the true uſe of the *capſula* and the water within it.

Here poſſibly it may be thought, that the *ligamentum ciliare* might as well have been made ſtronger, and have been inſerted into the cryſtalline itſelf, in which caſe it had been ſufficient to have drawn outwards and flattened the cryſtalline without all this *apparatus* of a *capſula* and water within it.

But this would not ſo well have anſwered the end propoſed. For the *ligamentum ciliare* ariſes from the edge of the *cornea* at its union with the *ſclerotis*, cloſe to the *uvea*; and conſequently when the longitudinal fibres of ſuch a ſtronger *ligamentum ciliare* had ſhortened themſelves, they muſt not only have drawn the cryſtalline outwards, but muſt have drawn the *cornea* inwards, that is, they muſt not only have leſſened the convexity of the cryſtalline, but muſt have increaſed the convexity of the *cornea*: And theſe two effects would have been contrary to one another, the flattening of the cryſtalline tending to ſuit the eye to remoter objects, and the increaſing the convexity of the *cornea* tending to ſuit the eye to nearer objects. Whereas the *ligamentum ciliare*, being made ſo weak, cannot ſenſibly affect the *cornea*, and yet by means of this admirable contrivance of a *capſula* and water within it, is ſufficient for the intended effect.

I need not take notice, that ſuch a ſtronger *ligamentum ciliare* might by its contraction have endangered the diſuniting the cryſtalline humour from the vitreous.

I had once thought, that both ſurfaces of the *capſula* were rendred leſs convex by its edge being drawn a little outwards, and had formed my computations from that notion. But upon conſidering the cloſe attachment of the hinder ſurface of the *capſula* to the membrane of the vitreous humour, particularly the firm adheſion of theſe two membranes at the edge of the *capſula*, as likewiſe the ſituation of the anterior and outer part of of the vitreous humour, ſuch as that it muſt neceſſarily obſtruct the drawing the edge of the *capſula* outwards, and eſpecially upon calling to mind the ſituation and inſertion of the *ligamentum ciliare*, I found the ſuiting the eye to diſtant objects could not be performed but by the flattening of the anterior ſurface only.

135. This may ſuffice to give a general notion of the manner, by which the eye alters its conformation according to the different diſtances of objects; but in order to ſhew that the means we have propoſed, are adequate to the intended purpoſe, it will be neceſſary to enter into particulars.

In Fig. 51, let *BAB* repreſent the *cornea*; *BSB* the *ſclerotica* joining to the *cornea* in *B* and *B*; *Bu*, *Bu* the *uvea*; *uu* the aperture of the pupil; *DCD* the anterior ſurface of the *capſula* of the cryſtalline humour; *DED* the hinder ſurface of the ſame *capſula*; *Bd*, *Bd* the *ligamentum ciliare*; *AS* the axis of the eye from the outer ſurface of the *cornea* to the *retina*; *AC* the axis of the *cornea* and aqueous humour; *CE* the axis of the *capſula*, the water within it, and the cryſtalline humour; *ES* the axis of the vitreous humour; *M* that point of the axis of the eye at which parallel rays will be collected into a point, or the principal focus of the eye; *f*, *L*, *I* the other foci of Art. 370. of this Book; *AP* the diſtance of an object from the eye,

when

when the pencils are collected into points on the *retina* at *S*.

Now let us ſuppoſe, as before in Art. 108, the radius of $BAB = 3,3294$; that of $DCD = 3,3081$; that of $DED = 2,5056$; $AC = 1,0358$; $CE = 1,8525$; the refraction at *A* as 4 to 3; that at *C* as 13 to 12; and that at *E* as 12 to 13.

Then will AM be 8,9993; the rectangle $LI \times fM = 49,2526$; and it remains to find MS, by deducting AM from AS.

136. Monſ. PETIT * from the meaſure of a ſingle eye makes $AS = 10,0578$: but this ſeems to differ greatly from the common length of an eye, and if there is no error of the preſs, could belong only to an eye very ſhortſighted.

In ſix eyes of adult perſons, which we have meaſured, the axis from outſide to outſide was as follows, 9,3; 9,8; 9,6; 9,3; 9,4; 9,0; the medium of which is 9,4.

From this length the thickneſs of the *ſclerotica* is to be deducted; and upon cutting thro' this coat at the termination of the axis, we placed two pins near its divided edge, and looking at them thro' a magnifying glaſs, the thickneſs of the coat ſeemed equal to one of the pins, whoſe diameter we before knew to be $=0,25$. Hence the axis of the eye, from the outer ſurface of the *cornea* to the *retina*, may be ſuppoſed uſually to be 9,15, and $MS = 0,1507$.

137. Hence, by Art. 370 of the Book of Opticks, $\frac{LI \times fM}{MS}$, or $PL = 326,7$, and $AP = 332$ tenths, or 33 inches nearly.

This therefore is the diſtance, at which an eye of theſe dimenſions, and with the refractions here ſuppoſed, muſt ſee an object with perfect diſtinctneſs, and without any ſtraining or effort of any of its parts, that is, with perfect eaſe; and may therefore juſtly be called the natural diſtance of ſuch an eye.

138. But we apprehend this diſtance is too large for common eyes, and that for two reaſons.

1. It ſeems reaſonable to ſuppoſe, that the diſtance, at which we uſually read a fair large print, is the diſtance we find by experience to be the moſt eaſy to us; and this is about 15 or 16 inches.

2. It is to be preſumed, that the natural diſtance at which we ſee diſtinctly with perfect eaſe, is but about double the leaſt diſtance we ſee diſtinctly at. For by Art. 8. of the *Remarks*, it requires nearly as great a change of conformation to leſſen the natural diſtance to one half, as to increaſe the natural diſtance to infinity. Whence it is reaſonable to conclude, that the natural diſtance is ſuch, as that no greater change of conformation is required to reduce it to the leaſt diſtance, than to increaſe it to the greateſt diſtance at which we can ſee diſtinctly. But if the natural diſtance be 33 inches, it will follow from the ſame Art. of the *Remarks*, that it requires above four times as great a change of conformation to reduce it to the leaſt diſtance of 5 or 6 inches, as to increaſe the natural diſtance to infinity. Whereas, if we ſuppoſe the natural diſtance to be about 15 or 16 inches, there will be required not much more change of conformation to ſee diſtinctly at about 6 inches, than to inlarge that diſtance to the greateſt diſtance *Perfect Viſion* can extend to.

139. This ſeems to make it neceſſary to increaſe ſome one at leaſt of the refractions we have before ſuppoſed. And indeed, as the aqueous humour is not a bare water, but an animal liquour in ſome degree charged with ſalts and ſulphurs, it ſeems reaſonable to allow it a refraction ſomewhat greater than that of 4 to 3, or more accurately than that of 80 $\frac{3}{8}$ to 60, which is a mean between the moſt refrangible and leaſt refrangible rays on paſſing out of air into water. See Sr Is. NEWTON's Opticks. pag. 114.

Natural diſtance of objects from the eye.

Let therefore the refraction of this humour and of the *cornea* be ſuppoſed that of 81 to 60, or of 27 to 20, inſtead of that before ſuppoſed of 80 to 60, and AM will now be 8,8202, MS 0,3298, and $LI \times fM$ will be 46,7621. Whence $PL = 141,8$, $AL = 5,2$, and AP, or the natural diſtance of the eye, will be 147 tenths, or 15 inches nearly, much the ſame as we find by experience the moſt eaſy diſtance to read at.

But here it may be proper to obſerve, that though this diſtance of 15 inches is the moſt eaſy to read at, when the print is large, or at leaſt of a middling ſize, yet it is not the eaſieſt diſtance to read at, when the print is ſmall.

For let a young perſon, but of full growth,

read

* Memoires de l' Acad. Royale 1728.

read a middling print, and you will find him hold the book at about 15 inches more or leſs from his eye. Then give him a book of a ſmaller print, and you will find he holds it nearer, perhaps about 12 or 13 inches.

The reaſon of this is plain. When the print is ſmall, the book is held nearer to inlarge the image upon the *retina*, that being reciprocally as *PL*, by Art. 376. of this book; ſo that by bringing the book nearer, the image is as large as that of the larger print was before. And to remedy the confuſion of the image, if it be conſiderable, either the *cornea* is rendred a little more convex by a gentle and ſmall contraction of its muſcular ring, or the pupil is a little contracted.

140. Having now ſettled the natural diſtance of objects from the eye, or the diſtance at which objects are moſt diſtinctly ſeen, when the eye is perfectly at reſt and at eaſe, without any the leaſt ſtrain or effort of any of its parts, we come next to examine whether the means we have propoſed for changing the conformation of the eye for viewing near objects, be ſufficient to accomodate that organ to *Perfect Viſion* at the leaſt diſtance, namely that of five or ſix inches. In order to which, beſides the refractions and meaſures we have already laid down, it will be neceſſary to give one or two meaſures of other parts.

141. The chord of the *cornea*, or *BB*, Fig. 51, is uſually, according to Monſ. PETIT, 5 lines *Paris* meaſure, that is in our meaſure, 4,4392. Hence the verſed ſine belonging to this chord, or that part of *AC* which is intercepted between *A* and *BB*, is 0,8481.

Let now the greater muſcular ring of the *uvea* contract itſelf, ſo as to leſſen its circumference about $\frac{1}{47}$ part, or in the proportion of 4,4392 to 4,5462: Then muſt the chord of the *cornea*, which is the diameter to that circumference, be leſſened in the ſame proportion, or from 4,4392 to 4,3462, by the edge of the *cornea* being drawn ſo much inwards. And as *BAB* the arc of the *cornea* continues of the ſame length as before, but is rendred only more convex, the radius to this arc and this chord will be 3, inſtead of 3,3294, being leſſened about one tenth part; and the verſed ſine of the *cornea* will be increaſed by 0,0835, that is, the line *AC* will be increaſed from 1,0358 to 1,1193, as will eaſily be found by computation.

142. Computing therefore by the new radius of the *cornea*=3, and this new line *AC*=1,1193, and uſing the other meaſures and refractions as before in Art. 139, we ſhall find *AM*=8,3594; *MS*=0,8741; *AL*=4,8970; *LI*×*fM*=41,8714. Whence *PL*=47,9; and *AP*=52,8, or a little more than 5 inches.

Therefore the means we have propoſed, for reducing the natural diſtance of the eye, to the ſmalleſt diſtance we ſee diſtinctly at, are ſufficient for that purpoſe, few grown perſons having *Perfect Viſion* nearer than between five and ſix inches. Tho' by a very little farther contraction of the muſcular ring the diſtance may ſtill be ſomewhat leſſened, where the *cornea* is ſufficiently flexible.

Nor can any juſt objection be drawn againſt the change of conformation we have here ſuppoſed in the eye, as being greater than can reaſonably be admitted. For the radius of the *cornea* alters only a tenth part, and this ariſes from the contracting the greater muſcular ring of the *uvea* only $\frac{1}{47}$ part, which is vaſtly leſs than the contraction of the leſſer muſcular ring, that being able to contract into half its dimenſion, when the eye is expoſed to a ſtrong light.

But poſſibly ſome doubt may ariſe about that circumference of the *cornea* into which the *uvea* is inſerted, whether by reaſon of its union with the *ſclerotica*, it can comply with the contraction of its muſcular ring, ſo as to be drawn inwards towards the pupil, and likewiſe to contract itſelf into a leſs circumference.

To this therefore we reply, that the ſpace, by which it approaches the pupil, is by our ſuppoſition very ſmall, being leſs than part of an inch: and that this ſmall motion is favoured by the obliquity of its junction with the *ſclerotica* obſerved by *Monſ. PETIT: And that the ſpace, by which that circumference ſhortens its length, is leſs than $\frac{1}{100}$ of an inch, which in a compreſſible and dilatable membrane is not hard to conceive.

But the quantity both of this approach to the pupil, and of this contraction in circumference, will be very conſiderably leſſened, if the arch *BAB* be ſuppoſed a little to ſtretch

* Memoires de l'Acad. Roy. des ſciences. 1728.

and

and dilate, when the points *B*, *B* are drawn inward. And this we take to be the truth of the caſe, though, not to tire our reader with too many *minutiæ*, we have above omitted this conſideration.

And laſtly, as we have above taken notice, Art. 134, that the elaſticity of the *capſula* and the tone of the *ligamentum ciliare* antagonize each other, it follows that, when the edge of the *cornea* is drawn inwards by the contraction of the *uvea*, and conſequently the *ligamentum ciliare* is relaxed, the *capſula* will then grow more convex. On which account a ſomewhat leſs convexity of the *cornea*, and a leſs contraction of the *uvea* will be neceſſary, than is above ſuppoſed.

143. We proceed now to examine into the ſufficiency of the means we have propoſed, to accommodate the eye to greater diſtances.

The chord of the cryſtalline humour, or *D D*, is according to Monſ. PETIT, † at a medium from 26 different eyes, in our meaſure, 3,7321. And the radius of the anterior ſurface being 3,3081, the verſed ſine to that chord is 0,5765. And the radius of the hinder ſurface being 2,5056, the verſed ſine to that ſame chord is 0,8355.

Now theſe two verſed ſines, 0,5765 and 0,8355 added together, make only 1,4120, falling ſhort of 1,8525 the axis of the cryſtalline by 0,4405, which plainly proves what we had before aſſerted, Art. 130, that the cryſtalline is compoſed of two ſegments of ſpheres joined together on the flat ſides, with the edge rounded off.

For a ſolid of the ſame dimenſions with the cryſtalline, I mean the compound cryſtalline conſiſting of the true cryſtalline, water and *capſula*, may be compoſed, either by prolonging the common chord of the two ſegments, till their verſed ſines added together make up the axis 1,8525, and then rounding off the edge till the common chord is again reduced to 3,7321:

Or by ſuppoſing two ſegments of the dimenſions aſſigned above to be clapped one on the upper ſurface, the other on the under ſurface of a cylinder having the ſame breadth with themſelves, and the altitude 0,4405, which added to the verſed ſines of the two ſegments makes up the axis 1,8525.

† Mem. 1730.

We have above taken notice, Art. 131, that the anterior ſurface of the *capſula* muſt probably have its middle part free from the inſertion of the *ligamentum ciliare*, to ſuch a breadth as to receive all the rays that paſs the pupil when moſt dilated. Now it is obvious, that none of the rays that paſs the pupil, can fall upon this middle part, although its breadth were conſiderably leſs than the diameter of the pupil, which hardly ever exceeds 2,22, or half the breadth of the *uvea*.

However, we ſhall here ſuppoſe the breadth of the anterior ſurface of the *capſula*, or *d d*, Fig. 51, to be 2,5, ſo that the right ſine of half the arc *d d* will be 1,25; the half arc will be 22°. 12′; and its verſed ſine will be 0,2452.

Let now the *ligamentum ciliare* contract itſelf ſo, as that the point *d* may be drawn forwards and outwards, and let its motion forwards, parallel to the axis of the eye, be about $\frac{1}{400}$ part of an inch, or 0,0255; and in conſequence of this motion let the vertex of the *capſula*, by Art. 134, be moved as much backwards. Then will the radius of this arch *d d* be increaſed from 3,3081 to 4,2000, and the verſed ſine will be leſſened from 0,2452 to 0,1941, the difference being 0,0511.

But hereby the axis of the aqueous humour, or *AC*, Fig. 51, will be increaſed by 0,0255, that is, from 1,0358 it will become 1,0613, and the axis of the whole cryſtalline, or *CE*, will be as much leſſened, that is from 1,8525 it will become 1,8270.

Then if we compute from this new radius 4,2000, and theſe new axes *AC* and *CE*, the other meaſures and refractions being the ſame as in Art. 139, we ſhall find *AM*=9,1209; whence *MS*=0,0291; and *LI*×*fM* will be 50,4316; and *AL*=5,4321. Whence *PL*=1733,0000, and *AP*=1738, or 14 feet, 5,8 inches.

Whence it appears, that the means we have here ſuppoſed, are fully ſufficient to enable the eye to extend its natural diſtance of 15 inches to that of 14 feet 5 inches, and that without any the leaſt motion of the cryſtalline humour itſelf, and with a very ſmall one of the anterior ſurface of the *capſula*, the point *d* and the vertex being moved only $\frac{1}{4000}$ part of an inch, and the intermediate parts moving leſs.

When

When the *ligamentum ciliare* ceaſes to contract, the *capſula*, whoſe parts, eſpecially between *d* and *d*, have been put a little upon the ſtretch, will by its elaſticity reſtore itſelf to its former dimenſions.

144. Having ſhown how the natural diſtance of an object from the eye to be ſeen with *Perfect Viſion*, may by rendring the *cornea* more convex be reduced to five or ſix inches, and on the other hand may by rendring the anterior ſurface of the *capſula* leſs convex be extended to 14 feet 5 inches, we are to obſerve, that though the eye is poſſeſſed of this power to extend *Perfect Viſion* from one of theſe limits to the other, yet it does not always exert that power to the utmoſt; but contents itſelf with attaining to *Diſtinct Viſion*, which as we have taken notice in Art. 7 and 103, cannot by the ſenſe be diſtinguiſhed from *Perfect Viſion*. And this muſt happen chiefly near the limits of *Perfect Viſion*, where the ſtraining either of the greater muſcular ring of the *uvea*, or the *ligamentum ciliare*, to the utmoſt they are capable of, muſt be ſomewhat laborious and uneaſy.

IV.

Indiſtinct viſion how rendred diſtinct.

145. We proceed now to the fourth point we propoſed to inquire into, namely, By what means Viſion, when it cannot be *Perfect*, is rendred *Diſtinct*, or at leaſt not ſo indiſtinct as it would otherwiſe be.

This we apprehend to be performed by two ſeveral ways.

1. The firſt is for the eye to apply the ſame power, as it uſes to obtain *Perfect Viſion*.

For inſtance, let us ſuppoſe the eye to be in a ſtate of inaction, and conſequently ſuited to have *Perfect Viſion* at the natural diſtance of 15 inches, and let any object, as a book of a ſmall print, be placed within 4 inches from the eye, and the eye be attentive to read it. Then immediately the *cornea* will be rendred ſo convex, as to ſuit the eye for ſeeing with *Perfect Viſion*, at ſome diſtance not leſs than five or ſix inches, and by this means the viſion at four inches, though it cannot be rendred perfectly diſtinct, yet will be leſs indiſtinct than it would otherwiſe be, and perhaps diſtinct enough to read the book with eaſe.

Again, let another object, as a playhouſe bill paſted againſt a wall, be preſented to the eye at the diſtance of ſixteen feet. Then as ſoon as we attempt to read this bill, the anterior ſurface of the *capſula* will be rendred flatter, ſo as to accomodate the eye to ſome diſtance not exceeding its utmoſt limit of 14 feet 5 inches; and though by this means viſion at the diſtance of 16 feet cannot be rendred perfectly diſtinct, yet will it be rendred leſs indiſtinct than before, and perhaps diſtinct enough to read the Bill with eaſe.

Here it may be proper to remark, that as, by Art. 133, the elaſticity of the *cornea* is an antagoniſt to the contraction of the greater muſcular ring of the *uvea*, and by Art. 134, the elaſticity of the *capſula* antagoniſes the contraction of the *ligamentum ciliare*, it follows that a greater contraction of either of theſe muſcles muſt be more laborious than a leſs contraction. Therefore in many caſes neither of them will be contracted to ſo great a degree as would be neceſſary to procure either *Perfect Viſion*, or viſion as little indiſtinct as poſſible, but to ſome leſſer degree only, ſuch as may procure *Viſion* ſufficiently *Diſtinct*.

For inſtance, if a young adult perſon in reading holds his book at 10 inches diſtance, it will not be neceſſary to contract the greater muſcular ring of the *uvea* ſo much, as to procure *Perfect Viſion* at the diſtance of 10 inches: for a middling print it may be enough to contract the *uvea* ſo much only, as would procure *Perfect Viſion* in caſe the book were at the diſtance of 13 or 14 inches; or at the diſtance of 11 or 12 inches, if the print be ſmall; and in this conformation of the eye he may ſee with ſufficient diſtinctneſs at 10 inches for the book to be eaſily read. And this leſſer contraction, being leſs laborious, will be uſed inſtead of the greater contraction, which is more fatiguing, eſpecially if he reads a great while.

And if he were to read at ſix inches diſtance, it will not be neceſſary to have *Perfect Viſion* at that diſtance; a leſs contraction of the *uvea* may ſuffice, ſuch as accommodates his eye to have *Perfect Viſion* at eight or nine inches diſtance, or poſſibly at ſeven inches diſtance, if the print be very ſmall.

For if the print be given, the leſſer diſtance he reads at, the more the *uvea* will contract: And if the diſtance be given, then the leſs the print, the more the *uvea* will contract. But it will never contract ſo much as to procure *Perfect Viſion*, at the diſtance of the book, but will ſtop at a degree of contraction

leſs

leſs laborious, and yet enough to render Viſion ſuffieicntly diſtinct; or the eye will not always receive one and the ſame conformation at the ſame diſtance; but the conformation will vary as the object is varied, though the diſtance be the ſame.

Likewiſe, in looking at an object at the diſtance of 14 feet, it will ſeldom be neceſſary for the *ligamentum ciliare* to contract ſo much as to procure *Perfect Viſion* at that diſtance. If the object be ſingle and uncompounded and ſufficiently large, or ſufficiently luminous, it may be ſufficient to contract the ligament ſo much only as would ſuffice to procure *Perfect Viſion*, at three or four foot diſtance. And if the object be compounded, but of few parts and thoſe large and eaſily diſtinguiſhable, it may not be neceſſary to contract the ligament more than would be ſufficient for *Perfect Viſion* at the diſtance of ſix or eight feet.

And in viewing objects much more remote than the diſtance *Perfect Viſion* can be extended to, it will not always be neceſſary to contract the *ligamentum ciliare* to the utmoſt, or to ſuch a degree as to procure *Perfect Viſion* at 14 feet five inches: but according to the largeneſs of the object, or of the parts it conſiſts of and to which the eye is attentive, ſometimes one, ſometimes another, leſſer and leſs laborious contraction of the ligament will ſuffice to ſhew the object with ſufficient diſtinctneſs, eſpecially when the pupil is contracted at the ſame time, as will often happen.

Conformation of the eye not always the ſame for the ſame diſtance.

So that in looking at very diſtant objects, the eye will not always have one and the ſame conformation, namely the flatteſt poſſible conformation of the *capſula*; but that conformation will be different for different objects at the ſame diſtance, as well as for the ſame object at different diſtances. And from this conſideration we are led to ſuſpect, that the Ingenious D^r^ [a] PORTERFIELD was miſtaken, when he fixed the utmoſt limit of his eye for *Perfect Viſion* at 27 inches.

2. The other way is the contraction of the pupil by the leſſer muſcular ring of the *uvea*. For by art. 382 of the Book of Opticks, the *radius of diſſipation* is, *cæteris paribus*, always proportional to the radius of the pupil. Conſequently, when the pupil is narrower, the *radius of diſſipation* and the *penumbra* ariſing from the diſſipation will be ſmaller, that is, viſion will be rendred either diſtinct, or at leaſt leſs indiſtinct than it would otherwiſe be.

146. Here it may be proper to take notice, that in a weak light the firſt of theſe means, namely the altering the convexity of the *cornea*, or of the *capſula*, muſt be uſed. For in ſuch a caſe the pupil is ſo far from contracting, that there is rather a neceſſity of dilating it to take in more light.

147. But in a ſtrong light the contraction of the pupil is chiefly made uſe of. For then that contraction anſwers two purpoſes; one to exclude an over great quantity of light, which would be offenſive to the eye; the other, to leſſen the indiſtinctneſs. And when the light is very intenſe, the pupil may contract ſo much as of itſelf to cauſe *Diſtinct Viſion*, and to render the other means altogether unneceſſary. So that theſe two ſeveral means of procuring diſtinct, or leſs indiſtinct viſion, may ſometimes be uſed jointly, that is, each in a moderate degree, and ſometimes ſingly.

Contraction of the pupil not always dependent on the will, or appearance of confuſion.

148. The degree, to which the pupil contracts, does not abſolutely depend either upon our will, or upon the ſenſation of confuſion in the object; but partly upon the degree of light.

This is eaſily proved in the following manner. By day-light take any book, and ſtanding about the middle of a room, with your back to the window, hold the book ſo near, as that the letters may appear indiſtinct, and yet not ſo much, but that you can read, though with ſome difficulty: Then turn your face to the light, and the book will be read with more eaſe. Again, holding the book at the ſame diſtance from your eye, go into the darkeſt part of the room, and ſtanding with your back to the light, you will find the book not at all legible: But upon coming to the window, with your face to the light, you will be able to read, eſpecially if the ſun ſhines, with great eaſe and diſtinctneſs.

Alſo, a perſon, who has been obliged for ſome years to uſe ſpectacles in reading, will in the ſunſhine be able to read very eaſily without them.

Whence it appears, that in a ſtrong light

[a] Medical Eſſays of Edinburg. Vol. IV. p. 169.

the pupil contracts to a greater degree, than it can be made to do by the act of our will, or by the ſenſation of confuſion only; conſequently it is a miſtake to think, that, upon ſeeing an object confuſedly, the pupil does always contract itſelf to the leaſt ſize it is capable of.

V.

Changes in the eye bycuſtom, or by age.

149. The fifth point we propoſed to examine into, was, What change is made in the eye either by habit and cuſtom, or by our growing into years.

In the eyes, as well as in all other parts of the body, the muſcles by conſtant exerciſe are enabled to contract themſelves with more ſtrength, and with more eaſe, and to a greater degree: And by diſuſing exerciſe they are reduced to leſs ſtrength, and perform their office with more difficulty, and in a leſs degree.

150. Alſo, the elaſtick parts of the eye, like thoſe of the reſt of the body, if they are often and long kept in tenſion, do more eaſily obey that tenſion, and do by degrees loſe ſomething of their elaſticity, ſo as not to be able to reſtore themſelves, or at leaſt not to do it with the ſame readineſs or to the ſame degree, when the tenſion ceaſes or abates: And if they are too ſeldom put in tenſion, they grow ſtiffer and are not ſo eaſily diſtended to the ſame degree.

151. From theſe cauſes ſeem to ariſe thoſe changes of the eye, that habit and cuſtom introduce.

Perſons, who are much and long accuſtomed to view remote objects, and but ſeldom to conſider near ones, ſee better at great diſtances, and not ſo well at near diſtances as other people.

This is often the caſe of Travellers, Sailors, Sportſmen, *&c.* In them the *ligamentum ciliare* being much uſed, comes by degrees, by art. 149, to acquire more ſtrength, ſo as to draw out the *capſula* of the cryſtalline farther, and thereby to reduce it to a leſs convexity than in other perſons. Whence they are enabled to ſee diſtinctly at greater diſtances than other perſons.

But the *capſula* being thus often put upon the ſtretch, and long detained ſo, will thereby in time loſe ſomewhat of its elaſticity, ſo as not to return to its natural convexity, when the contraction of the *ligamentum ciliare* ceaſes, by art. 150. And this is one reaſon, why theſe perſons do not ſee ſo well as the reſt of Mankind, at ſmall diſtances.

Alſo, the greater muſcular ring of the *uvea*, by diſuſe, will grow weaker, by art. 149: And the *cornea* by being ſeldom bent into a greater convexity, will in time grow ſtiffer and leſs capable of obeying the contraction of the muſcular ring, by art. 150, which is another cauſe why theſe perſons do not ſee ſo well at ſmall diſtances.

152. On the other hand, Perſons, who are much and long accuſtomed to view objects at ſmall diſtances, as Students in general, Watchmakers, Gravers, Painters in miniature, *&c.* ſee better at ſmall diſtances, and not ſo well at great diſtances, as the reſt of Mankind.

For in them the greater muſcular ring of the *uvea* contracts more eaſily and ſtrongly, by art. 149: And the *cornea*, by art. 150, more readily obeys the contraction of that ring. Whence they ſee better at ſmall diſtances.

But the *cornea*, by being thus often and long bent into a greater convexity, does by degrees loſe ſomething of its elaſticity, ſo as not to return to its natural convexity, when the muſcular ring ceaſes to act upon it, by art. 150. This is one cauſe of their not ſeeing ſo well at great diſtances.

Alſo, the *ligamentum ciliare*, being ſeldom employed to leſſen the convexity of the *capſula*, does by degrees become leſs capable of performing that office, by art. 149: And the *capſula*, being ſeldom drawn out and put into tenſion, muſt, by art. 150, loſe ſomething of its diſtenſile quality, ſo as leſs eaſily to comply with the action of the ligament. And this is another cauſe of their not ſeeing ſo well at great diſtances.

153. Perſons, who begin to uſe ſpectacles early, and keep conſtantly to the uſe of them, do in a little time find themſelves under a neceſſity of continuing to uſe them. For, by uſing ſpectacles, they have not occaſion to contract the greater muſcular ring, to bend the *cornea*, ſo much as before, and when they have forborn this contraction for ſome time, they are not ſo well able to perform it, as before they uſed ſpectacles.

154. In children the pupil is uſually more dilated than in grown perſons. This is eaſily ſeen; for in grown perſons the diameter of the

the pupil ſeldom appears equal to the breadth of the ring of the *uvea* on either ſide of it, that is, is ſeldom equal to one third of the breadth of the *cornea*, and is often much leſs, eſpecially in a good light. But in children the diameter of the pupil ſcarce ever appears ſo little as one third of the breadth of the *cornea*, and often exceeds half that breadth.

The reaſon of this we apprehend to be, that in children the *cornea* is extremely flexible, ſo as to be very eaſily bent by its muſcular ring, into any curvature that is neceſſary for ſeeing diſtinctly in reading, and conſequently their pupil has leſs occaſion to contract for *Diſtinct Viſion*.

But in grown perſons the *cornea* is ſomewhat ſtiffer and does not ſo eaſily bend, whence they have occaſion more frequently to contract the pupil.

And in elderly perſons the *cornea* grows ſtill more rigid, ſo that they can hardly read without ſpectacles, unleſs the print be very large, or the light be very ſtrong, ſo as to cauſe a great contraction of the pupil. It is for this reaſon they are obliged to hold the candle between the eye and the paper they read; and their doing ſo is a certain ſign that they begin to want ſpectacles.

155. Children read much nearer than grown perſons. This happens for two reaſons.

1. Their eyes are ſmaller, and the leaſt diſtance any eye can ſee diſtinctly at, is proportional to the length of the eye.

2. Their *cornea*, being very flexible, is eaſily accomodated to a leſs diſtance. And at a leſs diſtance the print appears larger, and is eaſier to read, than at a greater.

156. Elderly perſons ſee better at a great diſtance than younger people. The matter of fact I think is pretty generally agreed upon, and if queſtioned, may thus be examined. Let a perſon, whoſe ſight for near objects is impaired by age, obſerve the Moon at three or four days old, and take notice whether the illuminated part appears of a larger diameter than the dark part, and how much is the exceſs. And let him recollect, if he can, what was the appearance of thoſe two parts of the Moon, when he was much younger. Then, I am perſuaded, he will think the exceſs of the light part above the dark is either none at all, or much leſs than formerly. At leaſt this is my caſe. I well remember, that formerly the diameter of the light part of the Moon much exceeded that of the dark part: Whereas now the limbs of thoſe two parts appear nearly to unite in the ſame circle.

The cauſe why perſons advanced in years ſee better at a diſtance, has been reckoned to be the ſhrinking of the coats and humours of the eye. But this I think cannot be ſatisfactory. If the eye were to ſhrink in all parts proportionably, this would have a contrary effect, the eye would then become ſhortſighted, as has been obſerved of children, whoſe eyes are ſmaller than thoſe of men.

But the true reaſon I take to be this. The *cornea*, as it is of a rarer texture, and is more expoſed to the air than the *ſclerotica*, will in length of time ſhrink a little more than the *ſclerotica*, and will by that means grow a little flatter than it was before. For in Fig. 51, if the line *BuuB* continue of the ſame length, and the arc *BAB* be a little leſſened, the vertex of this arc at *A* muſt be a little depreſſed towards *C*, that is, the arc *BAB* muſt become flatter or leſs convex. And if the length of this arc of the *cornea* be leſſened by ſhrinking, about the $\frac{1}{150}$ part of an inch, this will be ſufficient to encreaſe the radius of the *cornea* from 3,3294 to 3,3500, and thereby to increaſe *AM*, in art. 143, from 9,1209 to 9,1497. And as by this ſhrinking of the *cornea*, its verſed ſine will be leſſened by 0,0070, and conſequently the axis of the eye will now be leſſened from 9,1500 to 9,1430, *AM* may ſomewhat exceed this axis, if the *capſula* of the cryſtalline be flattened as much as we have ſuppoſed in art. 143, and if it be ſomething leſs flattened, *AM* may be exactly equal to the axis, that is, parallel rays may be collected into a point upon the *retina*, or the fartheſt limit of *Perfect Viſion* may be extended to an infinite diſtance.

157. Elder perſons do not ſee ſo well at ſmall diſtances as thoſe of leſs age. This happens partly from the ſhrinking, and partly from the rigidity of the *cornea*, which increaſes with our age, and may carry out the neareſt limit of *Perfect Viſion* from 3 or 4 inches, as in children, and from about 5 or 6 inches in young adult perſons, to 20, 30, 40 inches, or a greater diſtance. And in this caſe the eye has no aſſiſtance in viewing near objects, but only from the contraction of the pupil, and this is not ſufficient for *Diſtinct Viſion*, unleſs in a ſtrong light.

If the arc of the *cornea* ſhrink $\frac{1}{200}$ part of an inch, this will remove the natural diſtance from 15 to 77 inches. And the *cornea* being now grown more rigid, the *uvea* will be leſs able to contract it into a greater convexity. While the *cornea* was more flexible, the *uvea* was able to render it ſo convex as to reduce the natural diſtance from 15 inches to 5, that is to a third part: but now probably the new natural diſtance of 77 inches can hardly be reduced to leſs than one half, that is to 38 or 39 inches.

Now as this is probably the caſe of many perſons above 50 years of age, and particularly my own, not to have *Perfect Viſion* at a diſtance leſs than 38 or 39 inches, it might be expected that ſuch perſons might have *Perfect Viſion* at an infinite diſtance, inaſmuch as we have juſt now ſhown, that upon the ſhrinking of the *cornea* only the $\frac{1}{1500}$ part of an inch, *Perfect Viſion* might be extended to an infinite diſtance, provided the *capſula* of the cryſtalline humour could be flattened to the ſame degree as was ſuppoſed in art. 143.

But perſons, whoſe *cornea* is thus flattened, are thereby enabled to ſee diſtinctly at much greater diſtances than before, without flattening the *capſula*; and therefore have leſs occaſion to contract the *ligamentum ciliare*; and that muſcle by diſuſe becomes leſs able to contract itſelf to the ſame degree as before, and conſequently this inability will hinder the extent of *Perfect Viſion* to the utmoſt diſtance it would otherwiſe reach to.

158. As the juices of grown animals are ſtronger and more loaded with animal ſalts and oils, than thoſe of younger ones, it is probable, that the refraction of the coats and humours of the eye becomes ſomething greater as we advance in age, than when we are children. For all ſulphureous ſubſtances refract more ſtrongly, *cæteris paribus*, than others. And Monſ. PETIT * obſerves, that the cryſtalline humour, which is at firſt perfectly limpid, has in adult perſons a yellowiſh tinge, which grows ſtronger as they advance in years.

The gradual increaſe of this yellowiſh tinge, and the increaſing refraction conſequent thereupon, may, I ſuppoſe, in ſome meaſure countervail the increaſing ſtiffneſs and flatneſs of the *cornea*. Otherwiſe we ſhould grow longſighted and want ſpectacles ſooner than we do.

VI.

159. The ſixth and laſt head of our enquiry is, What is the leaſt object, or angle, that the eye is capable of perceiving? The leaſt viſible angle.

The learned Dr. † HOOK aſſerts, that when an object ſubtends a leſs angle than a minute, it is to the generality of eyes wholly inviſible. And by the experiment related in art. 97 of this book, and one that I have made myſelf, within the limits of *Perfect Viſion*, I am inclined to think, that when the object is round, as a black circular ſpot upon a white ground, or a white ſpot upon a black ground, an eye muſt be exceeding good to perceive it under an angle much leſs than a minute.

160. But there are other caſes, in which a much leſs angle can be diſcerned by the eye, ſome of which we ſhall here conſider, after premiſing one obſervation, which ſeems neceſſary for explaining the reaſons of them.

In order to our perceiving the impreſſion made by an object upon any of our ſenſes, the impreſſion muſt be either of a certain degree of force, or of a certain degree of magnitude.

For inſtance, a very ſmall drop of dew, or rain, may fall upon the hand without our feeling it wet; and a very ſmall particle of ſugar may be laid upon the tongue without our taſting it ſweet; but a ſpark of fire of the ſame magnitude with the drop of rain, by falling upon the hand will ſenſibly affect it, becauſe the impreſſion of the ſpark of fire is of a greater force than that of the drop of water, or particle of ſugar.

And for the ſame reaſon, a ſtar, which appears only as a lucid point thro' a teleſcope, not ſubtending ſo much as an angle of one ſecond, is viſible to the eye, though a white or black ſpot of 25 or 30 ſeconds is not to be perceived.

161. Alſo, though one very ſmall drop of water will not ſenſibly affect the hand, yet a number of ſuch drops falling together, or one larger drop falling alone, will affect the hand with a ſenſe of wet, becauſe the quantity of the impreſſion is greater.

* Memoires de l'Acad. Roy. des Sciences. 1726, 1730. † Animadvers upon *Hevelius's Machina Coeleſtis* pag. 7, 8, and Poſthumous Works. pag. 97.

And

A line visible farther off than a ſpot of the ſame breadth.

And for the ſame reaſon, a line, of the ſame breadth with a circular ſpot, will be viſible at ſuch a diſtance, as the ſpot is not to be perceived at, becauſe the quantity of impreſſion from the line is greater than the quantity of impreſſion from the ſpot. And a longer line is viſible at a greater diſtance, than a ſhorter line of the ſame breadth.

162. It has been by ſome perſons ſuppoſed, that the impreſſions of viſible objects are received upon certain *villi* of the optick nerve, imagined to ſtand erect upon the *retina*, like the pile on velvet. And from the experiment that a ſpot leſs than one minute in diameter cannot be perceived by the eye, it would follow that the thickneſs of one of theſe *villi* is about $\frac{1}{5000}$ or $\frac{1}{6000}$ part of an inch.

163. But admitting the ſuppoſition, the diameter of one of theſe *villi* will be found to be much ſmaller.

For, I find, that a bit of ſilver wire, of the thickneſs of $\frac{1}{485}$ of an inch, laid upon a white paper, is viſible at the diſtance of 10 feet. Hence the angle ſubtended by the diameter of the wire is about $3'' \frac{1}{2}$. Conſequently, the thickneſs of a *villus* muſt be 17 times ſmaller, than in art. 162.

164. I took a ſingle filament of ſilk, and laid it cloſe to this bit of wire, then viewing them with a deep magnifying glaſs, I judged the diameter of the wire to be equal to 4 diameters of the ſilk. The diameter of the ſilk muſt therefore be about $\frac{1}{1340}$ part of an inch.

This ſilk and wire, being laid upon a white paper, were both viſible at the diſtance of 40 inches from the eye, and the ſilk appeared plainly leſs than the wire.

Here the ſilk ſubtended an angle of $2'' \frac{1}{2}$ only. Conſequently, the thickneſs of a *villus* is 24 times ſmaller than that determined in art. 162.

165. It will be objected perhaps, after DES CARTES's way of reaſoning, that there is no neceſſity of ſuppoſing the diameter of the *villi* to be ſo ſmall as the diameter of the ſilk. For if the diameter of a *villus* were 24 times as great as that image, ſo that the image took up only $\frac{1}{24}$ part of each *villus*, yet the whole of every *villus* the image fell upon, would be affected by the impreſſion upon that $\frac{1}{24}$ part. Conſequently, the ſilk muſt be equally perceivable, as if its image had taken up the whole of each *villus*.

We anſwer, If this be the caſe, then the ſilk ought to appear equal in breadth with the ſilver wire, whoſe image takes up only a ſixth part of each *villus*. But, in fact, the wire appears broader than the ſilk.

166. This greater viſibility of a line, than of a ſpot of the ſame breadth, within the limits of *Perfect Viſion*, ſeems to ariſe only from the cauſe we have here laid down, *viz.* the greater magnitude or quantity of the impreſſion upon the *retina* by the line, than by the ſpot. But without the limits of *Perfect Viſion* another cauſe concurs, whereby the difference of viſibility between the line and the ſpot is rendred much more conſiderable. For the impreſſion npon the *retina* made by the line, is then not only much greater, but likewiſe much ſtronger than that of the ſpot.

167. Let *ABC*, Fig. 52, repreſent a white circular ſpot upon a black ground, and let it be placed ſo much beyond the utmoſt limit of *Perfect Viſion*, as that the *radius of diſſipation*, art. 17, may conſiderably exceed the radius of the ſpot. Then as every point within this circle *ABC* diſſipates its rays over a circle conſiderably greater, and can receive no ſcattered light but only from the other points ſituated within the ſame circle *ABC*, it is plain that the whole circle *ABC* muſt appear conſiderably fainter than if it were ſeen by *Perfect Viſion*.

168. But, in Fig. 53, let the ſame circle *ABC* be contiguous to two other equal circles *DEF*, *GHI*, their centers being all ſituated in one right line, and let theſe three circles be viewed from the ſame diſtance as before. Then the circle *ABC* will not appear ſo faint and weak as before; becauſe it will receive part of the rays diſſipated from the adjacent circles *DEF*, *GHI*. And if the lines *GD*, *IF* be drawn touching theſe three circles, and the whole ſpace between thoſe tangents be made white, the circle *ABC* will be ſtill farther illuminated by the light which is diſſipated from the additional ſpaces compriſed between the circles and the tangents. But this will compoſe a line of the ſame breadth with the circle *ABC*, and by the addition of more circles this line may be extended to what length we pleaſe, and the whole line will be more illuminated than the ſingle circle *ABC*.

Therefore a line placed beyond the limits of *Perfect Viſion* will appear more ſtrongly illuminated, and will make an impreſſion of more

more force upon the *retina*, than a circular ſpot of the ſame breadth. Conſequently, upon this account, as well as upon that of the quantity of impreſſion, the line will be viſible at a greater diſtance than the ſpot.

169. As we have ſhown above, art. 163, 164, that a dark line upon a white ground is viſible, within the limits of *Perfect Viſion*, when it ſubtends an angle of no more than 2 or 3 ſeconds; and as there can be no reaſon to doubt but that a white line upon a dark ground muſt be viſible under as ſmall an angle; it might naturally be expected, that a white ſpace between two black parallel lines, the ſpace being of the ſame breadth as each of the lines, ſhould alſo be viſible within thoſe limits, when the ſpace ſubtends an angle of only 2 or 3 ſeconds, or that this ſpace ſhould be viſible at the ſame diſtance as one of the lines ſingly taken is perceivable. There might even be reaſon for thinking, that this ſpace, as it is white, ſhould be perceivable at a greater diſtance than a black line of the ſame breadth and length. But the caſe is far otherwiſe.

Different viſibility of ſingle and compound objects.

170. For in Fig. 54, let *AB* be two black lines drawn upon white paper with a ſpace between them equal in breadth to each of the black lines; and let *CD* be another black line drawn at ſome conſiderable diſtance from the two former, but equal in breadth and length with either of them. Then, if the paper be ſet againſt a wall, and you retire backwards from it, you will find that at ſome certain diſtance ſuited to your eye, even within the limits of *Perfect Viſion*, the white ſpace between the two lines *AB* will not be diſtinguiſhable, the two lines appearing as one broad line only; but at the ſame diſtance the ſingle line *CD* will be manifeſtly perceived, and will continue to be ſo, though you retire to a conſiderable diſtance farther backwards.

171. This will ſeem the more ſurprizing, when we conſider the following expetiment.

Upon the ſame paper with the two black lines *AB*, and the ſingle black line *CD*, let *IK*, in Fig. 55, be a white line, of the ſame length and breadth with *CD*, but lying between the two black ſpaces *IKEF*, *IKGH*, whoſe ſides next the white line are parallel to each other. Then, if the paper be fixed againſt a wall, and you retire backwards from it, the white line *IK* will be viſible at as great a diſtance as the black line *CD*, that is, at a much greater diſtance than the white line *AB* can be diſtinguiſhed.

172. Here it may reaſonably be aſked, Why the white line *AB* is not viſible at as great a diſtance as the black line *CD*, ſince thoſe two lines are equal both in length and breadth, and are both ſeen within the limits of *Perfect Viſion*? Alſo, Why is the white line *IK* ſeen at a greater diſtance than the white line *AB*, ſince theſe two white lines are equal in length and breadth, are both of them terminated by parallel black lines, and are both ſeen within the limits of *Perfect Viſion*?

173. In order to give a ſatisfactory anſwer to theſe queſtions, it will be neceſſary to premiſe another obſervation.

The more compounded any object is, or the more parts it conſiſts of, it will, *cæteris paribus*, be more difficult for the eye to perceive and diſtinguiſh its ſeveral parts.

For inſtance, it is ſomewhat difficult for the eye to judge how many figures are contained in the following numbers, 1111111111; 1000000000. But if we divide the figures in this manner, 11111,11111; 10000,00000; ſo as to conſtitute ſeveral objects leſs compounded, we can more eaſily eſtimate the number of figures contained in each of thoſe numbers; and more eaſily ſtill, if we thus divide them, 1,111,111,111; 1,000,000,000.

174. The cauſe of this, we apprehend, is the difficulty of keeping the eye perfectly ſteddy.

For in counting theſe figures, 1111111111, if the eye be arrived, for inſtance, at the fifth figure from the right hand, and by any imperceptible motion of the body, or involuntary fluctuation of the eye itſelf, its axis happen to be directed to the fourth or ſixth figure, or to the interval between the fifth figure, and either of theſe, we find ourſelves immediately at a loſs, not knowing from which figure to proceed in our counting, and are therefore obliged to begin again at the right hand.

That this is the true reaſon, may appear from its being eaſier to count the figures in the number 121212121212, than in the number 111111111111; and from its being ſtill eaſier to count them in the number 123123123123, or 123412341234, where from the diſſimilitude of the figures the eye more

more eaſily recovers its place when once loſt.

175. From the ſame cauſe of the inſtability of the eye it muſt be, *cæteris paribus*, more difficult to perceive and diſtinguiſh the parts of any compound object, when each of thoſe parts ſubtends a very ſmall angle, than to ſee a ſingle object of the ſame magnitude as one of thoſe parts.

For inſtance, the hour I. upon a dial-plate may be ſeen at ſuch a diſtance, as the hours II, III, IIII. are not to be diſtinguiſhed at, eſpecially if the obſerver be in motion, as in a coach, or on horſeback, or even in a boat upon the water. This may eaſily be experienced in looking at a dial where the intervals between the black or gilt ſtrokes are equal to the breadth of thoſe ſtrokes; and much more eaſily where the intervals are of a leſs breadth, which is a defect in large dials that are to be ſeen at a great diſtance. For in theſe, the intervals ought to be conſiderably broader than the ſtrokes.

Likewiſe, *AB*, in Fig. 54, is a compound object conſiſting of three parts, *viz.* the two black lines and the white line lying between them: But *CD* is a ſingle object conſiſting of one black line only upon a white ground: And *IK*, Fig. 55, is to be conſidered as a ſingle object conſiſting of one white line only upon a black ground.

Now in viewing either of theſe ſingle objects, if the eye be imperceptibly moved, all the effect from that motion will be only, that the object will be painted upon a different part of the *retina*; but wherever it be painted, there will be but one picture, ſingle and unconfounded with any other.

But in viewing the compound object *AB*, if the eye be ſuppoſed to fluctuate ever ſo little, the image of one or other of the black lines will be ſhifted on to that part of the *retina*, which was before poſſeſſed by the white line; and this muſt occaſion ſuch a dazzle in the eye, that the white line cannot be diſtinctly perceived and diſtinguiſhed from the black lines, which by a continual fluctuation will alternately occupy the ſpace of the white line, whence muſt ariſe an appearance of one broad dark line without any manifeſt ſeparation.

176. By trying this experiment with two pins of known diameters, ſet in a window againſt the ſky-light, with a ſpace between them equal in breadth to one of the pins, I find the diſtance between the pins can hardly be diſtinguiſhed, when it ſubtends an angle leſs than 40″, though one of the pins alone can be diſtinguiſhed under an angle vaſtly leſs.

177. But though a ſpace between two pins cannot be diſtinguiſhed by the eye, when it ſubtends a leſs angle than 40″, it would be a miſtake to think, that the eye muſt neceſſarily commit an error of 40″ in eſtimating the diſtance between two pins, when they are much farther off from one another.

For if the ſpace between the two pins be ſuppoſed to ſubtend an angle of 1°, and each of the pins to ſubtend an angle of 4″, which, by art. 163, 164, is greater than the leaſt angle the eye can diſtinguiſh, it is manifeſt, that the eye may judge of the place of each pin within 2″ at the moſt, and conſequantly the error committed in taking the angle between them cannot at the moſt exceed 4″, provided the inſtrument be ſufficiently exact.

178. And yet upon the like miſtake was founded the principal objection of Dr. Hook, againſt the accuracy of the cœleſtial obſervations made by the famous Hevelius.

179. By art. 149, a black ſpot upon a white ground, or a white ſpot upon a black ground, can hardly be perceived by the generality of eyes, when it ſubtends a leſs angle than one minute.

And if two black ſpots be made upon white paper with a ſpace between them equal in breadth to one of their diameters, that ſpace is not to be diſtinguiſhed, even within the limits of *Perfect Viſion*, under ſo ſmall an angle, as a ſingle ſpot of the ſame ſize can be diſtinguiſhed. To ſee the two ſpots diſtinct, therefore, the breadth of the ſpace between them muſt ſubtend an angle of more than a minute. It would be very difficult to make this experiment accurately within the limits of *Perfect Viſion*, becauſe the objects muſt be extremely ſmall; but by a rude tryal with ſquare bits of white paper placed upon a black ground, I judge that the leaſt angle, under which the interval between the two objects can be perceived, is at leaſt a quarter part greater than the leaſt angle under which a ſingle object can be perceived. So that an eye, which cannot perceive a ſingle object under

under a leſs angle than one minute, will not perceive the interval between two ſuch objects under a leſs angle than 75″.

180. Without the limits of *Perfect Viſion*, the diſtance at which a ſingle object ceaſes to be perceivable, will be much greater in proportion, than the diſtance at which a ſpace of equal breadth between two ſuch objects ceaſes to be perceivable.

For without theſe limits the image of each of the objects will be attended with a *penumbra*; and the *penumbræ* of the two near objects will take up part of the ſpace between them, and thereby render that ſpace leſs perceivable; but the *penumbra* will add to the breadth of the ſingle object, and will thereby make it more perceivable, unleſs its image be very faint.

Hevelius defended againſt Dr Hooke.

181. Dr. HOOK aſſerts, that the ſharpeſt eye cannot diſtinguiſh the interval between two ſtars, that are leſs than half a minute aſunder, and that not one eye in an hundred can diſtinguiſh an interval of leſs than a minute.

If we ſuppoſe therefore that HEVELIUS, whoſe eye undoubtedly was very good, as appears from his obſervations, and particularly from Dr. * HALLEY's teſtimony, could diſtinguiſh two ſtars that were one minute aſunder, it follows, from art. 64, that the *falſe image* of a ſtar to HEVELIUS's eye did not exceed half a minute.

But if 40″ be ſuppoſed the leaſt angle in a compound object, that HEVELIUS's eye could perceive in caſe of *Perfect Viſion*; then in order for him to perceive an interval between two ſtars that were 1′, or 60″ aſunder, 40″ at leaſt of that interval muſt have been clear of the two *falſe images* of the ſtars, ſo that the *falſe image* of each ſtar could at the moſt take up no more than 10″ of the interval between them. Conſequently, a ſingle ſtar to HEVELIUS's eye could appear only under an angle of 20″.

182. And from other conſiderations it is not improbable, that his eye muſt have been fitted to ſee a ſtar under ſo ſmall an angle as 20″, at the time of his controverſy with Dr. HOOK. For this noble Aſtronomer was then far advanced in years, and his eye muſt thereby have been ſomewhat flattened; and by a continued courſe of obſervations for near fifty years, it muſt have been as much adapted for viewing celeſtial objects diſtinctly, as practice and habit could make it.

183. Let us therefore conſider, what error in obſerving either the altitude of a ſtar, or the interval of two diſtant ſtars, muſt neceſſarily be committed by ſuch an eye; ſuppoſing the inſtrument he obſerved with to be perfectly exact.

And here we are firſt to take notice, that the ſights he made uſe of next his eye, were narrow rectangular ſlits ſo placed upon his inſtrument, that the length of the ſlit was always perpendicular to the plane of the angle he was to obſerve, and the breadth of the ſlit was in the ſame plane with that angle. So that by this means the eye received a ſufficient quantity of light along the ſlit to render the ſtar viſible, while at the ſame time that apparent diameter of the ſtar, which lay in the plane of the angle intended to be meaſured, was conſiderably leſſened. On which account, if not by reaſon of his age alone, we may reaſonably ſuppoſe a ſtar appeared to him under no greater angle than 20″, but rather leſs.

Let *AOB*, Fig. 56, be the altitude of the ſtar to be obſerved, whoſe center is *C*, and whoſe apparent vertical diameter is *Aa*, ſubtending an angle of 20″.

Here it is manifeſt, that if, inſtead of aiming at *C*, the center of the ſtar, or the middle of its light, he directed his ſight to the lower edge of the light at *a*, or to the upper edge of the light at *A*, he could err but 10″ either way, and that the greateſt difference between two obſervations of the ſame ſtar, could not exceed 20″.

But as he would undoubtedly aim at the middle of the light as near as he could, and we cannot eaſily ſuppoſe him to deviate from the middle more than half the radius of the ſtar, he could hardly commit an error of more than 5″ either way.

184. Let *Cc*, Fig. 57, be the interval between the centers of two ſtars *C*, *c*; and let *COc* be the angle which that interval ſubtends at the eye of the obſerver, and which HEVELIUS was to meaſure by his inſtrument; alſo, let *Aa*, *Bb* repreſent the apparent diameters of the two ſtars in the plane of the angle to be meaſured, each of them ſubtending an angle of 20″.

* Hevelii Annus Climactericus.

Then

Then if the axis of one obſerver's eye were directed to *c* the center of the ſtar *Bcb*, along the line *Oc*; and the axis of the other obſerver's eye were directed to *C* the center of the ſtar *ACa*, along the line *OC*; it is plain that the angle found by the inſtrument, muſt be the very angle *cOC* ſubtended by *Cc* the interval between the centers of the two ſtars, without any error.

Alſo, if the eye of one obſerver were directed to *b*, the right hand extremity of the ſtar *Bcb*; and the eye of the other obſerver were directed to *a*, the right hand extremity of the ſtar *ACa*; the angle *bOa*, found by the inſtrument, muſt be equal to the angle *cOC*, without any error.

So likewiſe, if the eye of each obſerver were directed to the left hand extremities of the two ſtars, *B* and *A*; the angle *BOA*, found by the inſtrument, muſt be preciſely equal to the angle *cOC*, without any error.

185. And if the two obſervers were ſo careleſs, and withal ſo unhappy, as to direct their eyes to the two remoteſt extremities of the two ſtars, *b* and *A*, the angle thereby found *bOA*, would exceed the true angle *cOC*, by an error of 20″, and no more.

And juſt the ſame error would ari e if they took their aim at the two neareſt extremities of the ſtars, *B* and *a*, the angle being then only 20″ two little.

186. But this could harldly happen, eſpecially to ſo experienced an obſerver as HEVELIUS, who would undoubtedly direct his line of colliſmation *Oc* to the middle of the light of the ſtar, as near as he could: and it can ſcarce be ſuppoſed that he could deviate from *c* the center of the ſtar *Bcb*, more than half the diſtance *cb*, or *cB*, which could alter the place of the ſtar no more than 5″. And if the ſame miſtake were committed the contrary way in the place of the other ſtar *ACa*, this could increaſe or diminiſh the interval between them by no more than 10″, ſuppoſing that interval to have been taken no more than once.

187. But by repeated obſervations, and taking a *medium* between them, as was his practice, he muſt ſettle the interval very near the truth; ſo that ſcarce any ſingle obſervation could differ from that *medium* more than 10″, and the reſt muſt generally approach the *medium* within 5″ or 6″, as appears to have been the caſe, both by comparing his obſervations one with another, and by the teſtimony given him by Dr HALLEY, who affirms that his obſervations made by plain ſights did not differ from the truth, *niſi contemnendâ minuti parte.* Therefore the obſervations of this illuſtrious Aſtronomer could not differ from the truth, 1, 2 or 3 minutes, nor was the largeneſs of his inſtruments, and the accuracy of their diviſions, a needleſs charge, or vain curioſity, much exceeding what nicety the eye could obſerve to, as was injuriouſly ſuggeſted by the learned Dr HOOK.

188. To what we have thought fit to ſay in vindication of this noble and renowned Aſtronomer, whoſe memory will always be honoured by ingenuous minds, it may perhaps be objected, that the eyes of thoſe aſſiſtants, who obſerved with him, might poſſibly not be ſo good as his own; and conſequently the error committed in taking the diſtances of two ſtars, might be conſiderably greater than what we here ſuppoſe.

We anſwer, that one of thoſe aſſiſtant obſervers was his Lady, probably not much younger than himſelf, and much accuſtomed to obſervation, as were likewiſe his other aſſiſtants: and if ſome of theſe were younger perſons, and conſequently might ſee a ſtar under a greater angle than 20″, by the bare eye, yet in looking through his ſights the angle might little exceed that magnitude: at leaſt, as they were trained up by him and uſed to obſervation, they could hardly deviate from the middle of the light ſo much as one half of the radius, as we have above ſuppoſed. And our liberality in this conceſſion may very well compenſate for the ſomewhat greater angle under which they might ſee the ſtar. To which we may add, that any error an aſſiſtant obſerver could commit, muſt affect the angle on one ſide only, and could not be doubled, as was above ſuppoſed. And upon the whole, the near agreement of his obſervations one with another, is a ſufficient proof, that his aſſiſtants were capable of obſerving with ſufficient accuracy.

189. Mr. *Huygens* in his *Syſtema Saturnium*, among many curious points, which with his uſual ſagacity and penetration he has happily diſcuſſed, to the ſatisfaction of the Learned World, Outer edge of Saturn's ring why inviſible.

World, propoſes a very difficult [a] queſtion, Why the outer edge of *Saturn*'s ring is not viſible in the form of *brachia*, when the earth is in the plane of the ring, *Saturn* then appearing round.

This he finds no other way of ſolving, than by ſuppoſing that this outer edge is of ſuch a nature, or is covered with ſuch a matter, as does not at all, or very little reflect the light that falls upon it from the Sun: and in this ſuppoſition he has been followed by ſome later Philoſophers.

190. Now, it has indeed been obſerved by Mr. [b] *Huygens* himſelf, as well as by ſignor *Caſſini* and ſome of our *Engliſh* Aſtronomers, that the outer part of the planes of the ring is leſs luminous than the inner part; and therefore it is not unlikely, that the outer edge of the ring may alſo be leſs luminous than the inner and brighter part of thoſe planes, or even than the body of *Saturn*, which is ſomething darker, at leaſt towards the outſide, than the inner and more lucid part of thoſe planes.

But that this outer edge ſhould be of ſo different a nature from thoſe parts of the upper and under plane, between which it lyes, and to which it is immediately contiguous, as that thoſe ſhould reflect the light in the manner they are found to do, and this ſhould reflect it very little, or not at all, is difficult to ſuppoſe.

191. It may therefore be worth while to try, whether this queſtion can be anſwered any other way, without admitting ſo hard a ſuppoſition.

Mr. *Huygens* ſuppoſes the thickneſs of the ring to be about 600 *German* miles, and as he makes the diameter of *Saturn* equal to 15 diameters of our Earth, which is about 2000 *German* miles over, this thickneſs of the ring muſt by his eſtimate be abont $\frac{1}{10}$ part of the diameter of *Saturn*.

Now the apparent diameter of *Saturn*, by the obſervations of the late accurate Aſtronomer, Mr. *James Pound*, is at his mean diſtance from the earth, 18″; therefore at his leaſt diſtance it muſt be 20″, and conſequently the greateſt apparent thickneſs of the ring muſt be eſtimated at $\frac{1}{50}$ part of 20″, when the earth is in the plane of the ring.

And if the teleſcope uſed by Mr. *Huygens* in theſe obſervations did, as he ſuppoſes, magnify an hundred times, the apparent thickneſs of the ring through that teleſcope muſt have been at the moſt $\frac{20}{50}''$ × 100, that is 40″.

Now we conceive, that in ſuch a compound body as *Saturn* appearing under an angle of 20″ × 100, or of 33′, that is, about the ſize of the full Moon, with two *brachia* proceeding from him of 40″ in apparent breadth, thoſe *brachia* cannot be very diſtinguiſhable, even though they were to reflect the light as ſtrongly as the body of the planet; much leſs, if they are not ſo luminous as the planet. And leſs ſtill can thoſe *brachia* be viſible, if when the Sun is in the plane of the ring, the Earth be in quadrature with him and *Saturn*, in which caſe the edge of the ring, being obliquely ſeen, will ſubtend an angle at the eye leſs than 40″ thro' the teleſcope.

But if the breadth of this edge be much leſs than $\frac{1}{10}$ part of *Saturn*'s diameter, as will appear probable from what follows; then it muſt have ſubtended a much leſs angle in Mr. *Huygens*'s teleſcope than 40″, ſo that the *brachia* muſt undoubtedly be inviſible, and *Saturn* ought to appear round, as he is found to do by obſervation.

192. It may be here objected, that in the year 1656, Mr. [c] *Huygens* ſaw the projection of this edge as a dark liſt upon the body of *Saturn*, at a time when the Earth and Sun were both nearly in the plane of the ring; and conſequently that the thickneſs of the ring is not ſo ſmall as to render it inviſible.

We anſwer, there is reaſon to ſuſpect this illuſtrious Aſtronomer was miſtaken, not in the obſervation itſelf, but in the judgment he made upon that obſervation.

That he ſaw a dark liſt traverſing the middle of the planet, is not to be doubted: but as a like dark liſt, or belt, has ſince been diſcovered about the middle of *Saturn*, at ſuch time as the Earth is out of the plane of the ring, ſo that the ring is plainly viſible, and the middle of *Saturn* appears within the eliplſe; we ſuſpect that this appearance, or ſuch part of it as was moſt diſtinguiſhable thro' his teleſcope of 23 feet, either alone, or jointly with the darkiſh line ariſing from the projection of the edge of the ring upon the body of *Saturn*, might be miſtaken for the

[a] Syſtem. Saturn. p. 61. [b] Coſmotheoros. p. 110. [c] Syſtem. Saturn. p. 16, 17, 61, 62.

projection

projection of that edge alone; which muſt cauſe him to conclude the thickneſs of the ring much greater than it really is.

193. And when the Earth and Sun were not exactly in the plane of the ring, but one was a little elevated above it, and the other a little depreſſed below it, or both were either a little elevated above it, or a little depreſſed below it, and yet ſo little that the plane of the ring was very faintly illuminated by the Sun; in all theſe caſes that plane of the ring, which was expoſed to the eye of the obſerver, would be projected upon the body of *Saturn*, as a narrow dark liſt, contiguous to the narrow dark liſt formed by the projection of the outer edge of the ring. And theſe two narrow dark liſts muſt appear as one, and might be taken by Mr. *Huygens* for the projection of the edge alone, which conſequently muſt have been reckoned much thicker than it really is. To which I muſt add, that when the Earth and Sun were on oppoſite ſides of the plane of the ring, or when both being on the ſame ſide, the Earth was a little more, or a little leſs out of that plane than the Sun, in both theſe caſes the eye might perceive a ſmall part of the ſhade caſt by the ring upon the body of the planet, which being contiguous to the other two narrow dark liſts would appear as one with them, and muſt increaſe the error.

194. But it will be farther objected to me, that when *Saturn* appears with *brachia*, or even with *anſæ*, in which caſe the ſame ſide of the ring is turned to the Earth, as is illuminated by the Sun, yet ſuch a dark liſt has been obſerved traverſing the body of *Saturn* ſometimes on the [a] northern part, and ſometimes on the [b] ſouthern, but always in ſuch a poſition as anſwers to the projection of the ring upon the body of the planet, and conſequently ariſing only from that projection.

I anſwer, that the outer part of the plane of the ring, which by Art. 190, in all probability is equally dark with the edge of the ring, will be projected upon the body of *Saturn*, contiguous to the projection of that edge, and together with it will form the appearance of one dark liſt.

To which we may add, that when the eye is any thing leſs elevated than the Sun above the plane of the ring, ſome part of the ſhadow muſt appear contiguous to the two projections abovementioned, and muſt increaſe the breadth of the liſt.

Conſequently, what was obſerved in this caſe by Mr. *Huygens*, was not the projection of the edge alone, but the united projection of that edge and of the outer and darker part of the plane of the ring, to which ſometimes a part of the ſhadow might be added: So that the ring may be much leſs in thickneſs than $\frac{1}{50}$ part of *Saturn*'s diameter, and might ſubtend a much leſs angle than 40″, when magnified an hundred times through the teleſcope.

195. *Galileo*, in his *Nuncius Sidereus*, propoſes an objection made by a great number of learned Perſons, againſt his new diſcovery, that the ſurface of the Moon was not even, but full of high mountains in all the brighter parts of it. Moon's edge why even, and not ragged.

If, ſay they, the limb of the Moon, which is generally very bright, be full of mountains, why does it appear even and ſmooth thro' the teleſcope, and not ragged like the teeth of a ſaw, or a wheel in clock-work?

To which that moſt accute and ingenious Philoſopher replies, that one range of hills only upon the very edge of the Moon might create ſuch an appearance as they ſuppoſed; but that ſeveral ranges of hills being placed one behind another, the tops of the nearer hills muſt be projected by the eye into the apertures or vallies between the hills of the outer ranges, and conſequently muſt cauſe the limb of the Moon to appear even, as it is found by obſervation.

196. To this it may be added, that if there were only one range of hills on the edge of the Moon, and the height of thoſe hills were about $\frac{1}{1000}$ part of the Moon's diameter, as he ſuppoſes in another place; then, the Moon's apparent diameter being about 30′, or 32′, the perpendicular height of thoſe hills, which alone is here to be conſidered, would be preſented to the naked eye under an angle of $\frac{3}{100}$ or $\frac{32}{1000}$ of a minute, and thro' his teleſcope magnifying about 30 times, would be preſented under an angle of $\frac{9}{10}$ or $\frac{96}{100}$ of a minute, which in ſuch a compound object could by art. 173, hardly be diſcerned. Much leſs could the ſmaller inequalities,

[a] Syſt. Saturn. p. 10, 11. [b] Ibid. p. 18, 21, 24.

which muſt be formed by ſeveral ranges of hills, be viſible thro' his or a much longer teleſcope.

Circle of diſſipation not uniformly luminous.

197. In accounting for the *phænomena* of indiſtinct Viſion above recited, we have made uſe of only this ſingle principle, that the rays of a pencil are not accurately collected into a point upon the *retina*, but occupy a circular ſpace thereon, which we have called the *circle of diſſipation*. This circle we have hitherto treated of, as if it were uniformly luminous, or as if the rays of the pencil were equally and uniformly ſpread over the whole circle. But in reality the fact is otherwiſe: the rays are not evenly diſpoſed all over this circle, but are denſer in ſome parts of it than in others. And though in many caſes, and indeed in moſt of the *phænomena* above related, this inequality of the denſity of the rays is not very conſiderable, nor occaſions any great change in the appearance, yet there are ſome few of them that will be remarkably affected by it; and there are ſome other very uncommon and ſurprizing appearances, that are not to be accounted for from the common laws of Opticks, and depend wholly upon this inequality of the rays in different parts of the *circle of diſſipation*.

A line indiſtinctly ſeen appears double.

198. Monſ. *De la Hire* * takes notice, that to ſhortſighted perſons, narrow objects at a diſtance, as the black lines for the hours upon a white ground in a large ſun-dial, appear double. This *phænomenon*, he obſerves, is one of the moſt difficult to account for; and finding no way of ſolving it by the common and known make of the eye, he is reduced to ſuppoſe, that in perſons, who ſee this appearance, the hinder ſurface of the cryſtalline humour is made after a peculiar manner, ſo that a perpendicular ſection thro' it reſembles the conchoide of *Nicomedes*.

But this is certainly a very hard ſuppoſition, and does not appear to have any foundation in nature, no cryſtalline of ſuch a make having ever yet been obſerved.

And if we ſhould even allow of this licence, which is ſo often taken by ingenious men, of framing an *hypotheſis* intirely out of their own brain, and without the leaſt appearance of foundation in the nature of things, in order to account for ſuch particular *phænomena* as they want to ſolve; yet this ſuppoſition of Monſ. *De la Hire* will be found utterly inſufficient to anſwer the different caſes of this *phænomenon*.

199. For a long narrow object, as one of the abovementioned lines upon a ſun-dial, or a ſign-iron, or a pole on the top of a houſe or ſcaffolding, or a narrow diſtant ſpire, when ſeen againſt the ſky-light, or the top of a broader ſpire, or a narrow interval between two chimnies, or one horn of the new Moon, or the whole illuminated part of the new Moon when very narrow, will appear double not only to ſhortſighted perſons, but to ſuch alſo as are longſighted, provided they apply a convex glaſs to the eye, ſo as to put themſelves into the caſe of a perſon naturally ſhortſighted. Conſequently this appearance is not occaſioned by any particular and unuſual form of the cryſtalline in ſhortſighted perſons.

Appears triple, quadruple, &c.

200. But theſe objects will not only appear double, which is the caſe Monſ. *De la Hire* wanted to account for. They will ſometimes appear triple, quadruple, quintuple, *&c.*

201. And the ſame appearances will occur, not only when the object is at too great a diſtance for *Diſtinct Viſion*, but alſo when it is too near for *Diſtinct Viſion*. Let any perſon whatſoever, whether ſhortſighted or longſighted it matters not, look upon a narrow black line drawn upon white paper, and held too near his eye to ſee the line diſtinctly, and he will ſometimes ſee the appearance of two, ſometimes of three or more dark lines, ſeparated by whitiſh lines.

202. This tryal will ſucceed ſtill better, if inſtead of a black line upon paper, a fine needle be held very near the eye, and viewed againſt the ſky-light, againſt a window, or againſt white paper ſtrongly illuminated. For the needle will appear as two, three, four, five, or more different needles, eſpecially towards the point.

203. But the beſt way I have met with of trying this experiment, is as follows. Take a parallel rule, and opening it to a ſmall aperture, hold it directly before your eye, ſo as to look at the ſky-light thro' the aperture. And let the diſtance you hold it at, at firſt, be the neareſt diſtance at which you can ſee it diſtinctly. At that diſtance the aperture will appear as one luminous line. But if you

* Traitè des differens accidens de la vuë. pag. 243.

bring

bring the parallel rule nearer to your eye, the aperture will appear double, or as two luminous lines, with a dark line between them. And accordingly as you vary the aperture, or the diſtance from your eye, you will ſee the appearance not only of two, three, four, five, *&c.* luminous and dark parallel lines, alternately, but of a greater number than you can count, eſpecially when you look thro' the aperture at the flame of a candle.

204. The parallel rule opened to a ſmall aperture affords the beſt means of trying the experiment beyond the bounds of *Diſtinct Viſion* likewiſe, by ſetting it in a window againſt the ſky-light, and viewing it thro' a convex glaſs, if you are longſighted, or with the bare eye, if ſhortſighted. For then you will have all the ſame appearances as in the preceding article.

205. Likewiſe when a broad luminous or white body is ſeen againſt a dark ground, as a piece of white paper, the tallow or wax in a candle next the flame, or the lower part of the flame itſelf, or when the ſky-light is looked at contiguous to the dark edge of a building, and the object is either too near or too far off for *Diſtinct Viſion*, the apparent edge of the luminous body will be bounded by a dark line, without which will be a luminous line parallel to it; and ſometimes there will appear two or more ſuch dark and luminous lines alternately, like ſo many fringes on the outſide of the luminous body.

206. And when a dark body is ſeen againſt a light ground, as the edge of a building againſt the ſky-light, or a flat rule againſt white paper, or againſt the ſky-light, and the object is too near or too remote to be diſtinctly ſeen, the dark body will appear fringed with luminous and dark edges alternately. And all thoſe appearances may be made to happen to any eye, by uſing a concave or convex glaſs as circumſtances ſhall require. But the experiment will ſucceed beſt, when the light of the luminous body, or that againſt which the dark body is ſeen, is neither very ſtrong, nor very weak.

207. As, I ſuppoſe, it will readily be allowed me, that the appearances mentioned in the eight preceding articles, are only different caſes, or varieties, of one and the ſame *phænomenon* diverſified by different circumſtances, and muſt ariſe from one and the ſame cauſe; and that the *hypotheſis* of Monſ. *De la Hire* is utterly inſufficient for their ſolution, ſince there is no eye whatſoever to which they may not all be made to happen, at leaſt by means of a convex glaſs for one, or concave glaſs for another, I ſhall proceed to lay down what I apprehend to be the cauſe of them, and that not an imaginary or fictitious cauſe, but ſuch an one whoſe exiſtence has been demonſtrably proved, long before thoſe appearances were taken notice of.

Cauſed by the Fits of eaſy refraction and reflexion of light,

208. Sir * ISAAC NEWTON has demonſtrated, that the rays of light are not, in all parts of their progreſs, in the ſame diſpoſition to be tranſmitted from one tranſparent medium into another, but that ſometimes a ray, which is tranſmitted thro' the ſurface of the ſecond medium, would be reflected back from that ſame ſurface, if the ray had a very little farther to go before it fell upon that ſurface. This change of diſpoſition in the rays of light to be either tranſmitted by refraction, or to be reflected by the ſurface of a tranſparent medium, he calls their *Fits of eaſy Refraction* and *Fits of eaſy Reflexion:* and he proves, that theſe Fits ſucceed each other alternately at very ſmall intervals in the progreſs of the rays.

209. Thus if *A*, Fig. 58, be a luminous point, or center of a pencil of rays, which iſſuing from the point *A* fall upon *BaD* a refracting ſurface interceding two pellucid mediums of different denſity; and the ray *Aa*, perpendicular to that ſurface, be in a *Fit of eaſy refraction*, when it arrives at the point *a*; it will paſs on and be tranſmitted thro' the medium *BDF*. And if the ray *Ab*, whoſe paſſage from the point *A* to *b* is a little longer than that of the ray *Aa* from *A* to *a*, be in a *Fit of eaſy Reflexion*, when it arrives at the point *b*; it will not be tranſmitted thro' the medium *BDF*, but will be reflected back again into the medium *BDA*. And if we ſuppoſe *Ab* to be the neareſt ray to *Aa*, that is in a *Fit of eaſy reflexion*; and *Ac* the neareſt ray to *Ab*, which is in a *Fit of eaſy refraction*; and *Ad* the next ray which is in a *Fit of eaſy reflexion*; and *Ae* the next ray which is in a *Fit of eaſy refraction*; and *Af* the next ray which is in a *Fit of eaſy re-*

* Opt. Book II.

flexion:

flexion: Then all the rays between *a* and *b* will be tranſmitted thro' the medium *BDF*; all the rays between *b* and *c* will be reflected back; all the rays between *c* and *d* will be tranſmitted; all the rays between *d* and *e* will be reflected back; and all the rays between *e* and *f* will be tranſmitted. And thus we may imagine a great number of parcels of rays, which will be tranſmitted or reflected alternately by the ſurface *B a D*.

210. Now if all the parcels of rays tranſmitted thro' the ſurface *B a D* be accurately collected into a point, or focus, there will no other conſequence ariſe from the other parcels of rays having been reflected backwards, than that this focus will be leſs luminous than it would have been, had every parcel of rays been tranſmitted to it.

211. But if the parcels of tranſmitted rays be received upon a plane *MN*, before they arrive at their focus, *F*, this plane will be diſtinguiſhed alternately into luminous and dark ſpaces, luminous where the parcels of tranſmitted rays fall upon it, and dark where the other parcels of rays ought to have fallen, had they not been reflected backwards at the ſurface *BaD*, as is manifeſt from Fig. 58 and 59, in which laſt the middle circle is ſurrounded by ſeveral rings luminous and dark alternately. And it is to be remarked, that thoſe parcels of rays which fall upon the right hand ſide of the ſurface *BaD*, do likewiſe fall upon the right hand ſide of the plane *MN*, and conſtitute the right hand ſide of the rings.

212. If the tranſmitted rays meet with the plane *mn*, Fig. 60, after they have paſſed the focus, the effect will be juſt the ſame as in the preceding article, except only that the parcels of rays, which fall upon the right hand ſide of the ſurface *BaD*, will now, after decuſſating in the focus, fall upon the left ſide of the plane *mn*, and will conſtitute the left ſide of the rings in Fig. 61, as is manifeſt from inſpection of the figures.

213. Here it is to be remarked, that the middle part of the image, in the four laſt figures, will not always be luminous as is there repreſented; but will ſometimes be dark, as in Fig. 62, that is, when the midddlemoſt rays about *Aa* are in a *Fit of eaſy reflexion* upon their arrival at the ſurface *BaD*.

214. As theſe effects of the *Fits of eaſy refraction* and *eaſy reflexion* of the rays of light muſt happen at all refracting ſurfaces whatſoever, it is manifeſt that, when a pencil of light falls upon the *cornea*, ſome parcels of its rays will be tranſmitted into the eye, while other parcels are reflected back into the air; and conſequently that the picture of a luminous point, if it were received upon a plane placed before the cryſtalline, would conſiſt of a middle circle, ſurrounded with rings dark and luminous alternately, as in Fig. 59, 61, 62.

215. But the tranſmitted rays, which conſtitute the luminous rings, will, upon their arrival at the anterior ſurface of the cryſtalline, be ſome of them in *Fits of eaſy refraction*, and ſome of them in *Fits of eaſy reflexion*, that is, ſome of them will be tranſmitted, and others will be reflected back. The conſequence of this will be, that the luminous rings ariſing from the refraction at the the *cornea*, will now be ſubdivided into narrower rings dark and luminous alternately; and ſome of theſe narrower lucid rings will fall upon the ſpaces poſſeſſed by the dark rings cauſed by the reflexion of the *cornea*, and will ſubdivide them likewiſe into dark and lucid rings alternately; and ſuch would be the picture of a lucid point, when received upon a plane within the cryſtalline.

216. But when the tranſmitted rays arrive at the hinder ſurface of the cryſtalline, thoſe lucid and dark rings will again be ſubdivided, as at the anterior ſurface, by which means the dark or luminous rings will ſometimes be rendred ſmaller and more in number than before, and ſometimes by the junction of new dark rings to the former dark rings, or of new lucid rings to the former lucid rings, thoſe rings will be rendred larger and reduced to a leſſer number.

Such therefore muſt be the image of a lucid point upon the *retina*, when that point is either too near, or too remote to be ſeen by *Diſtinct Viſion*.

217. As the interval between a *Fit of eaſy tranſmiſſion* and the next *Fit of eaſy reflexion* is exceeding ſmall, being in air but about $\frac{1}{1800000}$ part of an inch, and in water but about the $\frac{1}{2400000}$ part of an inch, it is obvious that the leaſt motion of the body, or of the eye, or of any of the parts within it, muſt make a change in theſe rings, making dark parts lucid, and lucid parts dark, whence muſt ariſe an infinite variety in the number, magnitude and order of the rings.

218. From

Appearance of a lucid point by indiſtinct Vicn.

218. From what has been ſaid, it plainly appears, that when a lucid point is too far off, or too near, for *Perfect Viſion*, its image upon the *retina*, will be diverſified with light and dark rings alternately, after the manner repreſented in Fig. 59, 61, 62, or, the *circle of diſſipation*, which we have hitherto conſidered as uniformly luminous, will not be ſo, but will be alternately divided into light and dark rings, ſometimes more, ſometimes fewer, ſometimes broader, ſometimes narrower.

219. When any parcel of theſe rings alternately light and dark, are ſo very narrow and cloſe together, as that they cannot ſingly be perceived, they will all appear as one ring, which our ſenſe will judge of as light or dark, according as the lucid rings in that parcel for number and breadth either exceed or fall ſhort of the rings adjacent.

220. It may be proved, that when rays iſſue from ſo diſtant a point as to unite in the eye before the *retina*, their denſity in the ſeveral lucid rings decreaſes all the way from the center to the circumference, and conſequently that the outer lucid rings are leſs luminous than the inner ones.

Therefore the middle part of the image of a ſtar will be the ſtrongeſt, and conſequently may be viſible when ſurrounded with ſuch a light as effaces the weaker appearance of the outermoſt lucid rings.

Stars appear ſmalleſt by day-light.

For this reaſon ſtars muſt appear ſmaller while day-light continues, than they do at night. For the day-light effacing their outer lucid rings, deprives them of their *crines ſeu fulgores adſcititii*, as *Galileo* [a] calls them, or the *radii adventitii* as they are termed by Mr. *Horrox*. [b]

So likewiſe a ſtar appearing within the Moon's edge, muſt have the outer part of its image effaced by the ſtronger light of the Moon, and conſequently the ſtar thus leſſened may appear at ſome diſtance within the limb, as was obſerved by Monſ. *De la Hire*, Art. 69.

Why ſtars twinkle.

221. If by Art. 213, the middle part of the image of a ſtar be changed from light to dark, and the adjacent ring be at the ſame time changed from dark to light, as muſt happen from the leaſt motion of the eye towards or from the ſtar, this will occaſion ſuch an appearance as we call the ſcintillation or twinkling of the ſtars.

222. And if the axis of the eye do not continue directly and ſteddily pointed at the ſtar, but have the leaſt nutation to the right or left, or upwards or downwards, this will diſturb the uniform appearance, by cauſing the light to project out from the lucid rings into the darſt rings, whereby the continuity of the rings will be broken. And if theſe nutations to different parts do ſwiftly ſucceed one another, this will make the light ſeem to project out different ways at the ſame time, that is, it will occaſion what we call the radiation of a ſtar. Why radiate.

223. In our former articles about the fixed ſtars from Art. 61 to 69, a correction is to be made by leſſening the *faint falſe image*, probably about one half, or at leaſt one third for moſt eyes; excepting only the caſe where two ſtars are near enough for part of their *faint falſe images* to coincide: for there, an outer lucid ring of one ſtar, tho' too weak to affect the eye ſingly, yet by coinciding with the oppoſite lucid ring of the other ſtar, may become ſtrong enough to be perceived.

224. When light paſſes out of one tranſparent medium into another of a different refraction, and the rays are partly tranſmitted and partly reflected back at the ſurface of the ſecond medium; at incidences near the perpendicular the quantity of the tranſmitted rays will much exceed the quantity of reflected ones. Proportion of refracted and reflected rays near the perpendicular.

Upon a table I placed two candles of equal height and burning with equal brightneſs, at equal diſtances from a ſheet of white paper: and ſet a book upright between one of the candles and the paper, in ſuch manner as to cut off the light of that candle from one half of the paper, while the other half was illuminated by both candles. By this means one half of the paper was nearly twice as luminous as the other half. Then I took a flat, thin piece of clear glaſs, and held it in ſuch manner, as to look perpendicularly thro' it at the more luminous half of the paper with one eye, the other being ſhut, and to ſee the darker half of the paper with the ſame eye beſide the glaſs. In doing thus, I obſerved that the more luminous half of the paper, ſeen perpendicularly thro' the glaſs, appeared much brighter than the other half of the paper ſeen with the bare eye on the ſide of the glaſs. Conſequently leſs than half the

[a] Nuntius Sidereus, pag. 23. [b] Venus in ſole viſa,

light

light was reflected back from the two ſurfaces of the glaſs, and more than half was tranſmitted thro' thoſe two ſurfaces to the eye.

Taking then ſuch another piece of glaſs, I held it parallel to the firſt piece, ſo as to look thro' them both at the more luminous half of the paper, which ſtill appeared conſiderably whiter than the other half of the paper ſeen with the bare eye beſide the glaſſes; ſo that leſs than half the light was reflected back from the two glaſſes.

In looking thro' three ſuch glaſſes in the ſame manner, the more luminous half of the paper had now ſomething of a greeniſh caſt, the glaſſes not being of a perfect clearneſs; but ſtill that half appeared full as light, if not lighter, than the other half of the paper ſeen with the bare eye beſide the glaſſes. Conſequently not more than half the light was reflected back from the three glaſſes put together.

Now if we ſuppoſe a certain part of the incident light to be reflected back from the firſt ſurface of the firſt glaſs, and the remainder of the light to be tranſmitted thro' that firſt ſurface and to fall upon the ſecond ſurface; and a proportionable part of that remaining light to be reflected back at the ſecond ſurface, and the reſt to be tranſmitted to the firſt ſurface of the ſecond glaſs; and a proportionable part of the remaining light to be reflected back at its incidence upon that ſurface, and upon every one of the three following ſurfaces of the glaſſes; and one half of the whole light to be reflected back from the ſix ſurfaces put together, and the remaining half to be tranſmitted thro' the laſt ſurface to the eye: Then calling the whole light 1, and the part reflected back at the firſt ſurface, x, we ſhall have $x = \frac{\sqrt[6]{2}-1}{\sqrt[6]{2}} = 0{,}1091$,

that is, about $\frac{1}{10}\frac{1}{0}$ of the incident light is reflected back at the firſt ſurface, and about $\frac{8}{10}\frac{9}{0}$ is tranſmitted thro' that ſurface.

At oblique incidences.

225. As the incident rays deviate more and more from the perpendicular, the proportion between the quantity of reflected rays and the quantity of rays tranſmitted will increaſe more and more: but leſs than half the quantity of the incident rays will be reflected, and more than half will be tranſmitted thro' ſubſtances tolerably clear, unleſs the angle of incidence be very great.

In the experiment of the laſt article, if the ſingle glaſs, inſtead of being perpendicular to the axis of the eye, be gradually more and more inclined to that axis, the brighter half of the paper ſeen thro' the glaſs will gradually appear leſs and leſs luminous, and will more and more approach to the ſame degree of whiteneſs with the other half of the paper, which is illuminated with one candle only and viewed with the bare eye. And at ſome certain degree of obliquity the one half of the paper ſeen thro' the glaſs, and the other half of the paper ſeen with the bare eye will appear equally luminous: But even then only about $\frac{2}{1}\frac{9}{0}\frac{}{0}$ of the incident light is reflected back from the firſt ſurface, x being equal to $\frac{\sqrt{2}-1}{\sqrt{2}}$; and about $\frac{2}{1}\frac{1}{0}\frac{}{0}$ is reflected back from the ſecond ſurface. And this, I think, does not happen, till the angle of incidence is about 70 or 80 degrees.

By this article the lucid parts near the center in Fig. 59, 61, 62, will bear a greater proportion to their contiguous dark parts, than the lucid parts near the circumference to the dark parts contiguous to them.

Obſervations on the Fits of eaſy refraction and reflexion.

226. Here it may be of uſe, not only for the better underſtanding of what is to follow, but for the fuller comprehenſion of this curious doctrine of the *Fits of eaſy refraction* and *eaſy reflexion*, to lay down two or three obſervations, which though not explicitely delivered by Sir ISAAC NEWTON, yet may eaſily be deduced from his Theory, eſpecially when compared with the two preceding articles.

1. When a ray of light is ſaid to be in the *Fit of eaſy refraction*, it is not meant, that the ray muſt neceſſarily be tranſmitted thro' every pellucid medium whatſoever, and at any whatſoever obliquity of incidence upon the ſurface of that medium; but only that the ray is more eaſily tranſmitted, and more difficulty reflected, when in that Fit, than when it is in the *Fit of eaſy reflexion*. And when a ray is ſaid to be in the *Fit of eaſy reflexion*, it is not meant, that the ray muſt neceſſarily be reflected back from every pellucid medium whatſoever, and at any whatſoever obliquity of incidence upon the ſurface of that medium; but only that the ray is more eaſily reflected, and more difficulty tranſmitted, when in that Fit, than in the

Fit of eaſy refraction. For this reaſon theſe Fits are not called, abſolutely, Fits of refraction, and Fits of reflexion, but Fits of *eaſy* refraction, and Fits of *eaſy* reflexion.

2. During the time, or during the part of the progreſs of a ray of light, that the ray is in the *Fit of eaſy refraction*, or in the *Fit of eaſy reflexion*, it is not conſtantly in juſt the ſame diſpoſition to be refracted, or to be reflected: But there is one inſtant of time, or one point in the progreſs of the ray, at which it is in the middle of the *Fit of eaſy refraction*, and is then the moſt diſpoſed to be refracted, and the leaſt diſpoſed to be reflected; and from that inſtant, or that point, it becomes gradually leſs diſpoſed to be refracted, and more diſpoſed to be reflected, till another inſtant, or point, at which the ray is in the middle of the *Fit of eaſy reflexion*, and is then the leaſt diſpoſed to be refracted, and the moſt diſpoſed to be reflected; and from this ſecond inſtant, or point, it becomes gradually more diſpoſed to be refracted, and leſs diſpoſed to be reflected, till it arrives at a third inſtant, or point, like the firſt, when the ray is again in the middle of the *Fit of eaſy refraction*, or is the moſt diſpoſed to be refracted, and the leaſt diſpoſed to be reflected.

To explain this, let the right line *AAAA*, Fig. 63, repreſent part of the progreſs of a ray of light, in which line ſeveral points as *A*, *A*, *A*, *A*, are taken at equal diſtances from each other, and other points *E*, *E*, *E*, are taken at equal diſtances from each other, and from the points *A*,*A*,*A*,*A*; and let us ſuppoſe that in every one of the points *A*,*A*,*A*,*A*, the ray is moſt ſtrongly diſpoſed to be refracted, and in every one of the points *E*,*E*,*E*, it is moſt ſtrongly diſpoſed to be reflected; that is, in each point *A* let the ray be in the middle of the *Fit of eaſy refraction*, and in each point *E* let it be in the middle of the *Fit of eaſy reflexion.*

Then will the diſtance *AA* repreſent the interval between the middle points of two ſubſequent *Fits of eaſy refraction*, *EE* the interval between the middle points of two ſubſequent *Fits of eaſy reflexion*, and *AE* the interval between the middle point of a *Fit of eaſy refraction* and the middle point of the ſubſequent *Fit of eaſy reflexion.*

And the diſpoſition of the ray to be refracted, being ſtrongeſt in the middle of the *Fit of eaſy refraction*, or in the point *A*, and weakeſt in the point *E*, will by degrees diminiſh all the way from *A* to *E*; and will again by the ſame degrees increaſe all the way from *E* to *A*. And the diſpoſition of the ray to be reflected, being ſtrongeſt in the middle of the *Fit of eaſy reflexion*, or in the point *E*, and weakeſt in the point *A*, will gradually diminiſh all the way from *E* to *A*, and will again gradually increaſe all the way from *A* to *E*.

3. That part of the progreſs, in which a ray is in the *Fit of eaſy refraction*, is longer or ſhorter, as the medium it falls upon has a leſs or greater refractive power, and its ſurface is leſs or more obliquely ſituated with regard to the progreſs of the ray: And that part of the progreſs, in which the ray is in the *Fit of eaſy reflexion*, is longer or ſhorter, as the medium the ray falls upon, has a greater or leſs refractive power, and its ſurface is more or leſs obliquely ſituated with regard to the progreſs of the ray.

In every interval *AE* let a point be taken as *a*, ſo that the ray falling upon a medium of a given refracting power, and with a given obliquity, in the point *a*, ſhall be tranſmitted thro' that medium; but would have been reflected back, had it fallen upon the medium in any point between *a* and *E*. Then will the ray be in the *Fit of eaſy refraction* with regard to that medium and that obliquity of incidence, throughout the ſpace *a A a*; and will be in the *Fit of eaſy reflexion* throughout the ſpace *aE a*; ſo that the beginning of the *Fit of eaſy refraction* will happen in *a*, the middle in *A*, and the end in the ſubſequent *a*; and the beginning of the *Fit of eaſy reflexion* will happen in this laſt point *a*, the middle in *E*, and the end in the following point *a*: or the interval *AE* will be ſo divided in the point *a*, that throughout the ſpace *Aa* the ray will be in the *Fit of eaſy refraction*, and throughout the ſpace *aE* it will be in the *Fit of eaſy reflexion.*

But if the refracting power of the medium be leſſened, or if its obliquity to the progreſs of the rays be leſſened, the ray will be refracted, tho' it meet the ſurface of the medium below the point *a*, nearer to *E*; or the point *a*, at which the *Fit of eaſy refraction* ends and the *Fit of eaſy reflexion* begins, muſt be removed nearer to *E*, that is, *Aa* will be longer, and *aE* will be ſhorter, or

a A a the part of the progreſs, in which the ray is in the *Fit of eaſy refraction*, will be longer, and *a E a* the part of the progreſs, in which the ray is in the *Fit of eaſy reflexion*, will be ſhorter.

And juſt the contrary will happen, if the refracting power of the medium, or its obliquity to the progreſs of the rays be increaſed, the point *a* then being removed nearer to *A*, whereby *A a* or *a A a* is made ſhorter, and *a E* or *a E a* is made longer.

Intervals of thoſe fits how divided.

227. If parallel rays, moving with an uniform velocity, fall upon a plane ſurface of a tranſparent medium with ſuch obliquity, that one half of the rays are tranſmitted thro' that ſurface, and the other half are reflected back; every ray muſt be in the *Fit of eaſy refraction* thro' one half of its progreſs, and in the *Fit of eaſy reflexion* thro' the other half of its progreſs.

For, ſince by the ſuppoſition the rays move with an uniform velocity, there muſt be as many rays in any one point of their reſpective intervals *A E* between the *Fits of eaſy refraction* and *reflexion*, as in any other point of the ſame intervals. Conſequently, the number of rays in the ſpace *A a* muſt be to the number of rays in the ſpace *a E*, as *A a* to *a E*. But ſince half the rays are refracted and half reflected, the number of rays in the ſpace *A a* is equal to the number of rays in the ſpace *a E*: therefore *A a* is equal to *a E*, that is, the rays are in the *Fit of eaſy refraction* through one half of their progreſs, and in the *Fit of eaſy reflexion* through the other half of their progreſs.

228. But if the obliquity of the plane ſurface be ſo much leſſened, as that three fourths of the incident rays are now tranſmitted thro' it, and only one quarter of them reflected, then every ray muſt be in the *Fit of eaſy refraction*, with regard to this obliquity of the plane ſurface, through three fourths of its progreſs, and in the *Fit of eaſy reflexion* through one quarter of its progreſs. That is, the interval *A E*, between the two points of eaſieſt refraction and eaſieſt reflexion, muſt be ſo divided in *a*, that *A a* may be three fourths of that interval, and *a E* one fourth.

229. And in general, if the interval *A E* repreſent the whole quantity of the incident light, *A a* the quantity of the refracted light, and *a E* the quantity of reflected light; *A a* will be that part of the interval, in which every ray is in the *Fit of eaſy refraction*; and *a E* that part of the interval, in which every ray is in the *Fit of eaſy reflexion*.

In theſe three articles we have ſuppoſed the rays of light to move with an uniform velocity: but if they move more ſwiftly when in the *Fit of eaſy refraction*, and more ſlowly when in the *Fit of eaſy reflexion*, as Sir ISAAC NEWTON ſeems to have thought; we muſt then imagine the line *A A A A*, inſtead of the progreſs of a ray of light, to repreſent the time of that progreſs, *A* the inſtant of time when the ray is moſt ſtrongly diſpoſed to be refracted, *E* the inſtant when it is moſt ſtrongly diſpoſed to be reflected, *A E* the interval of time between thoſe two inſtants, and *a* the inſtant when the ray paſſes out of the *Fit of eaſy refraction* into the *Fit of eaſy reflexion*, and the conſequences will be juſt the ſame.

230. It follows from the preceding article, jointly with art. 224, that, in the caſe of perpendicular, or nearly perpendicular incidence of light from air upon glaſs, or from glaſs upon air, a ray of light is in the *Fit of eaſy refraction* thro' $\frac{89}{100}$ of the interval *A E*, and in the *Fit of eaſy reflexion* thro' no more than $\frac{11}{100}$ of that interval.

231. If two mediums differ leſs in their refractive power than air and glaſs, a ray of light, in paſſing out of one of thoſe mediums into the other almoſt perpendicularly, will be in the *Fit of eaſy refraction* more than $\frac{89}{100}$ of the interval *A E*, and in the *Fit of eaſy reflexion* leſs than $\frac{11}{100}$ of that interval.

232. Sir ISAAC NEWTON has [a] obſerved, that thoſe ſurfaces of tranſparent bodies reflect the greateſt quantity of light, which have the greateſt refracting power: And till this matter ſhall be more accurately ſettled by experiment, it may probably be ſuppoſed, that the quantity of light reflected back at the ſurface of any tranſparent body is proportional to the refracting power of that body, or to the ſquare of [b] *B R*.

233. Hence we may infer, that the ſquare of *B R*, or the refracting power of glaſs being to that of water, as [c] 1,4450 to 0,7845, and the quantity of light reflected back at a per-

[a] Opticks Book II. Part III. Prop. I. [b] Ibid. Prop. X. [c] Ibid.

pendicular

pendicular incidence upon glaſs being [a] 0,1091, the quantity of light reflected back at a perpendicular incidence upon water will be 0,0592, or nearly $\frac{6}{100}$ of the incident light. Conſequently, a ray of light falling perpendicularly from air upon water, or from water upon air, will be in the *Fit of eaſy reflexion* thro' only $\frac{6}{1000}$ of the interval AE, Fig. 63, and in the *Fit of eaſy refraction* thro' $\frac{994}{1000}$ of that interval.

234. Likewiſe, the refraction of the aqueous humour of the eye, or of the *cornea*, being that of 27 to 20, by art. 110, and conſequently its refracting power, or the ſquare of BR being 0,8225, the quantity of light reflected back at a perpendicular incidence upon the *cornea* will be 0,064 of the incident light.

235. Alſo, the refraction out of the aqueous humour into the cryſtalline being that of 13 to 12, by art. 111, and conſequently the refracting power of the cryſtalline humour in this caſe being 0,1736, the quantity of light reflected at a perpendicular incidence from the aqueous humour upon the cryſtalline will be 0,0135 of the incident light. And the ſame proportion of the incident light will be reflected at the hinder ſurface of the cryſtalline, the refraction out of that humour into the vitreous being the ſame as that out of the aqueous humour into the cryſtalline.

236. Hence it follows, that if the ſurface of the *cornea* and the two ſurfaces of the cryſtalline were not ſpherical, but plane, like the piece of glaſs uſed in the preceding [b] experiment, then in looking directly at any object, $\frac{64}{1000}$ parts of the incident light would be reflected back at the *cornea*, and the remainder would be tranſmitted to the anterior ſurface of the cryſtalline; and of this remainder $\frac{135}{1000}$ parts would be reflected back at that firſt ſurface, and the remaining light would be tranſmitted to the ſecond ſurface; where again $\frac{135}{1000}$ parts of the remaining light would be reflected back, and the *reſiduum* would be tranſmitted to the *retina*. So that if the whole light incident upon the *cornea* be called 1, the quantity reflected back at the three ſurfaces put together would be 0,0891, or $\frac{1}{11}$ part nearly, and the quantity tranſmitted to the *retina* would be 0,9109, or about $\frac{10}{11}$.

But as theſe three ſurfaces are not plane, but ſpherical, on which account the greateſt part of the incident rays will fall upon them with ſomewhat greater obliquity, and conſequently a part ſomewhat greater will be reflected, we ſhall not be far from the truth, if we ſuppoſe $\frac{1}{10}$ part to be reflected back at all the three ſurfaces put together, and $\frac{9}{10}$ to be tranſmitted to the *retina*.

How much light loſt in viſion by thoſe fits.

237. From the preceding article jointly with art. 210, we may probably collect, that in looking at an object between the limits of *Perfect Viſion*, it appears leſs luminous by about $\frac{1}{10}$ part, than it would do were all the light incident upon the *cornea* tranſmitted to the *retina*.

238. And much the ſame thing happens in looking at a broad object, although too near or too far off for *Perfect Viſion*. For then every pencil will likewiſe loſe $\frac{1}{10}$ part of its light by the three reflexions above-mentioned, and the remaining $\frac{9}{10}$ of the light will be ſcattered into the *circle of diſſipation*: But every pencil within the *falſe image*, art. 18, 72, will receive as much light from the adjoining pencils, as it beſtows upon them by this diſſipation. Therefore the center of every pencil, or, every point of the *falſe image* of the object upon the *retina*, will have juſt as much light thrown upon it, as if the object had been ſeen by *Perfect Viſion*, that is, $\frac{9}{10}$ of the whole incident light.

Appearance of a lucid phyſical line by indiſtinct Viſion.

239. A lucid phyſical line upon a dark ground, when much too near, or much too far off for *Diſtinct Viſion*, will appear ſometimes as three, ſometimes as five, ſeven, or more lucid parallel lines, with two, four, ſix, or more dark parallel lines between them.

For a lucid point, when indiſtinctly ſeen, will by art. 211, 212, 213, exhibit the appearance of a circle ſurrounded with dark and lucid rings alternately, and this muſt happen to every phyſical point in the lucid line.

Let therefore AaA, Fig 64, be a lucid phyſical line, a ſome point in that line from which a pencil of rays flows to the *cornea*. And let a portion of rays in the middle of this pencil be in the *Fit of eaſy refraction* at their incidence upon the *cornea*, and be thereby tranſmitted to the anterior ſurface of the cryſtalline; where let them again be in the *Fit of eaſy refraction*, and be thereby tranſmit-

[a] Art. 224. [b] Ibid.

ted to the hinder ſurface of the cryſtalline. And at that hinder ſurface let the ſame middle portion of rays be again in the *Fit of eaſy refraction*, and let the middlemoſt ray of this portion, *Aa*, Fig. 58, be preciſely in the middle of the Fit, or in one of the points *A*, Fig. 63; and let the outermoſt ray, as *Ab*, or *Aβ*, Fig. 58, be at the end of the Fit, or in one of the points *a*, Fig. 63; and let this portion of rays be thereby tranſmitted to the *retina*, and form thereon the lucid circle *abβ*, Fig. 64. And let this lucid circle be ſurrounded, by art. 211, 212, with a dark ring, *bcγβ*, and this again with the lucid ring *cdδγ*, as repreſented in figure 64; and through the points *b*, *c*, *d*, *β*, *γ*, *δ*, draw the lines *BbB*, *CcC*, &c. parallel to the line *AaA*.

Then I ſay, inſtead of the one lucid line *AaA*, will appear three lucid lines *BBBB*, *CDDC*, and ΓΔΔΓ, ſeparated from one another by the darkiſh lines *BCCB*, and *BΓΓB*.

For if a like middle portion of rays, in every pencil flowing from the lucid line *AaA*, be ſuppoſed to be in the *Fit of eaſy refraction* at their incidence upon the *cornea*, and upon the anterior and hinder ſurface of the cryſtalline, and at this laſt ſurface the middlemoſt ray be preciſely in the middle of the Fit, and the outermoſt rays in the end of the Fit, ſo as that every point in the line *AaA* may form an image upon the *retina* exactly like that of the point *a*; then the lines *BbB*, *BβB*, muſt be tangents to all the lucid circles formed by thoſe points: Alſo, the lines *BbB*, *CcC*, on the one hand, and *BβB*, ΓγΓ, on the other, muſt be tangents to the firſt dark ring formed by each of thoſe points; and the lines *CcC*, *DdD* on the one hand, and ΓγΓ, ΔδΔ on the other, muſt be tangents to the firſt lucid ring formed by every one of thoſe points.

From which it follows, that, as the ſpace 1*b*1 1*β* 1, which is cut off from the image of the point *a*, by the tangents *BbB*, *BβB*, has more light in proportion to its magnitude, than the ſpace 1*b*1 2*c*2, which is cut off from the ſame image by the tangents *BbB*, *CcC*; the whole ſpace compriſed between the tangents *BbB*, *BβB*, will be more luminous than the whole ſpace compriſed between the tangents *BbB*, *CcC*, that is, the whole ſpace *BbBBβB* will appear as a lucid line, and the whole ſpace *BbBCcC* will appear as a dark line.

In like manner it will be found that the ſpaces *CcCDdD*, ΓγΓΔδΔ, ought to appear as lucid lines; and the ſpace *BβBΓγΓ* muſt appear as a dark line.

In this caſe therefore, inſtead of one phyſical lucid line *AaA*, we ſhall have the appearance of three lucid lines ſeparated by dark lines parallel to them; and if the lgiht of the point *a* be ſtrong enough to form more lucid rings than are here repreſented, and the breadth of thoſe lucid rings be not too little to be perceived, art. 174, we may have the appearance of 5, 7, or more lucid lines ſeparated by parallel dark lines.

240. But when the lucid line *AaA* is of any conſiderable length, it is not poſſible that in every pencil iſſuing from that line, things can happen exactly as we have ſuppoſed. Although a middle portion of rays in every pencil were to be tranſmitted through every one of the three ſurfaces abovementioned to the *retina*, and were to form thereon a lucid circle like *abβ*; yet the middlemoſt ray cannot in every pencil be preciſely in the middle of the *Fit of eaſy refraction*, at its incidence upon the hinder ſurface of the cryſtalline. In ſome pencils the middlemoſt ray will not be come to the middle of the Fit, in others it will be paſt the middle of the Fit, at its incidence upon that ſurface.

Alſo, this middle portion of rays, which we have ſuppoſed to be wholly tranſmitted thro' each of the three refracting ſurfaces, and thereby to form on the *retina* the lucid circle *abβ*, cannot in every pencil be ſo tranſmitted. In ſome pencils a part of the middle portion of rays muſt neceſſarily be in the *Fit of eaſy reflexion*, at its incidence upon one or other of thoſe ſurfaces, and conſequently cannot be tranſmitted to the *retina*.

We are therefore to conſider what alterations in the *phænomenon* will ariſe from theſe changes in our ſuppoſitions.

241. In thoſe pencils where the middle portion of rays is wholly tranſmitted to the *retina*, but the middlemoſt ray is not yet arrived to the middle of the *Fit of eaſy refraction*, at its incidence upon the hinder ſurface of the cryſtalline, it is manifeſt that the luminous circle *abβ*, will be inlarged and will ſwell farther outwards, and the dark and luminous

rings,

rings, together with the dark and luminous lines ariſing from them, will by this means be alſo carried farther outwards.

And in thoſe pencils where, at the time of this incidence, the middlemoſt ray is paſt the middle of the *Fit of eaſy refraction*, the luminous circle $ab\beta$ will be leſſened, and by this means the dark and luminous rings ſurrounding it, and the dark and luminous lines ariſing from thoſe rings, will be drawn farther inwards.

But as the number of pencils in the one caſe muſt be equal to the number of pencils in the other, theſe two contrary effects will very nearly balance each other, and conſequently the *phænomenon* will thereby undergo very little alteration, except only that the edges of the luminous and dark lines, where they entrench upon each other, will thereby be rendred ſomewhat leſs diſtinct than they would otherwiſe appear.

242. We come next to conſider thoſe pencils, where the middle portion of the rays, which we have ſuppoſed to form the luminous circle $ab\beta$, are not wholly tranſmitted to the *retina*; but ſome part of them, being in the *Fit of eaſy reflexion* at their incidence upon one or other of the three ſurfaces abovementioned, are reflected back and never arrive at the *retina*.

And here we are to remember that, by art. 236, hardly more than $\frac{1}{10}$ part of this middle portion of rays, taking all the pencils one with another, can be reflected back by the three ſurfaces put together.

Now in ſuch pencils, where this tenth part of the rays of the middle portion, were it not reflected back, muſt have fallen upon the middle part of the luminous circle, there muſt be a ſmall dark ſpot in the middle of the image, as *a*, Fig. 65; and the luminous circle will by this means be changed into a luminous ring, $b\beta$, whoſe breadth will be nearly equal to the radius of the luminous circle in Fig. 64. Conſequently, this luminous ring, and the luminous line *BBBB*, will extend farther outwards than in Fig. 64; and the dark and lucid rings, bc, cd, and the dark and lucid lines ariſing from thoſe rings will alſo be carried farther outwards, and will be rendred ſomewhat narrower.

And in ſuch pencils, where this tenth part of the rays of the middle portion, were it not reflected back, muſt have fallen upon the outer part of the luminous circle, that circle and the luminous line *BBBB* will thereby be leſſened, and the dark and lucid ring ſurrounding it, and the dark and lucid lines ariſing from thoſe rings, will be removed farther inwards, and will be rendred ſomething broader, as in Fig. 66.

But as the number of pencils in one of theſe caſes muſt be nearly equal to the number of pencils in the other; and the effect of the one is contrary to that of the other; the change in the *phænomenon* will be little more, than to cauſe ſome farther indiſtinctneſs in the edges of the luminous and dark lines.

243. In ſuch pencils where this tenth part of the middle portion of the rays is not all contiguous, but is divided into three or more disjoined parcels, which, had they not been reflected back at the three ſurfaces, muſt have fallen upon ſeparate ſpaces in the luminous circle, it is manifeſt, that the change in the *phænomenon* will be ſtill leſs than in the two preceding articles, only the luminous circle, and the lucid line ariſing from it, will be rendred ſomething leſs luminous. And the dark ring, and the dark lines ariſing from it, will be rendred ſomething leſs dark, in caſe the rays that tend thereto, be not all reflected, but ſome parts of them be tranſmitted.

244. This would be the caſe of a lucid phyſical line, ſuch as one might imagine to be compoſed by a great number of ſtars diſpoſed in a right line and contiguous to each other. The middle line would be always luminous, ſo that the number of lucid lines would always be odd, and the number of dark lines between them would always be even.

Appearance of a narrow lucid line by indiſtinct Viſion.

245. A lucid line upon a dark ground, of a ſmall ſenſible breadth, but whoſe *true image* upon the *retina* is not ſo broad as the *circle of diſſipation*, when too near, or too far off for *Diſtinct Viſion*, will ſometimes appear as two, ſometimes as three, four, five or more lucid parallel lines, with dark parallel lines between them.

Let *a*, Fig. 67, be a point in the middle of the breadth of the lucid line, and let *e* and *i* be the two extreme points in the breadth of the ſame line, and let the axis of the eye be directed to the point *a*.

Then will the axis of the pencil of rays flowing from the point *a*, be perpendicular to the *cornea* and to both ſurfaces of the cryſtalline humour, and the image of that point

point upon the *retina* will be agreeable to what is repreſented in Fig. 59, 61, or 62: And the phyſical line, which runs thro' the middle of the lucid line, and paſſes thro' the point *a*, will form an image upon the *retina* like that repreſented in Fig. 64.

But the axis of the pencil of rays flowing from the point *e*, will fall obliquely upon the *cornea*, and upon both ſurfaces of the cryſtalline humour. Conſequently, not only the center of the image of the point *e* will fall upon a different part of the *retina* from the center of the image of the point *a*, and within the *circle of diſſipation* of the point *a*; but the image of the point *e* will not be ſimilar to the image of the point *a*, ſo that ſome lucid parts of one will fall upon ſome dark parts of the other, and on the contrary. And if a phyſical line be conceived to paſs thro' *e*, parallel to the phyſical line which paſſes thro' *a*, ſome lucid parts of the image of one phyſical line will fall upon ſome dark parts of the image of the other. And what we have ſaid of the image of the phyſical line paſſing thro' *e*, will be likewiſe true of the image of the phyſical line paſſing thro' the point *i*.

And accordingly as the breadth of the lucid line is greater or leſs, or theſe two points *e* and *i* are more or leſs diſtant from the point *a*, the dark lines of the images of the phyſical lines paſſing thro' thoſe points, may either coincide in the middle of the *retina*, in which caſe there will appear a dark line in the middle of the image: Or, they may be contiguous only, or elſe leave a ſenſible ſpace between them, in either of which caſes a lucid line will appear in the middle.

And as the ſame reaſoning will extend to the other points on the inſide of *e* and *i*, it is manifeſt, that the appearance of the whole lucid line may conſiſt of more or fewer, narrower or broader, lucid and dark lines, as in art. 203, 204, with infinite variety.

Other appearances by indiſtinct Viſion.

246. From the ſame cauſe, and by a like way of reaſoning it will follow, that when a broad lucid ſurface, as *BBbb*, Fig. 68, is contiguous to a broad dark ſurface as *BBdd*, and is ſeen by indiſtinct viſion, the lucid ſurface will appear bordered with a dark line as *BB* Γ Γ, and that again with a lucid line beyond it; and ſometimes there will be two or more of theſe dark and lucid lines alternately, as was related in art. 205.

247. Or the broad dark ſurface *BBdd*, contiguous to a broad lucid ſurface *BBbb*, when ſeen by indiſtinct viſion, will appear narrower than when ſeen by *Diſtinct Viſion*, that is, from *BBdd* it will be reduced to Δ Δ *dd*, and the line Δ Δ will be the apparent edge of the dark ſurface, which will be bordered with the lucid line Δ Δ Γ Γ, and that again with the dark line Γ Γ *BB*, and ſometimes there will appear two or more of theſe lucid and dark lines alternately, as in art. 206.

248. From which it may eaſily be conceived, that when the dark ſurface is very narrow, as *A*, Fig. 69, it may appear divided into two dark lines with a lucid line between them, as *B*; or as three dark lines with two lucid lines between them, as *C*; or there may appear four, five, or more dark lines ſeparated by lucid lines, as was mentioned in art. 199, 200, 201, 202.

But in all theſe caſes it is to be obſerved, that viſion muſt not be a little only, but greatly indiſtinct, and that the light muſt neither be exceeding ſtrong, nor very weak. When the light is too ſtrong, the eye is too much dazzled to perceive the appearance plainly; when too weak, the lucid lines between the dark ones are too faint to be diſcerned.

Experiments to confirm the Theory.

249. If the account we have given of theſe appearances, from a cauſe that was before known really to exiſt in nature, and which we have ſhown to be ſufficient to produce the *phænomena* above recited, ſhould ſeem to ſtand in need of any farther confirmation, it may receive a very ſtrong one from the following experiments.

Let *Aa*, Fig. 70, be a lucid line formed by a narrow aperture of a parallel rule held too near the eye for *Diſtinct Viſion*, and ſeen againſt the ſkylight or againſt white paper ſtrongly illuminated; and let Δ Δ *DD* be the appearance of that lucid line conſiſting of three luminous lines and two dark ones; and let Δ Δ *EE* and *DDEE* be the appearance of the two parts of the parallel rule itſelf. Alſo, let *FFGG* be a flat rule held near the eye, on the right hand ſide, and parallel to the edge of the parallel rule.

Then if the flat rule *FFGG* be gradually moved tranſverſly, keeping always its parallel poſition, as its edge *FF* comes a-croſs the edge of the pupil, the apparent edge of the parallel

parallel rule *DD* will advance to meet it, and will obliterate firſt the lucid line *DDCC*; and as the edge of the flat rule advances by degrees farther and farther a-croſs the pupil, the ſame edge of the parallel rule *DD* will advance farther and farther to meet it, obliterating the dark line *CCBB*, then the lucid line *BBBB*, , then the other dark line *BB*ΓΓ, and laſtly, part of the other lucid line ΓΓΔΔ, of which it will at laſt only leave ſo much as is equal in breadth to the lucid line *Aa*.

250. And if the flat rule be thus moved reciprocally forwards and backwards a-croſs the pupil, the apparent edge of the parallel rule *DD* will likewiſe appear to move reciprocally forwards and backwards, either advancing to meet the edge *FF*, or receding from it, ſometimes obliterating the lucid and dark lines of the image, ſometimes ſuffering them to re-appear; and that half of the parallel rule *DDEE* ſometimes appearing broader, ſometimes narrower. And juſt the reverſe will happen, if the flat rule be placed on the left ſide of the parallel rule, and be drawn the contrary way a-croſs the eye.

251. Now this is preciſely what ought to happen from the theory above laid down.

For, from art. 211, Fig. 58, 59, it is plain, that when an object is ſeen too near for the rays of each pencil to be united upon the *retina*, thoſe rays of every pencil, which fall on the right hand part of the *cornea*, muſt likewiſe fall on the right hand part of the *retina*, and conſequently muſt, from the known laws of viſion, conſtitute the left hand part of the appearance; and on the contrary.

So that, when the edge of the rule, *FF*, is drawn a-croſs the *cornea*, it firſt intercepts thoſe rays of every pencil, which fall the moſt to the right hand upon the *retina*, and which conſtitute the part of the appearance the moſt to the left hand, that is, the lucid line *DDCC*, which being obliterated, the apparent edge of the parallel rule muſt now be *CC*, inſtead of *DD*.

And the edge *FF*, by moving farther along the *cornea* will intercept the parcel of rays from every pencil, which fall next on the right hand part of the *cornea* and *retina*, and which conſtitute the dark line *CCBB*, dark in compariſon of the adjoining lucid lines, but more luminous by means of theſe rays, than the ſides of the parallel rule. And this dark line being obliterated, the apparent edge of the parallel rule will now be advanced from *CC* to *BB*.

And after the ſame manner it is eaſy to conceive, that by the continued motion of the flat rule a-croſs the *cornea*, the luminous line *BBBB*, the dark line *BB*ΓΓ, and the greater part of the luminous line ΓΓΔΔ muſt likewiſe be obliterated.

And at laſt, when the remaining aperture, or uncovered part of the *cornea*, is left very narrow, the edges of the parallel rule and the aperture between them muſt appear diſtinct, juſt as if they were ſeen thro' a narrow ſlit, or a pinhole in a card, and conſequently the lucid line muſt appear of its true and proper magnitude.

252. If the parallel rule be placed at too great a diſtance for *Diſtinct Viſion*, then upon drawing the edge of the flat rule *FF* a-croſs the *cornea*, the right hand ſide of the parallel rule will grow gradually broader, the edge ΔΔ ſeeming to advance the ſame way with *FF*, and thereby obliterating firſt the lucid line ΔΔΓΓ, then the dark line ΓΓ*BB*, the lucid line *BBBB*, the dark line *BBCC*, and laſtly reducing the lucid line *CCDD* to the magnitude of the lucid line *Aa*, when the edges of the two ſides of the parallel rule will appear diſtinct. This is eaſy to conceive from art. 212, and Fig. 60, 61. And juſt the reverſe of this will happen, if the flat rule be placed at firſt on the left ſide, and be drawn the contrary way a-croſs the eye.

253. As in both theſe caſes, when the edge *FF* is reciprocally and quickly moved backwards and forwards, the tremulous motion, art. 250, of the edge *DD*, or ΔΔ, is very conſpicuous, this affords one means of diſcovering when viſion is conſiderably indiſtinct, and thereby of fixing the limits of *Perfect Viſion* within ſome latitude.

254. After the ſame manner, and for the ſame reaſons, when the edge ΔΔ of a dark ſurface, Fig. 68, appears bordered with a lucid line, as ΔΔΓΓ, and a dark line without that, as ΓΓ*BB*, if the eye be too near, then upon placing the edge of a flat rule on the right hand, and drawing it a-croſs the *cornea*, the apparent edge ΔΔ will advance to meet the flat rule, and will thereby obliterate firſt the lucid line, and then the dark one, after which that edge will appear diſtinct in the line *BB*, contiguous to the lucid ſurface *BBbb*.

And

And if the eye be too far off, the flat rule must be placed on the left hand, and then upon drawing it the contrary way a-croſs the *cornea*, the edge Δ Δ will advance along with it, obliterating firſt the lucid and then the dark line as before, till it arrives at the ſituation *BB*, when the confines of the dark and lucid ſurface will appear diſtinct.

255. Alſo, when a narrow dark object, as *A*, Fig. 69, appears double, as *B*, or treble as *C*, if the edge of the flat rule be moved a-croſs the *cornea*, parallel to the image, then if the eye be too near, the oppoſite dark line will ſeem to advance towards the rule, obliterating the lucid or dark lines in its way, till it comes into the ſame ſituation with the dark line neareſt the rule, when it will appear as one only diſtinct line.

But if the eye be too far off, the dark line neareſt to the rule will advance along with it, obliterating the lucid or dark lines in its way, till it comes into the ſituation of the fartheſt dark line, when it will appear diſtinct and as one line only.

256. By the ſame means of covering part of the pupil with the edge of a flat rule, or with the finger only held parallel to the ſide of a book, a perſon who needs ſpectacles, or another who is ſhort-ſighted, may read at ſuch a diſtance, that the book would otherwiſe be wholly illegible by the ſpreading of the *penumbræ* between the letters. For the aperture of the pupil being now leſſened, the *circle of diſſipation*, and the *penumbræ* ariſing therefrom between the letters, will be leſſened in the ſame proportion. And if the light be ſtrong enough to bear covering the greateſt part of the pupil, then the *penumbræ* above and below the letters will likewiſe be conſiderably leſſened, which will render the book ſtill more eaſily legible.

257. A farther proof of the truth of the theory above delivered may be taken from the following experiment.

In looking at a lucid or dark line, which on account of its being too near, or too remote from the eye to be ſeen diſtinctly, exhibits the *phænomenon* of Fig. 69, or 70, move your head gradually ſideways to the right hand or the left, that the rays from the object, which fall upon the *cornea*, may vary both the length of their paſſage and likewiſe the obliquity of their incidence, and conſequently may vary their *Fits of eaſy refraction* and *eaſy reflexion*.

Then you will ſee the lucid and dark lines of the image perpetually changing their places, their magnitude, and their ſtrength, and ſeeming to roll over one another, as they ought to do by this theory.

258. This ſubject has unexpectedly carried me out to ſo great a length, that I ſhould now think it high time to conclude, were it not proper to advertiſe the curious reader of an appearance, that in making theſe experiments may poſſibly miſlead him, as at firſt it did me. An extraordinary appearance.

In looking at the confines of a dark and a lucid ſurface ſeen by indiſtinct viſion, there will ſometimes appear on the border of the lucid ſurface, without the utmoſt dark line of the image, or to the right hand of *BB*, Fig. 68, a line of ſome breadth much more luminous than the reſt of the lucid ſurface. And on the border of the dark ſurface, within the utmoſt lucid line of the image, or to the left hand of Δ Δ, will appear a line of ſome breadth much darker than the reſt of that ſurface.

This appearance I at firſt imagined to proceed from the ſame cauſe, as the other *phænomena* recited in the preceding articles, namely the viciſſitude of the *Fits of eaſy refraction* and *eaſy reflexion* in the rays of light; but finding no way of accounting ſatisfactorily for it from that cauſe, I ſuſpected ſome miſtake, and therefore applied myſelf to conſider that appearance more attentively.

259. Upon white paper I drew a rectangle as *ABCD*, Fig. 71, which with ink I made very black; then viewing it at too ſmall a diſtance for *Diſtinct Viſion*, I ſaw it bordered on all ſides by two luminous and two dark lines alternately. But after looking attentively upon it for a little time, I perceived another luminous border without the utmoſt dark perimeter *a b c d*, which new border was much brighter than the reſt of the white paper. Not owing to the Fits of eaſy refraction and reflexion.

Now, upon turning my eye a little towards that part of the new luminous border, which lay without the dark line *cd*, I found that border to increaſe in breadth, and to grow brighter than before. And in like manner, when I directed my eye to the new luminous border without the utmoſt dark line *ab*, or *ad*, or *cb*, I found the reſpective luminous borders to increaſe in breadth and in the ſtrength of their light.

This

This put me upon turning my eye from the black rectangle to a distant part of the white paper, when immediately a luminous rectangle much brighter than the rest of the paper, and of the same dimensions nearly with the black rectangle, was presented to my eye, and continued for some time, after which it gradually vanished.

But to a different cause.

260. This occasioned me to reflect upon some like appearances I had met with before, either from my own observation, or the relation of some of my acquaintance, and to judge that both the one and the other must proceed from a different cause than what I had hitherto supposed. But for farther confirmation, I put on a pair of spectacles, by means of which the black rectangle appeared very distinct, and free from the alternate lucid and dark edges that had appeared before.

But after viewing the rectangle attentively for some time, such a luminous border as is above described, brighter than the rest of the paper, began to appear round it; and to which side soever of the rectangle I directed my eyes, the luminous border on that side appeared brighter and broader than before. Then turning my eyes quite away from the black rectangle to a distant part of the white paper, a bright rectangle, of the same size with the dark one, immediately appeared and continued for some time, after which it gradually vanished away.

261. I then drew two other black rectangles upon white paper, of the same height and breadth, and parallel to each other, with a narrow white space between them, as represented in Fig. 72.

Then viewing them attentively for some time, till the bright luminous border began to appear, I suddenly transferred my eyes to a distant part of the white paper, where immediately I perceived the appearance of a dark space between two bright rectangles, of equal magnitude with the white space and the two dark rectangles, respectively, in Fig. 72.

And as this equally happened, whether the object was seen by indistinct vision with the bare eyes, or by *Distinct Vision* through the spectacles, it was now manifest, both from this and the experiment of the preceding article, that this new appearance was no way owing to the indistinctness of the sight, or to the *Fits of easy refraction* and *reflexion*, but to another cause, which the intelligent reader may by this time guess at, and which will more plainly appear when I have related the other observations of myself and friends, which I hinted at in art. 260.

Other appearances of a like nature.

262. A person sitting to be shaved against a light sash window, fixed his eyes intently upon the window for some time, and afterwards shutting them, he had now the appearance of a window similar to that he saw before: only the glass panes were dark, and the wood between them was luminous.

263. Another fixed his eyes for some time upon the end of his pen black with ink, and held against white paper; and upon suddenly moving his pen, without stirring his eyes, he saw the appearance of a bright luminous end of a pen upon the same part of the paper. And since I was told of this appearance, I have often made the same observation.

264. In looking at the setting sun, in a coach upon the road, when the light is not so strong as much to offend the sight, if the eyes be suddenly directed to a distant part of the sky, we have the appearance of one, two, three or more dark circles of about the same size with the sun, upon different parts of the sky.

265. A person considering the reason of the vast length of the image of the setting sun seen by reflexion from the *Thames*, looked so long upon it, that when he took off his eye, every thing appeared to him with a long dark beam upon it for above a quarter of an hour.

266. Another, after looking too long at the Sun eclipsed, saw a dark spot upon every thing for several months after.

The general reason of these appearances.

267. These, and many more *phænomena* of a like kind, seem to depend upon this principle, that when we have been for some time affected with one sensation, as soon as we cease to be so affected, a contrary sensation is apt to arise in us, sometimes of itself, and sometimes from such causes, as at another time would not produce that sensation at all, or at least not to the same degree.

Every body knows, that the sudden cessation of intense pain after some continuance, is immediately followed with sensible pleasure.

In coming out of a strong light, into a room with the window-shutters almost closed, we have immediately a sensation of darkness; and this continues much longer than the pupil requires to dilate and accomodate itself to

that weak degree of light, which is almoſt inſtantly done.

But after ſtaying ſome time in a much darker place, the ſame room, which appeared dark before, will be ſufficiently light.

In coming out of a cold bath, the intenſe cold is immediately ſucceeded by a glowing heat. But this, as well as the effects recited in art. 265, 266, is partly owing to other cauſes than the bare ceaſing of the contrary ſenſation.

If we drink a diſh of coffee, or thea, without ſugar, and afterwards taſte of another diſh with but little ſugar in it, this laſt will appear very ſweet.

But if we firſt eat any thing very ſweet, a diſh of coffee or thea moderately ſweetened, will taſte very bitter; and it were eaſy to bring a great number of like examples.

268. But ſuch a change of ſenſation is not always general, ſo as that the whole organ of one of our ſenſes is affected by it: it is oftentimes partial, ſo that from one and the ſame cauſe, one part of the organ undergoes a change of ſenſation, while the other part perceives no change.

For inſtance, the ſkin of our bodies, to ſpeak looſely, may be conſidered as the whole organ of feeling, with reſpect to the heat and cold of the air we are environed with, or of the water we may happen to be immerſed in: And the whole *retina* may be conſidered as the organ of ſight.

But when we are in a moderate degree of warmth, a gentle, mild breeze of air may not affect our faces, which yet would ſtrike us with a ſenſe of chillneſs, if our naked bodies were expoſed to it.

And a tepid bath, as that of *Buxton*, may feel neither warm nor cold to our hands, and yet upon immerſion may feel cold to our bodies.

269. And this ſenſation may not only be partial, affecting one part of the organ, and not at all affecting the other: but one part of the organ may be affected one way, while another part is the contrary way affected by the ſame cauſe.

Thus the hand may be in ſuch a temper, that upon dipping it into the bath of *Buxton*, the water ſhall feel warm: but upon immerſion of the whole body, the ſame water ſhall feel cold.

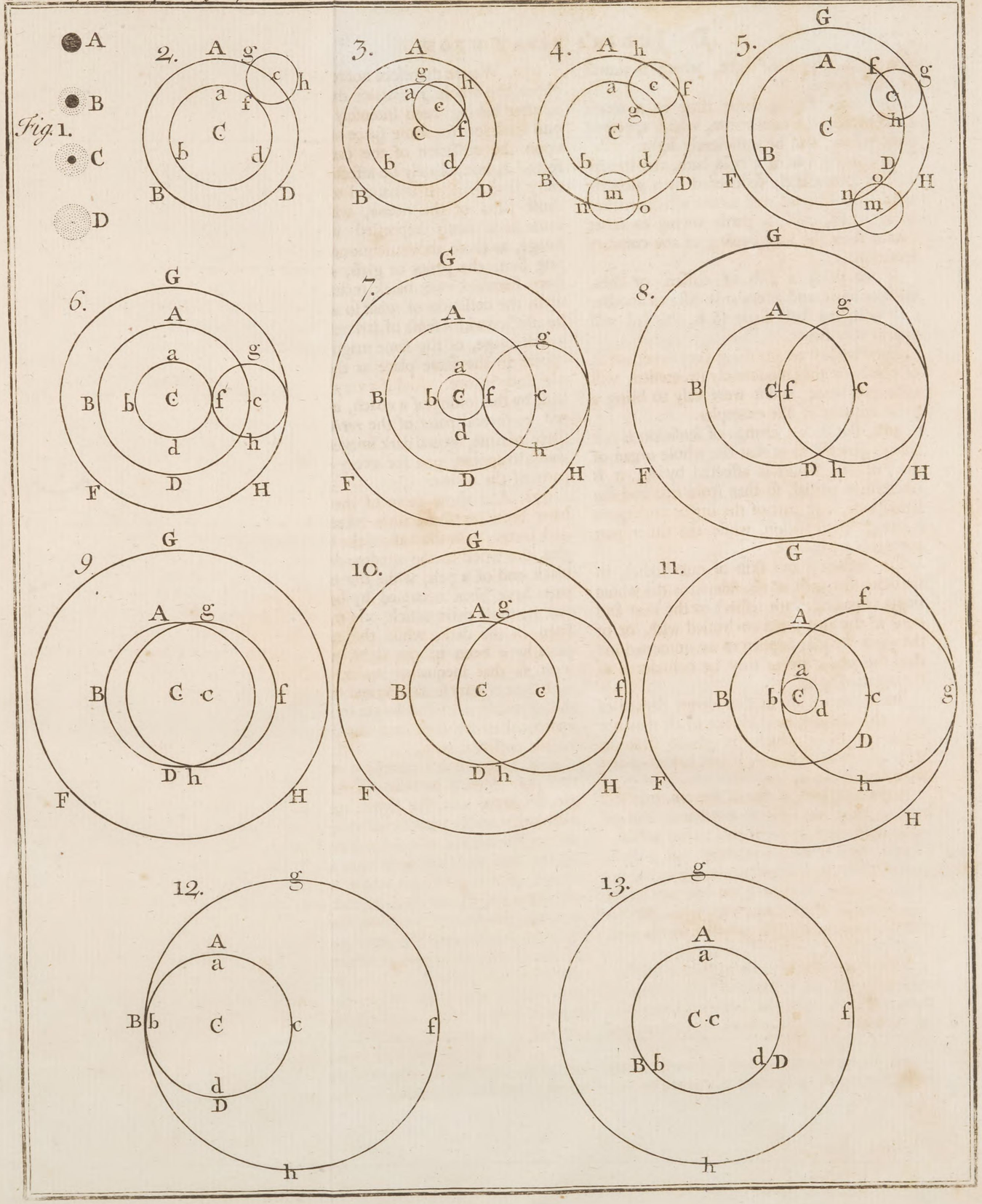
And the air will ſometimes feel warm to our faces, that would feel cold to our naked bodies.

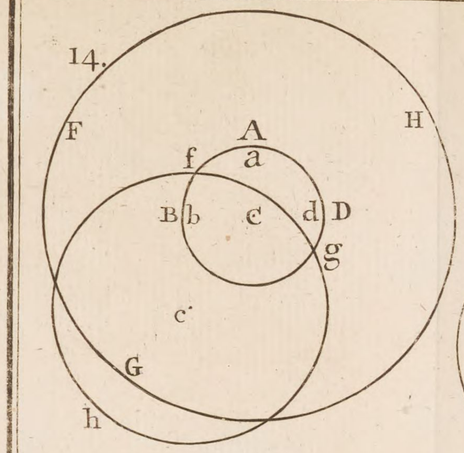
270. We are therefore not to wonder, that either the whole *retina*, or that any part of it, after having been intenſely affected with one ſenſation for ſome ſpace of time, ſhould upon the ceſſation of the cauſe that ſo affected it, immediately be affected with a contrary ſenſation. It is natural to expect, that thoſe parts of the *retina*, which have for ſome time been impreſſed with a bright image, as thoſe abovementioned of the ſetting Sun, the panes of glaſs, or the white ſpace between two black rectangles, ſhould upon the ceſſation of what ſo affected them, be affected with a ſenſe of darkneſs, that is, that a dark image, of the ſame magnitude, ſhould appear in the ſame place as the bright one had done before. And if a very bright image had, by the jolting of a coach, been transferred to ſeveral parts of the *retina* in a ſhort ſpace of time, ſeveral dark images ought afterwards to appear, one for every one of thoſe parts of the *retina*.

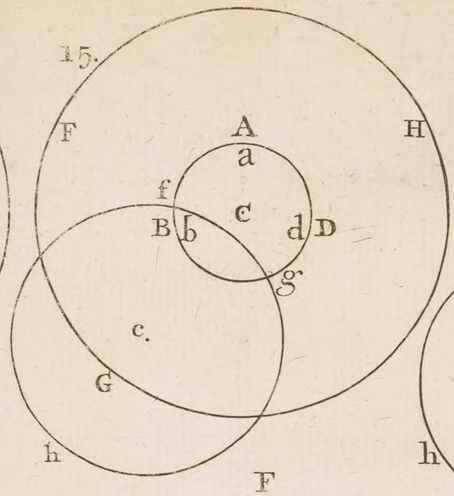
271. And thoſe parts of the *retina*, that have been for ſome time taken up with a dark image, as in the caſe of the black rectangles, the wood in the window-frame, or the black end of a pen, while the neighbouring parts have been occupied by bright images, that is, thoſe parts which, one may ſay, have been in the dark, while the neighbouring parts have been in the light, will now, as ſoon as that inequality ceaſes, be affected with the contrary ſenſation; and this will cauſe the appearance of the bright images, ſimilar and equal to the dark ones thoſe parts were before poſſeſſed with.

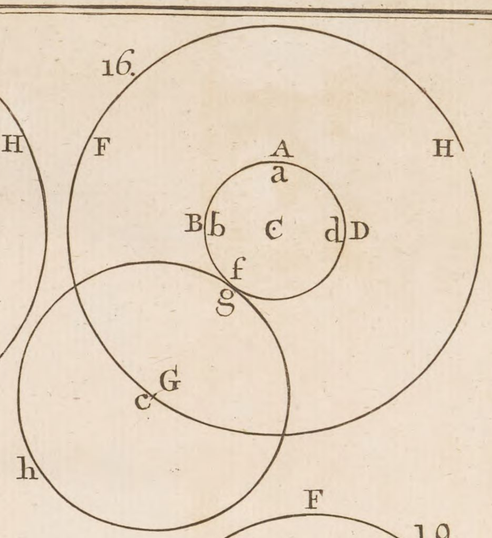
272. Hence it is a neceſſary conſequence, that after viewing intenſely the black rectangle, art. 259, 260, for ſome time, upon the leaſt unavoidable nutation of the head, ſome part of the *retina*, which had been occupied by the dark and faint lucid lines without the image of that black rectangle, would now be impreſſed by the light of the white paper, and this ſudden tranſition from darkneſs to light would cauſe the light upon that part of the *retina*, to appear ſtronger than upon the other patrs, which had all along received the light of the white paper, that is, a luminous border would appear, brighter than the reſt of the paper.

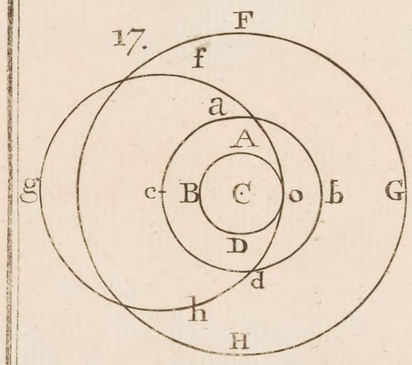
273. And when the eye was a little diverted to one ſide, in order to conſider more attentively this luminous border, by this means

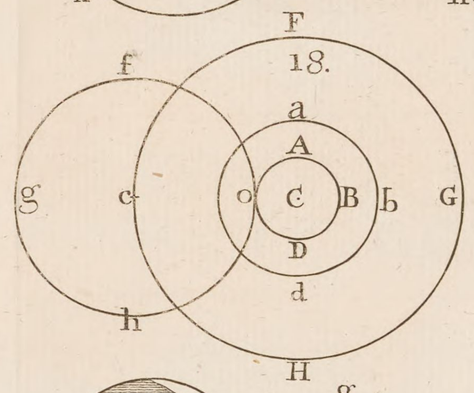

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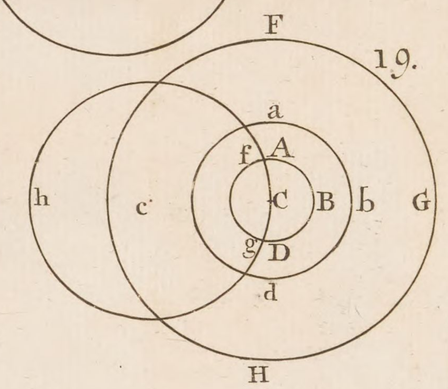

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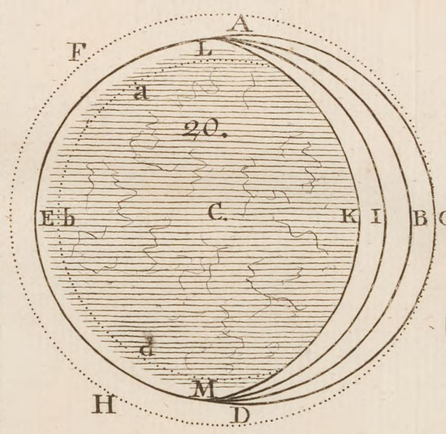

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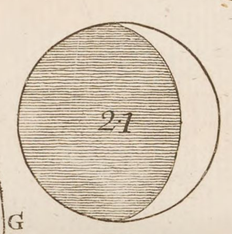

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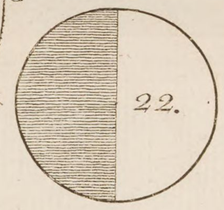

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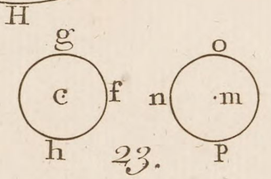

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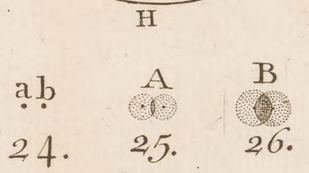

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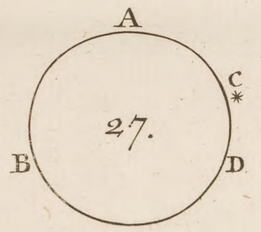

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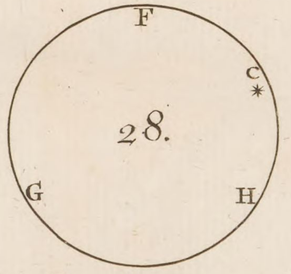

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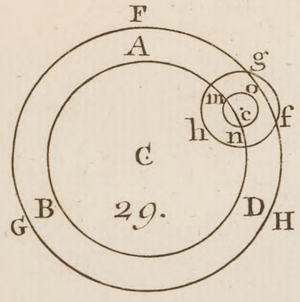

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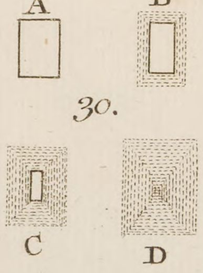

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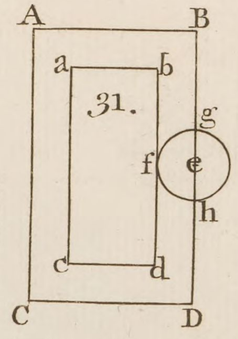

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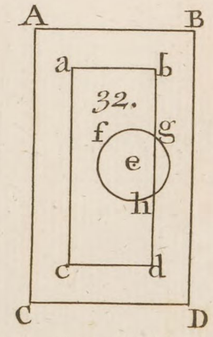

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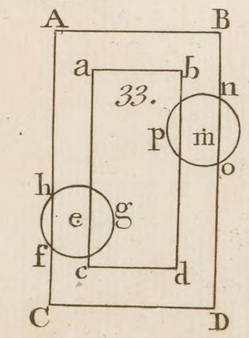

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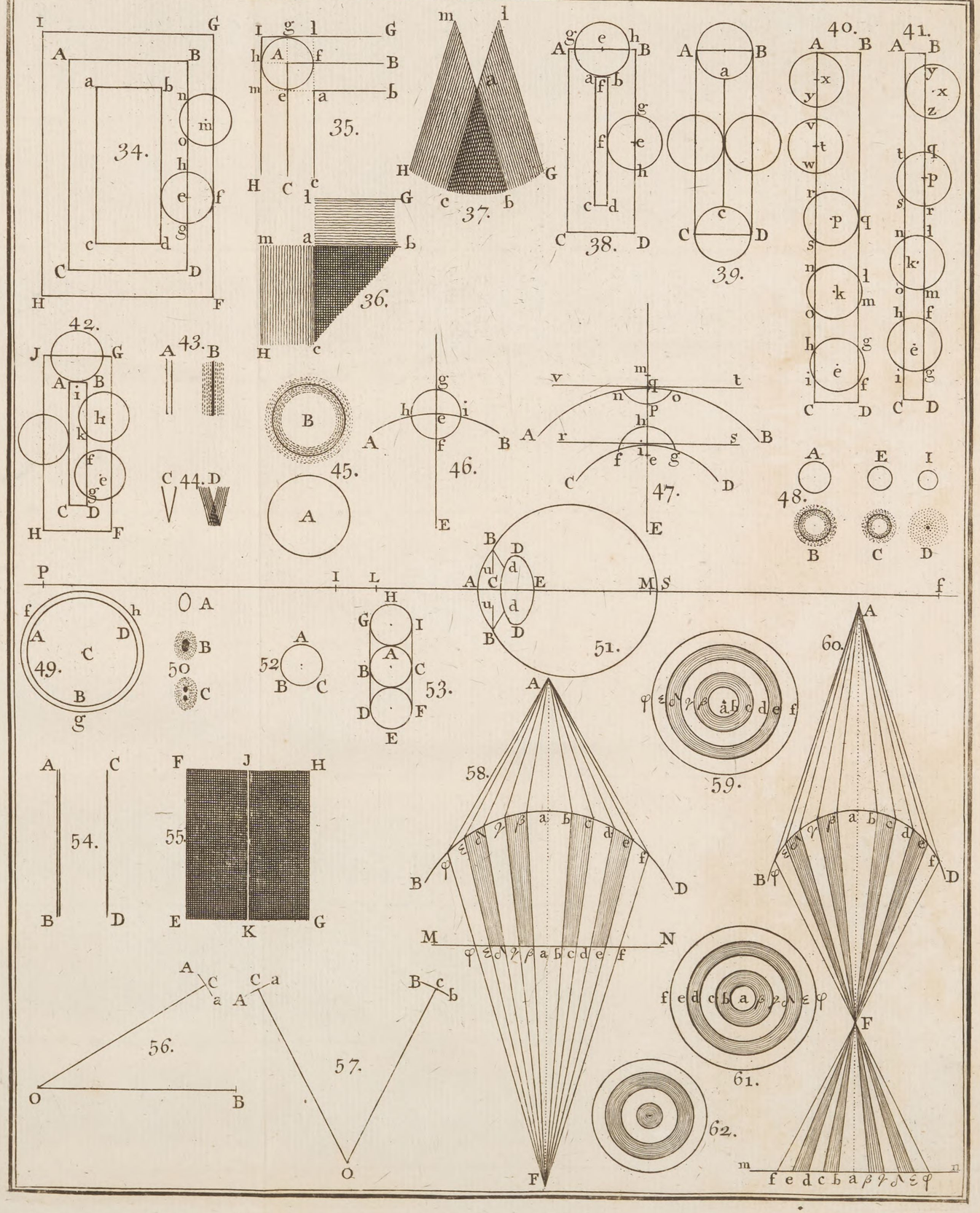
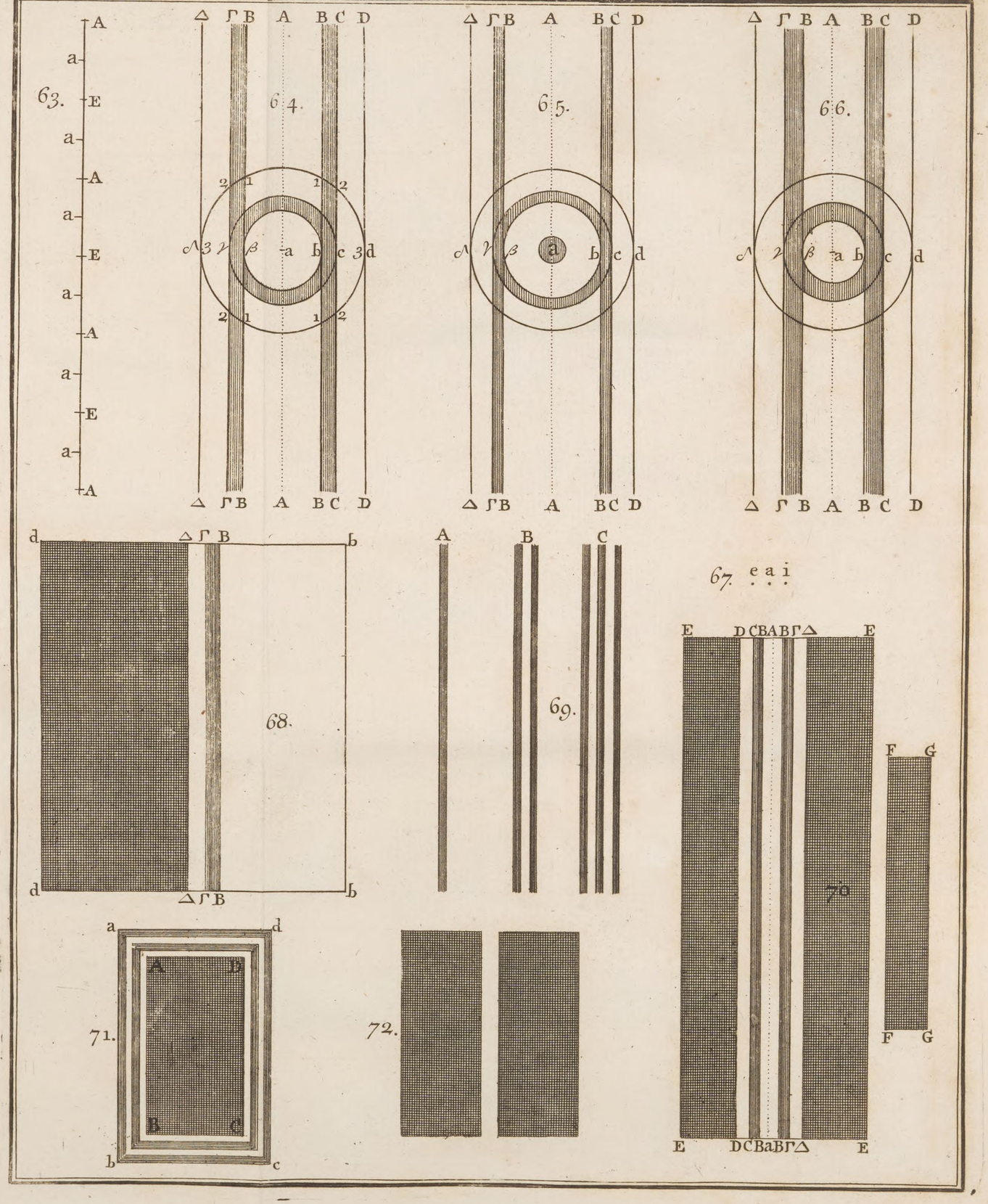
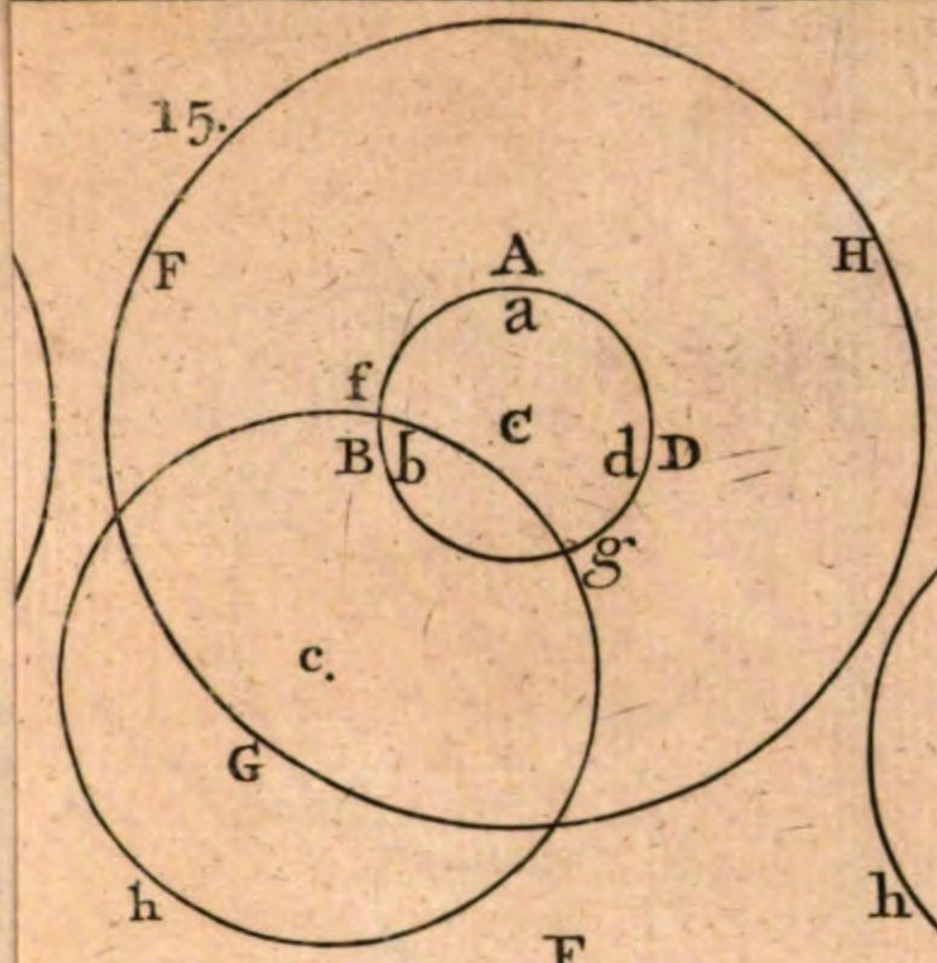

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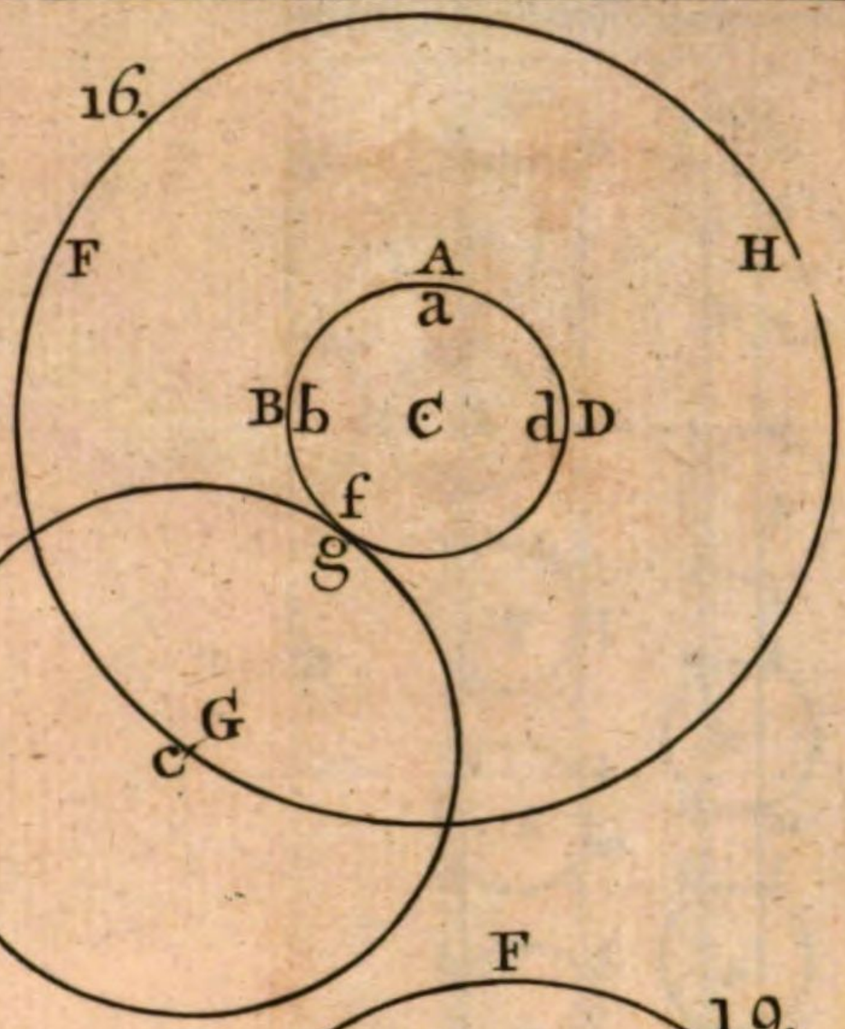
Plate 20. end of the Essay.



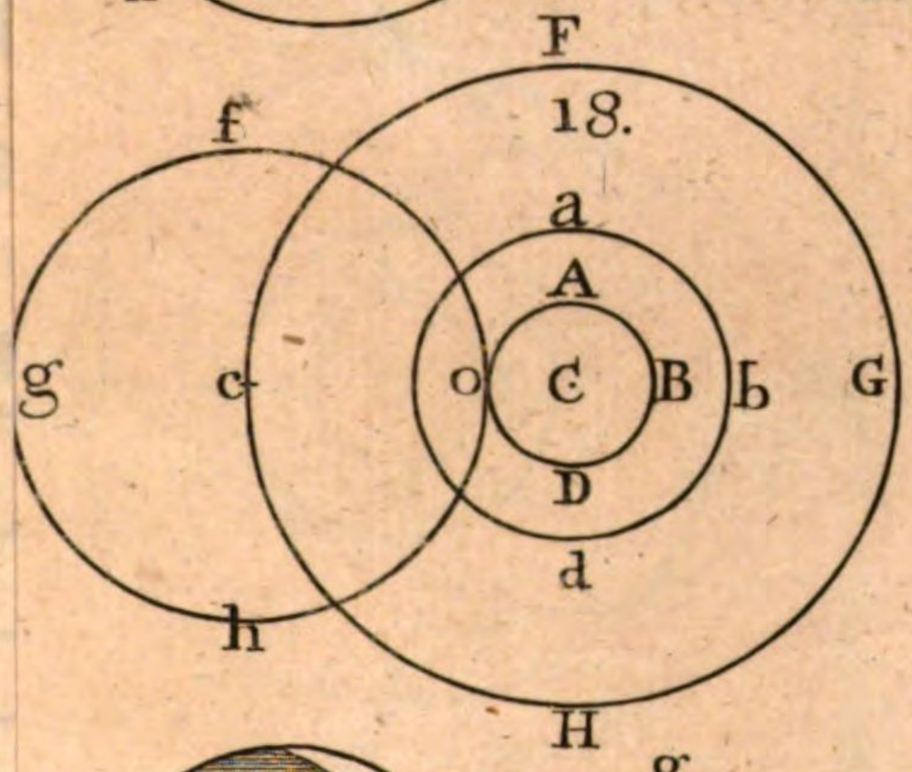
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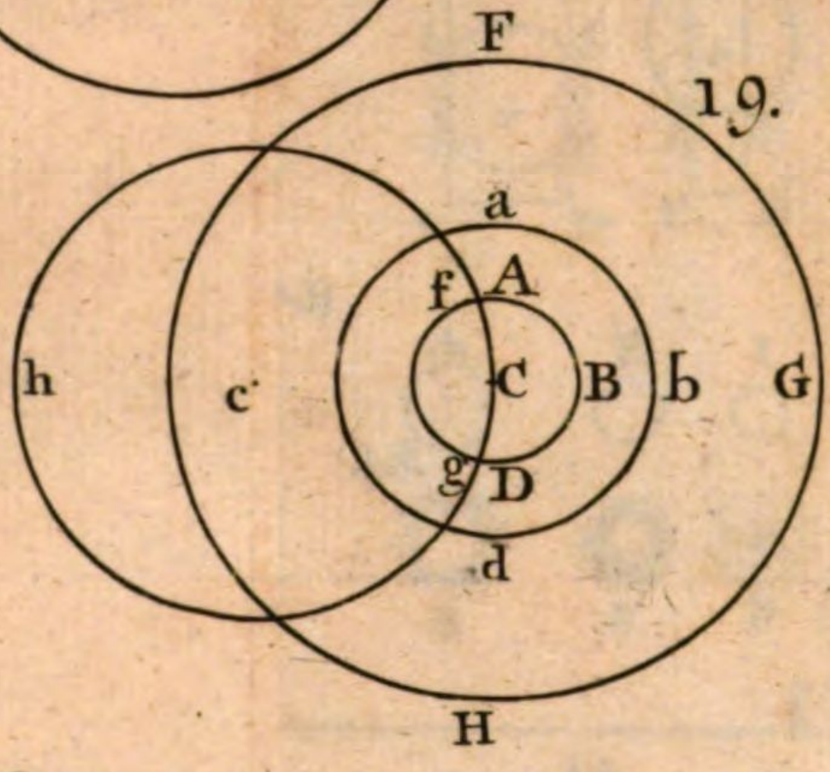
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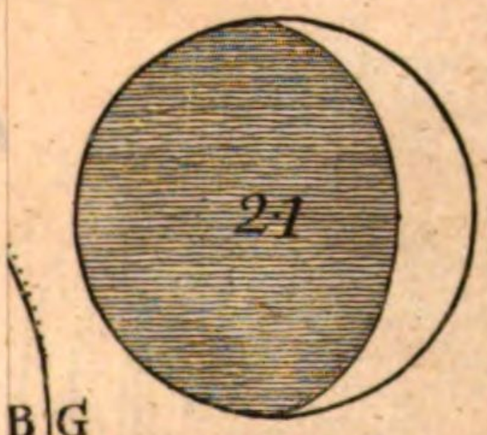

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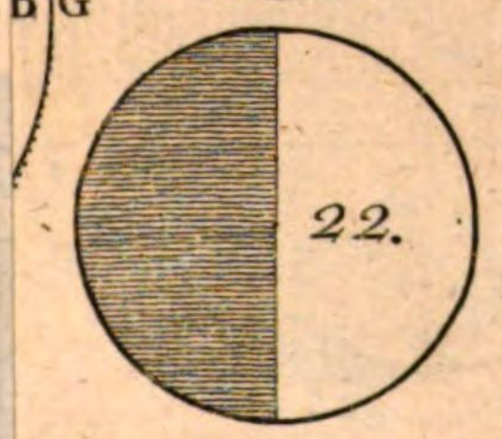
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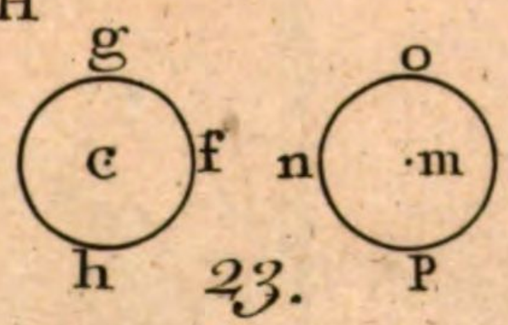

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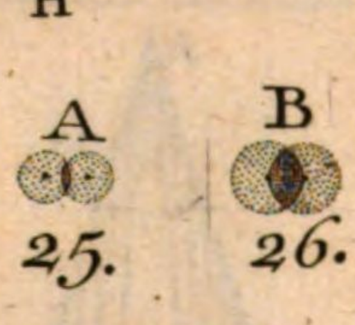

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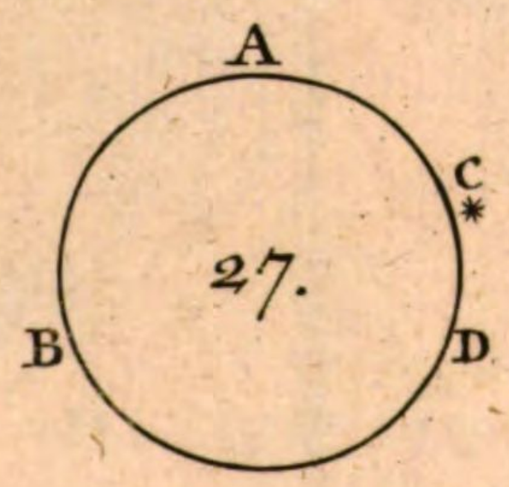

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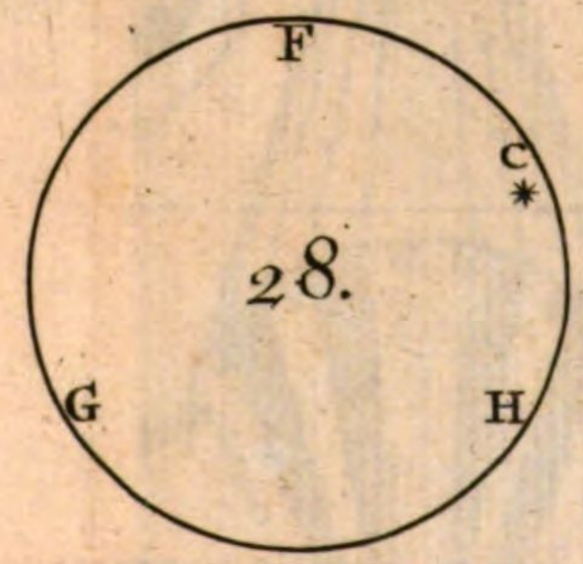

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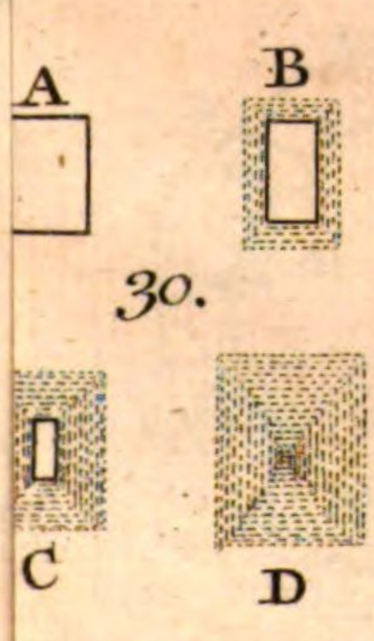

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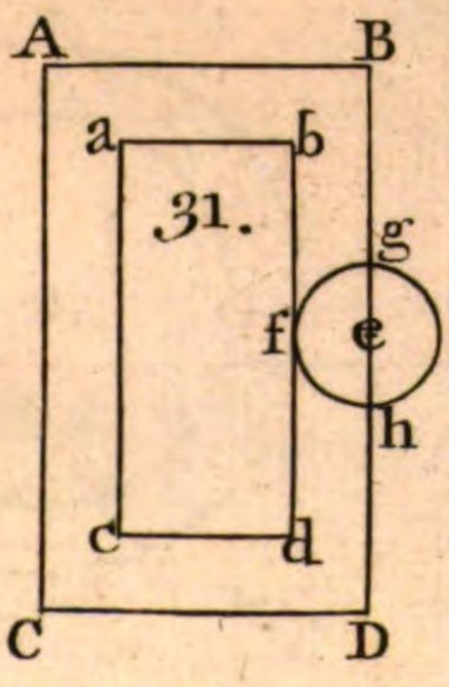
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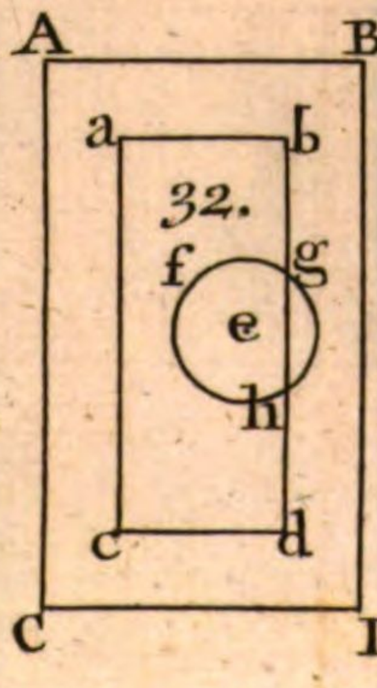

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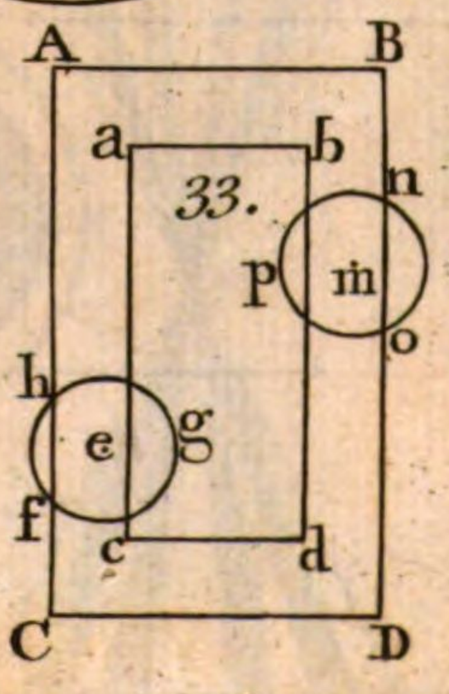

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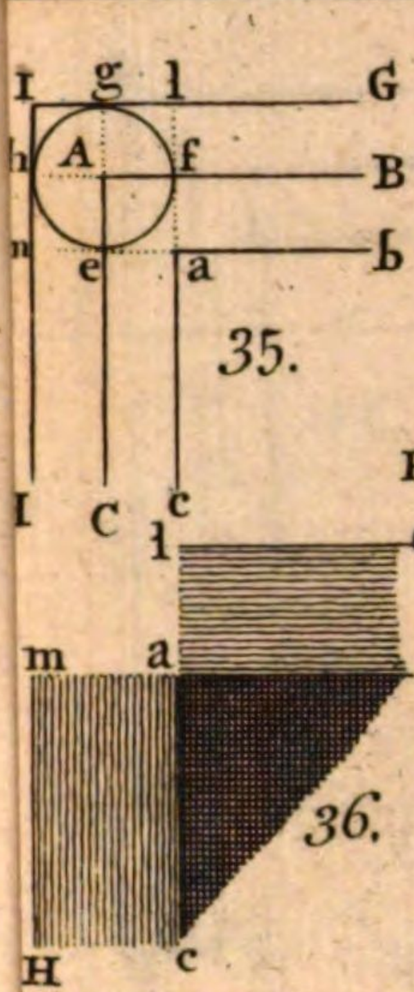

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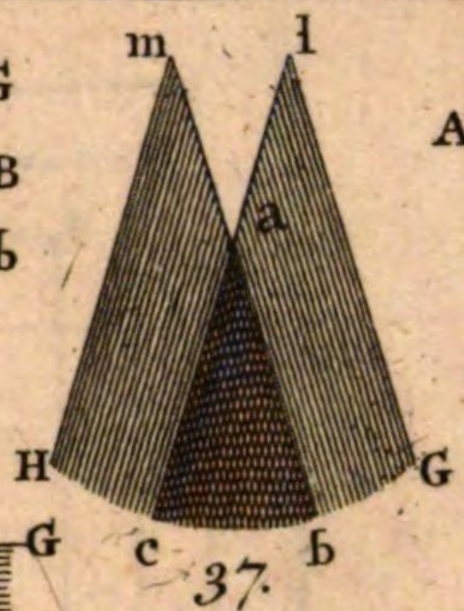

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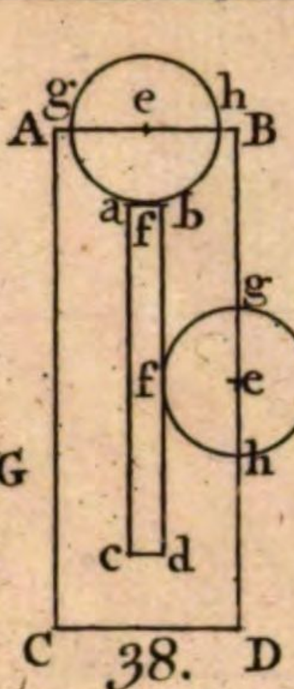

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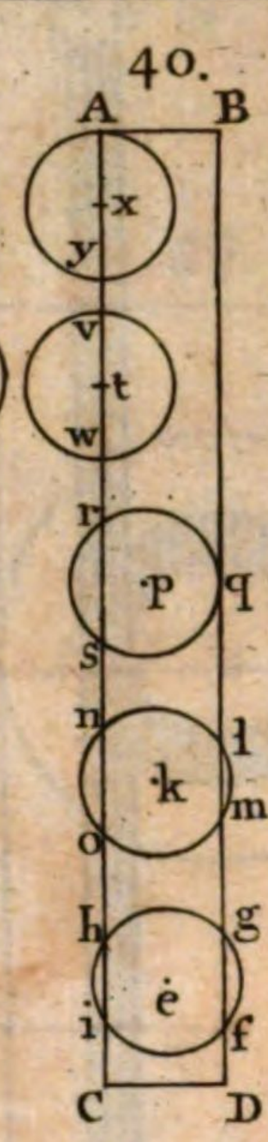
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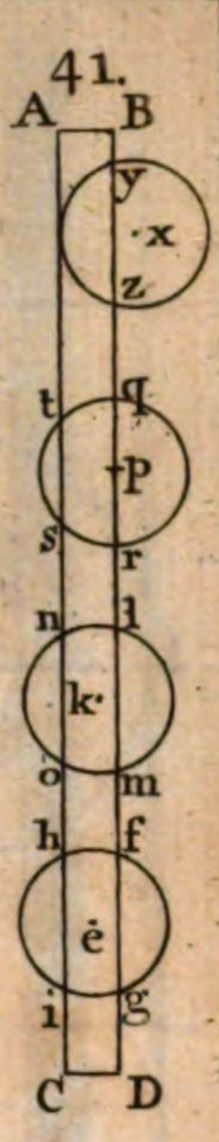

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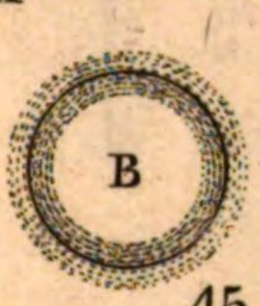

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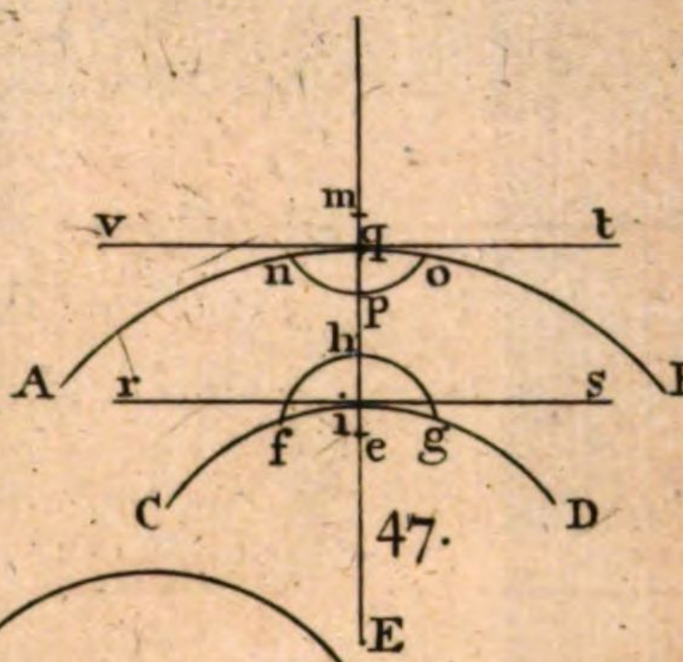


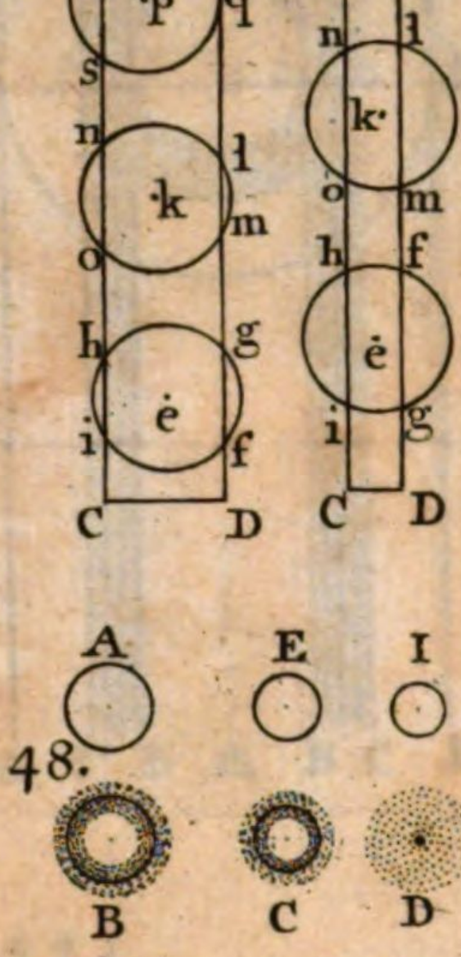
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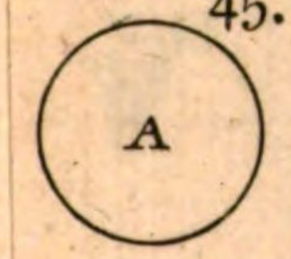


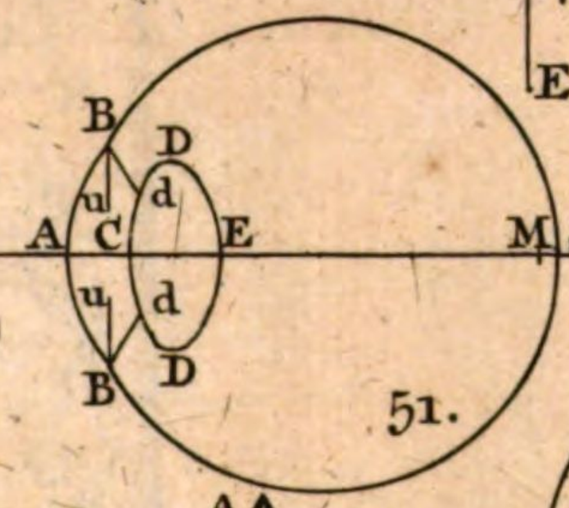

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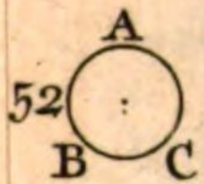

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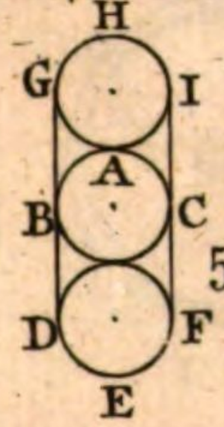

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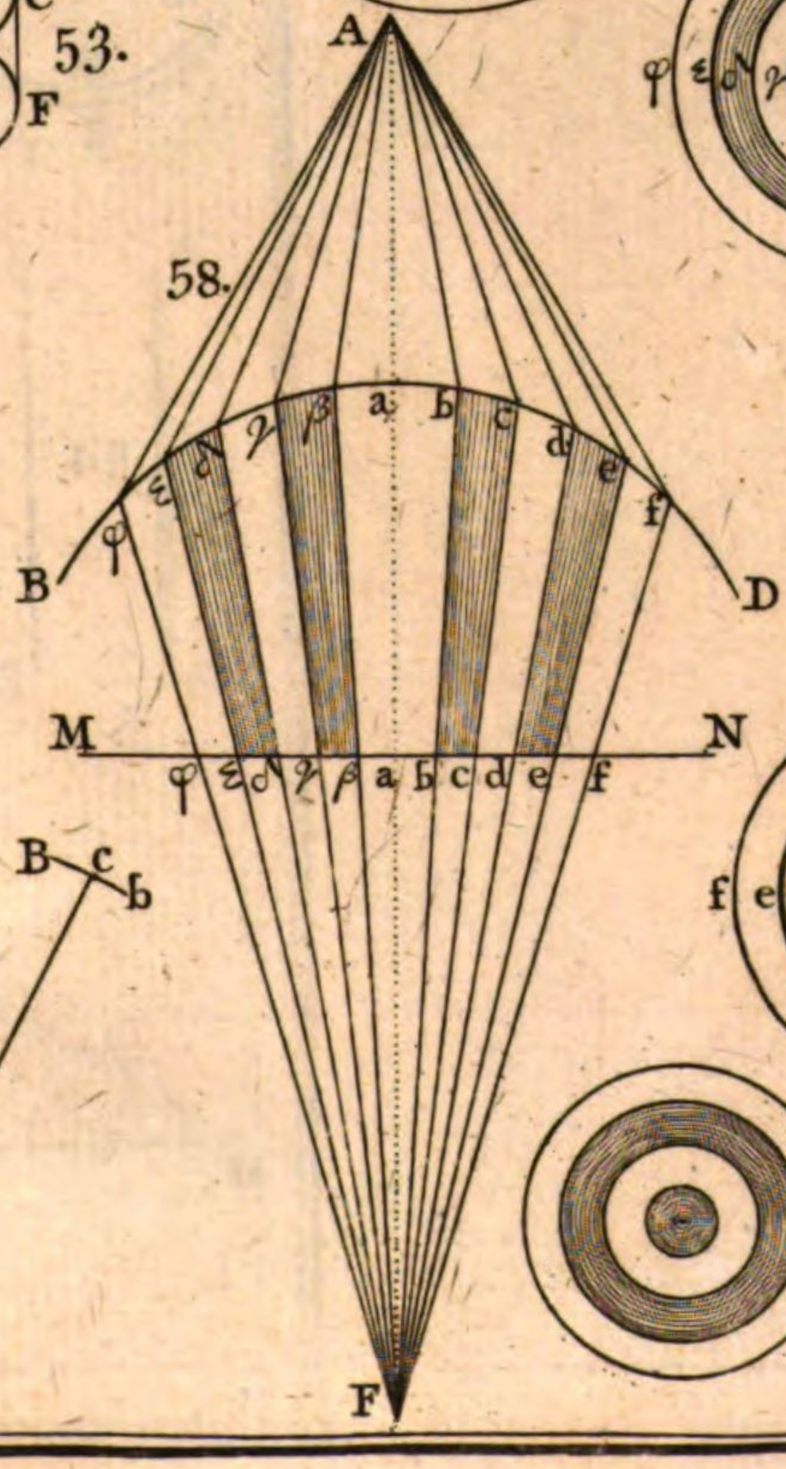

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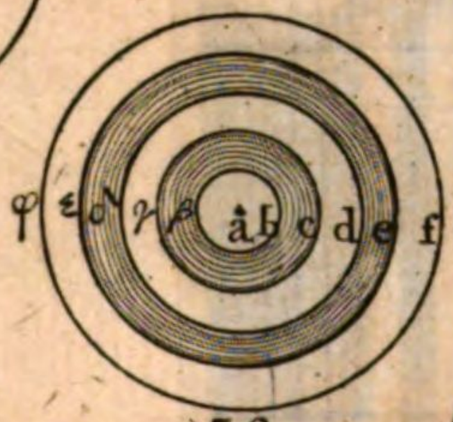

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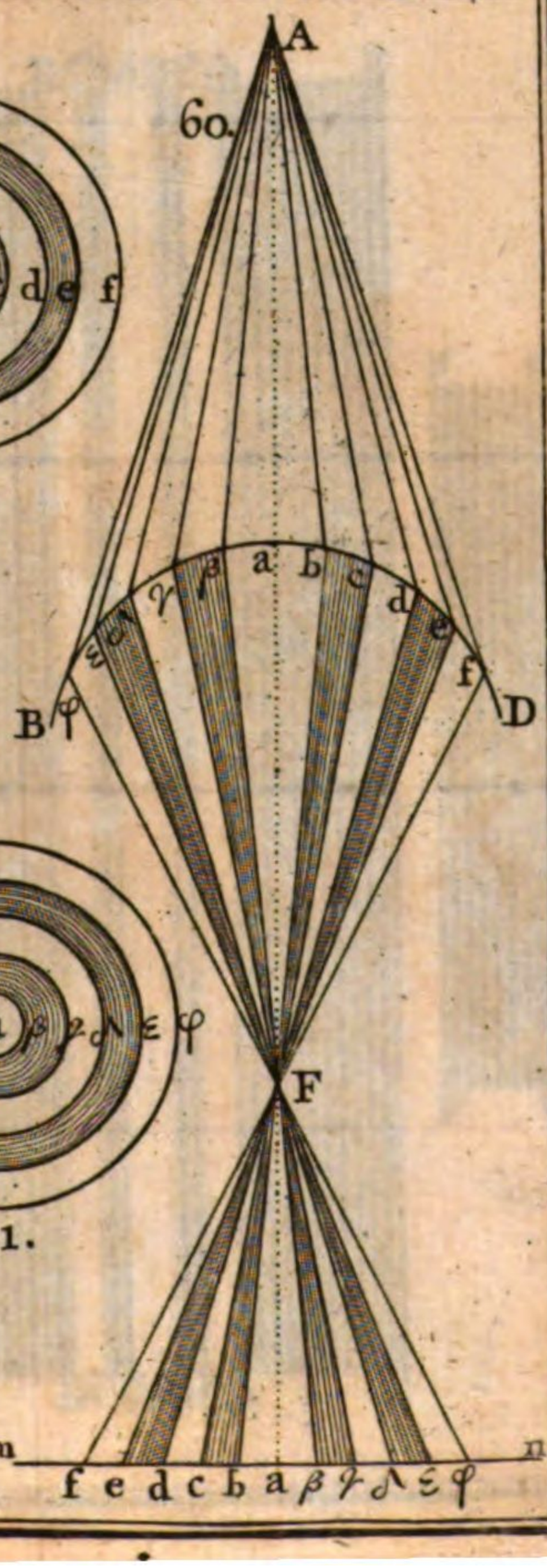

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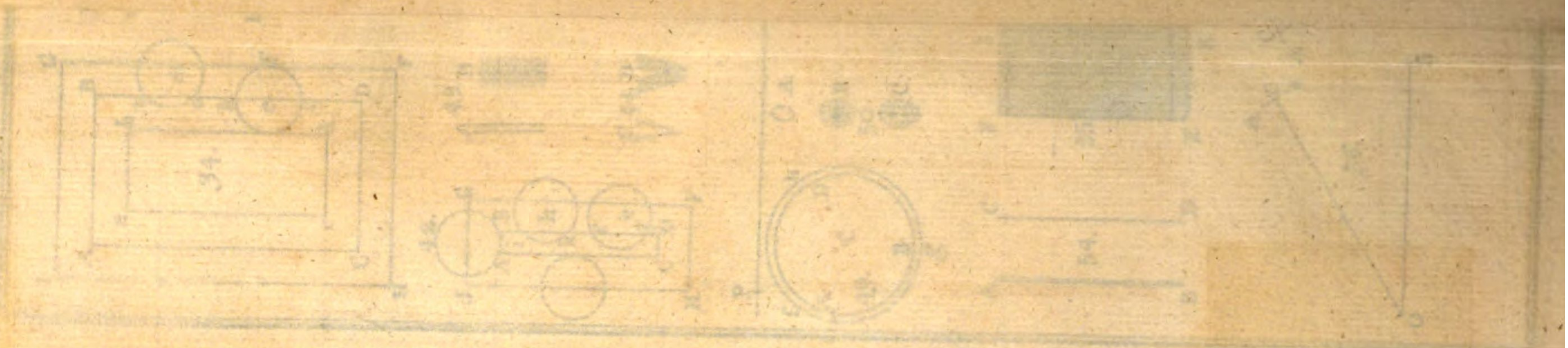

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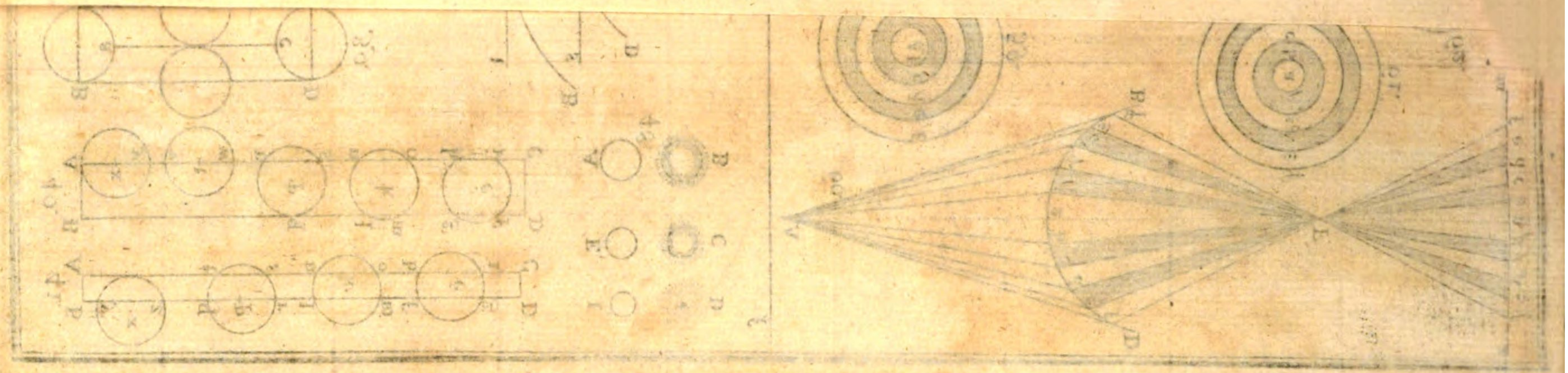

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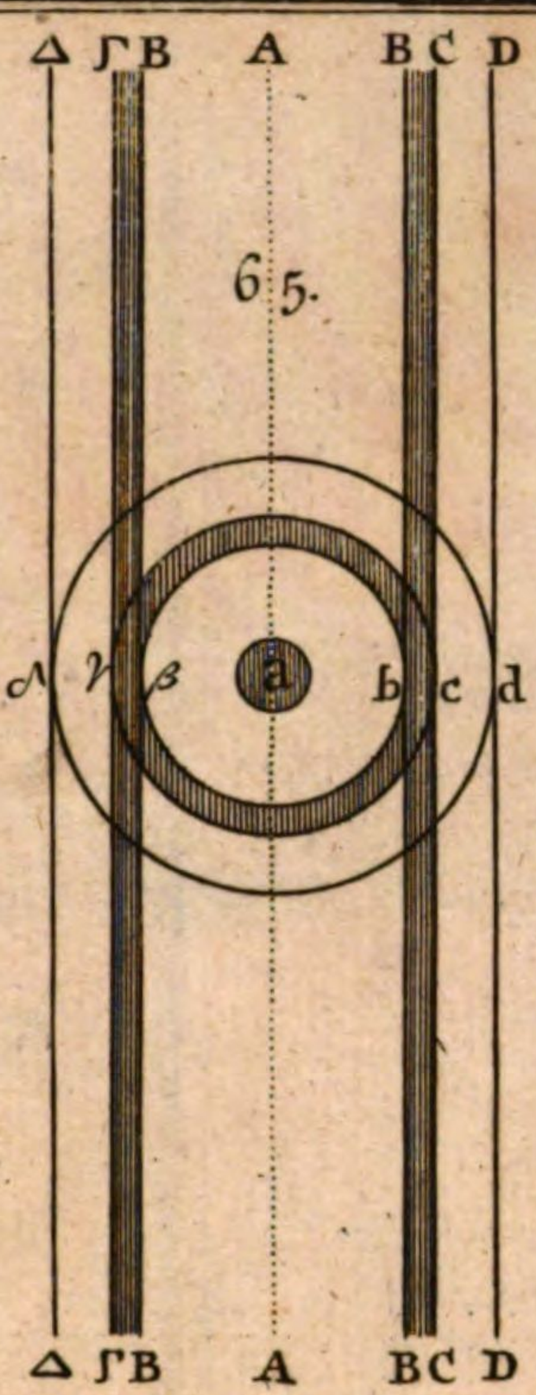




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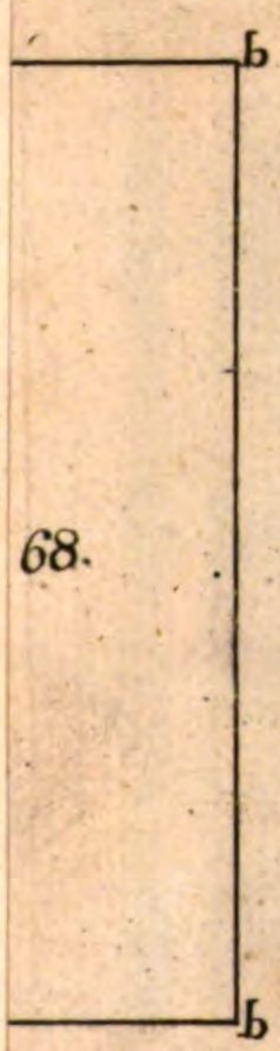


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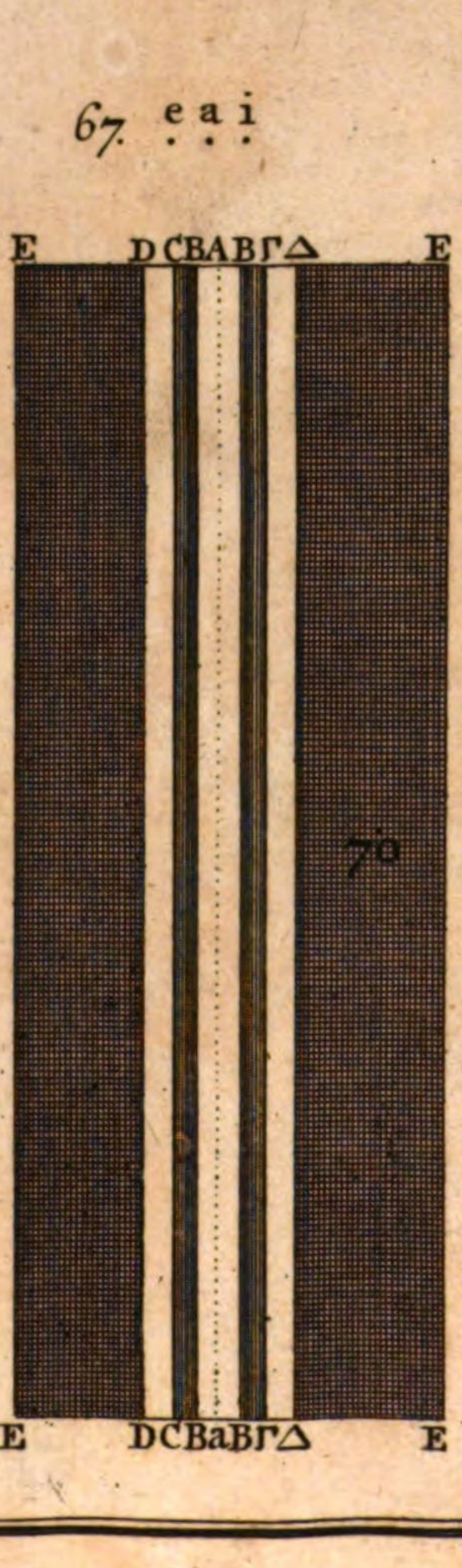


68.



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67.





72.


D C E L 7
E
D C E L 7
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7 L B V B G D
7 L B V B G D

means a part of the *retina*, which was before poſſeſſed with part of the image of the black rectangle itſelf, would now be impreſſed with the light of the white paper, and by having undergone a greater degree of darkneſs than the former part, it would be affected with a ſtill ſtronger ſenſation of light, that is, the luminous border would not only appear broader, but ſtronger than before.

274. It will be ſo eaſy to apply what I have here ſaid of the luminous border, to the dark border mentioned in art. 258, that I ſhall ſpend no more time about it: and indeed I ſhould hardly have been thus particular about the luminous border, but that I cannot help ſuſpecting, that even the Great Sir ISAAC NEWTON himſelf was once led into a miſtake by a like appearance. At leaſt, I do not find, he has any where accounted for the extraordinary ſtrength of the ring of light, next the central black ſpot, ſpoken of in Obſ. 23. Part. I. of his ſecond Book of Opticks, nor do I ſee any way of accounting for it, but from ſome ſuch conſideration.

FINIS.

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The numbers with no letter before them refer to the Articles in the four Books, with r *before them to the Articles of the Remarks, and with* e *before them to the Articles of the Eſſay at the end of the Work.*

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